Effect of Assumptions Used to Calculate Bottom-Hole Pressures in Gas Wells

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ABSTRACT

The general energy equation, including change in kinetic energy, was solved by numerical integration and used to evaluate simplifying assumptions and application practices over a wide range of conditions. When extreme conditions were encountered, sizable errors were caused by large integration intervals, application of Simpson's rule and neglecting change in kinetic energy. A maximum error of only 1.31 percent was caused by assuming temperature and compressibility constants at their average value. It was discovered that a discontinuity can develop in the integral for the injection case. This discontinuity indicates a point of zero pressure change and is an inflection point in the pressure traverse.

INTRODUCTION

When a pressure in a gas well is to be calculated, one of the first decisions is to select a method of calculation. In many instances, this selection becomes a problem because the literature, at best, provides an evaluation of any method for only a limited range of conditions. Once a method has been selected, a question often arises as to the size of the calsulation interval which should be used. The question regarding calculation interval arises because an analytic solution is not obtainable and approximate solutions must be used.

This paper presents an evaluation of major assumptions and application practices of probably the two most widely used methods for calculating steady-state single-phase gas well pressures. The two methods are Cullender and Smith[1-6] (numerical integration), and average temperature and compressibility.[2-4,7] The Cullender-Smith method assumes that change in kinetic energy is negligible and is normally applied in two steps with a Simpson's rule correcton. The average temperature and compressibility method, in addition to neglecting kinetic energy change, asumes that temperature and compressibility are constant at their average values. This method is normally applied for wellhead shut-in pressures of less than 2,000 psi, and in one step.

Computer programs were written to compute bottom-hole pressure with and without the assumptions, using various approaches. Values of input parameters investigated are shown in Table 1. Flow rate was limited to a maximum of 5,000 and 10,000 Mcf/D for tubing sizes of 1.610 and 1.995 in. ID, respectively. Flow rate was also limited to 10,000 Mcf/D for a tubing size of 2.441 in. ID when wellhead flowing pressure was 100 psia. These limitations were imposed on flow rate so as not to exceed sonic velocity. The z factor routine available necessitated limiting bottom-hole temperature to 240F and wellhead pressure to 3,000 psia.

Pressures were compared on the basis of percent deviation from the trapezoidal integration of Eq. 1 or 2 at 100-ft intervals. A preliminary investigation indicated that a 1,000-ft interval solution would differ from a 50-ft interval solution by less than 0.25 percent; therefore, the 100-ft interval was chosen for a base. For the purpose of comparison, deviations less than 1 percent were considered insignificant.

EQUATIONS

Cullender and Smith give the equation for calculating pressure in a dry gas well, neglecting kinetic energy change, as

$$\frac{1{,}000\,\gamma L}{53.33} = \int_{p_{tf}}^{p_{wf}} \frac{p/Tz}{\dfrac{2.6665\,f q_{sc}^2}{d^5} + \dfrac{D}{L}\dfrac{(p/Tz)^2}{1{,}000}}\,d(p) \quad . \quad . \quad (1)$$

If change in kinetic energy is considered, Eq. 1 becomes

$$1{,}000\,L = \int_{p_{tf}}^{p_{wf}} \frac{\dfrac{53.33}{\gamma}\dfrac{p}{Tz} + \dfrac{111.1\,q_{sc}^2}{d^4 p}}{\dfrac{2.6665\,f q_{sc}^2}{d^5} + \dfrac{D}{L}\dfrac{(p/Tz)^2}{1{,}000}}\,d(p) \quad , \quad . \quad (2)$$

where $111.1\,q_{sc}^2/d^4p$ = kinetic energy term. Eqs. 1 and 2 can be evaluated numerically at specific depths using the trapezoidal rule as shown by Cullender and Smith.

If change in kinetic energy is neglected and temperature and compressibility are assumed constant at their average values, Eq. 1 can be integrated to give the average temperature and compressibility equation,

$$p_{wf}^2 = e^s p_{tf}^2 + \frac{L}{D}\left[\frac{2.6665 f}{d^5} q_{sc}\,\bar{T}\,\bar{z}\right]^2 (e^s - 1) \quad , \quad . \quad (3)$$

where pressure squared is in thousands. Eq. 3 can be

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[1]References given at end of paper.

TABLE 1—VALUES OF PARAMETERS USED IN INVESTIGATION

Parameter	Values Used in Investigation					
Flow rate, Mcf/D	0	500	1,000	5,000	10,000	20,000
Wellhead pressure, psia	100	500	1,000	2,000	3,000	
Flow string ID, in.	1.610	1.995	2.441	4.000	4.950	
Specific gravity	0.550	0 650	0.750			
Wellhead temperature, °F	40	100				
Temperature gradient, °/ft	0.006	0.014				
Integration interval, ft	100	500	1,000	2,000	5,000	
Total depth, ft	10,000					

solved by making a one-step trial-and-error solution on $\bar{z}$.

Assumptions common to all three equations as used in this investigation are (1) steady-state turbulent flow, (2) gas as the single-phase flowing fluid, (3) straight-line temperature gradient and (4) friction factor constant over total length of pipe.

EFFECT OF SIZE OF INTEGRATION INTERVAL

Pressures were calculated using Eq. 2 for trapezoidal integration intervals of 100, 500, 1,000, 2,000 and 5,000 ft for a total of 228,336 pressures. The interval size of 100 ft accounted for 162,408 of these. Intervals larger than 100 ft accounted for the remaining 65,928 pressures.

Only 414 or 0.63 percent of the 65,928 pressures showed deviations greater than 1 percent. These deviations occurred at interval sizes of 2,000 and 5,000 ft and at wellhead pressures of 1,000 psia and less. The maximum deviation was 8.62 percent. In general, deviation is a maximum at high flow rate, low wellhead pressure, high specific gravity, low temperature and high temperature gradient. Fig. 1 is an example of how percent deviation increases with increasing integration interval size. Figs. 2 through 4 are examples of how percent deviation due to integration interval is influenced by flow rate, wellhead pressure and depth, respectively. Fig. 4 demonstrates that maximum deviation occurs at the first interval, regardless of the size.

EFFECT OF APPLYING SIMPSON'S RULE

In flowing gas wells, pressure is not always a linear function of depth. The nonlinear function occurs at low pressures and relatively high flow rates. This is caused by the second term in the denominator of Eq. 1 or 2 being insignificant at the surface but becoming significant with depth. When a nonlinear function exists, the size of the integration interval for Eqs. 1 and 2 should be reduced. Cullender and Smith suggested that application of Simpson's rule to a two-step calculation would give the approximate equivalent of a four-step calculation, even though pressure intervals may be unequal.

This investigation shows that application of Simpson's rule to a nonlinear pressure function tends to produce a pressure lower than a trapezoidal integration with small intervals. Fig. 5 shows an example of such a situation. The low reservoir pressure is caused by unequal pressure intervals. Pressure intervals in the upper portion of the wellbore will be larger than pressure intervals in the lower portion. In this situation, Simpson's rule improperly weights values of the integrand in Eq. 1 or 2. Cullender and Smith were apparently dealing with conditions where the error caused by Simpson's rule was approximately equal to the error caused by large intervals in the trapezoidal integration. These errors are opposite in sign.

EFFECT OF KINETIC ENERGY

If the kinetic energy term is neglected in Eq. 2, the integrand will be too small and the calculated pressure change too large. An example of neglecting kinetic energy change is given in Table 2. Using an integration interval of 100 ft, pressures with and without kinetic energy were calculated for all combinations of other parameters shown

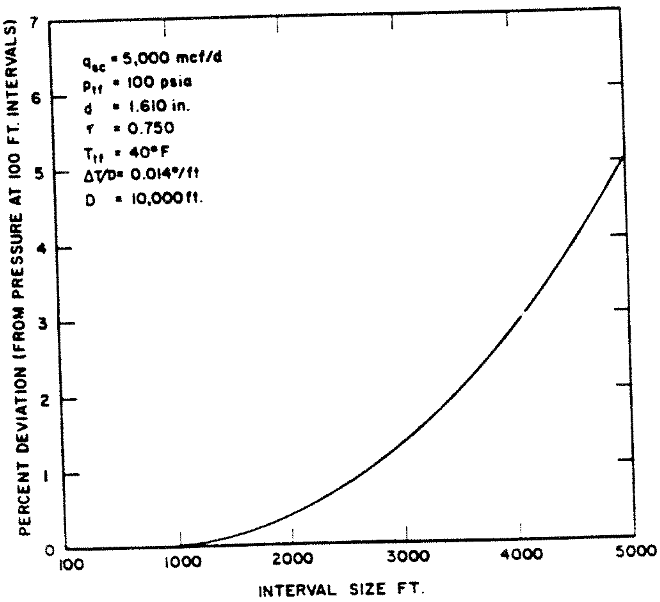

Fig. 1—Effect of Integration Interval.

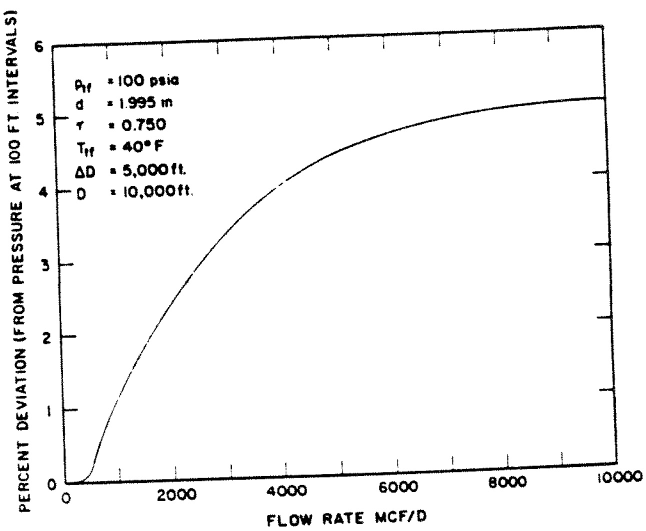

Fig. 2—Effect of Flow Rate on Deviation Due to Integration Interval.

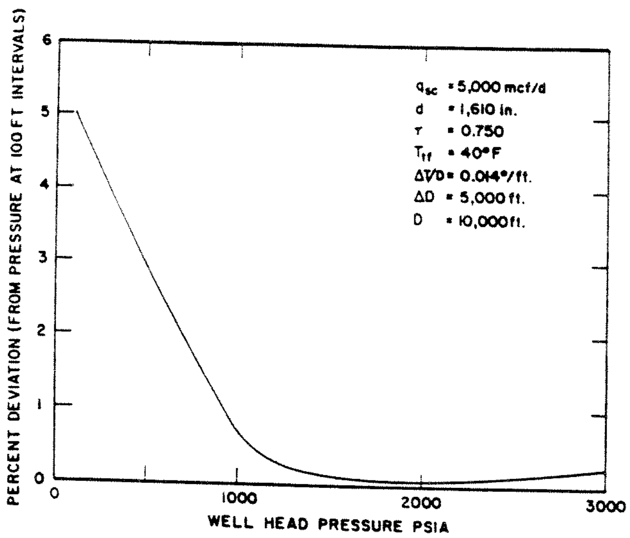

FIG. 3—EFFECT OF PRESSURE ON DEVIATION DUE TO INTEGRATION INTERVAL SIZE.

in Table 1 for 162,408 pressures. Of these pressures, 1,405 (0.87 percent) showed a deviation greater than 1 percent. Deviations greater than 1 percent did not occur below 4,000 ft, nor at wellhead pressures above 100 psia. The maximum deviation of 9.12 percent occurred at

Depth	100 ft
Flow rate	10,000 Mcf/D
Wellhead pressure	100 psia
Tubing diameter	1.995 in.
Specific gravity	0.750
Wellhead temperature	100F
Temperature gradient	0.014F/ft

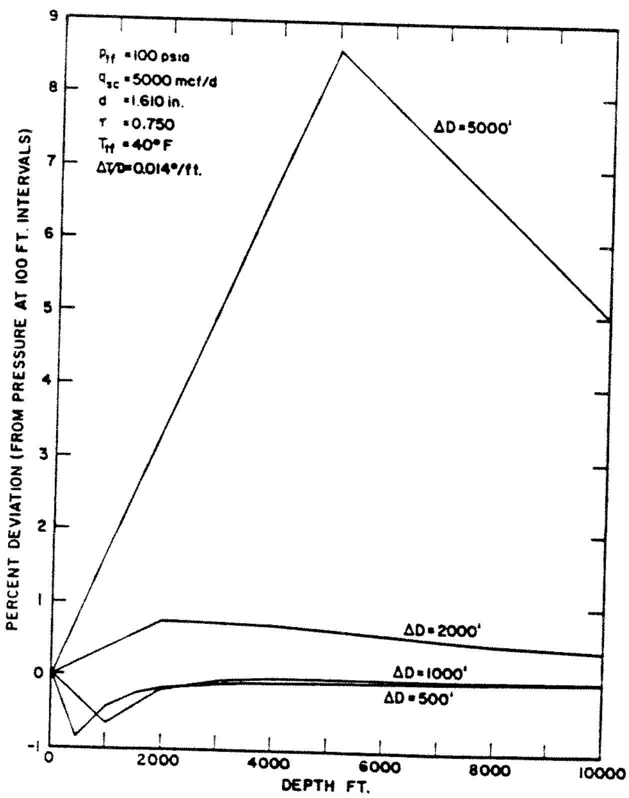

FIG. 4—EFFECT OF DEPTH ON DEVIATION DUE TO INTEGRATION INTERVAL SIZE.

TABLE 2—BOTTOM-HOLE PRESSURE COMPARISON WITH AND WITHOUT KINETIC ENERGY FROM 100-FT INTERVAL CALCULATIONS

q_{sc} = 10,000 Mcf/D Tbg ID = 1.995 in. T_{tf} = 100F

p_{tf} = 100 psia $\gamma = 0.750$ $\frac{\Delta T}{D} = 0.014°/\text{ft}$

Depth (ft)	Pressure With Kinetic Energy (psia)	Pressure Without Kinetic Energy (psia)	Percent Deviation
0	100.0	100.0	—
100	221.3	241.5	9.12
200	305.7	325.8	6.57
300	372.8	392.0	5.15
400	430.1	448.3	4.23
500	480.8	498.3	3.63
1,000	681.7	696.5	2.17
2,000	973.0	985.4	1.27
3,000	1,207.0	1,218.2	0.92
4,000	1,414.7	1,425.2	0.74
5,000	1,607.6	1,617.7	0.62
6,000	1,791.2	1,801.0	0.54
7,000	1,968.6	1,978.3	0.49
8,000	2,141.8	2,151.3	0.44
9,000	2,311.9	2,321.3	0.40
10,000	2,479.6	2,489.1	0.38

Bottom-hole flowing pressure:

With kinetic energy	221.3 psia
Without kinetic energy	241.5 psia.

The deviation in this example became less than 1 percent at a depth of 2,800 ft. This set of conditions is not likely to be encountered in actual practice.

Eq. 2 shows that the kinetic energy term is proportional to flow rate squared and inversely proportional to pressure and the fourth power of diameter.

EFFECT OF AVERAGE TEMPERATURE AND COMPRESSIBILITY

Eq. 3 was solved for depths of 4,000, 6,000, 8,000 and 10,000 ft. Values of other parameters used are shown in

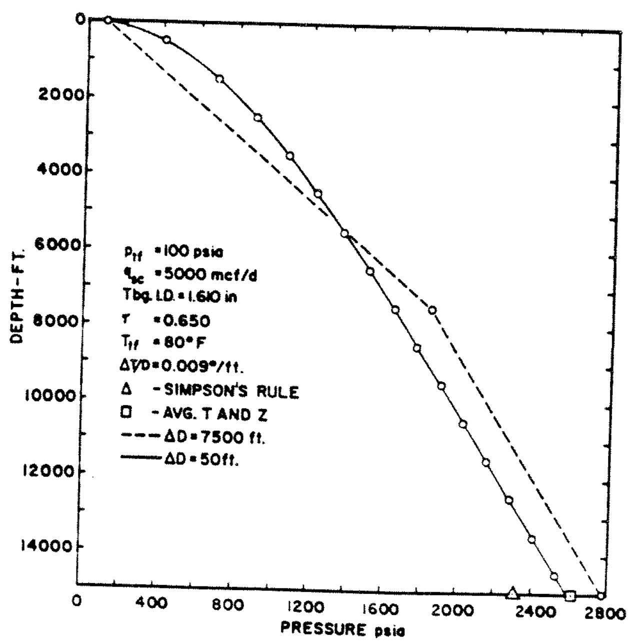

FIG. 5—COMPARISON OF SOLUTIONS FOR BOTTOM-HOLE PRESSURE.

Table 1. A total of 6,432 pressures were calculated from Eq. 3. When compared with pressures from the trapezoidal integration at 100-ft intervals and neglecting kinetic energy, 42 (0.65 percent) of the pressures deviated by more than 1 percent. All of these deviations except one occurred for conditions of

Wellhead pressure	2,000 and 3,000 psia
Wellhead temperature	40F
Temperature gradient	0.014 F/ft
Specific gravity	0.750
Depth	10,000 ft.

The maximum deviation of 1.31 percent occurred at a wellhead pressure of 2,000 psia and a flow rate of zero. Deviations for a wellhead pressure of 3,000 psia were less than those for 2,000 psia. This indicates that deviations will probably not increase for pressures above 3,000 psi. It should be noted that the maximum deviation caused by assuming average temperature and compressibility is considerably less than that caused by neglecting kinetic energy, large integration intervals or application of Simpson's rule (Fig. 5).

INJECTION CASE

For injection, D/L in Eqs. 1 and 2 is negative. In this case, the denominator of the integrand can become zero and the integrand goes to infinity. Fig. 6 is a plot of integrand I as a function of depth and shows the discontinuity which can develop. When a discontinuity occurs, change in pressure over that one interval may be assumed equal to zero and becomes an inflection point in the pressure traverse.

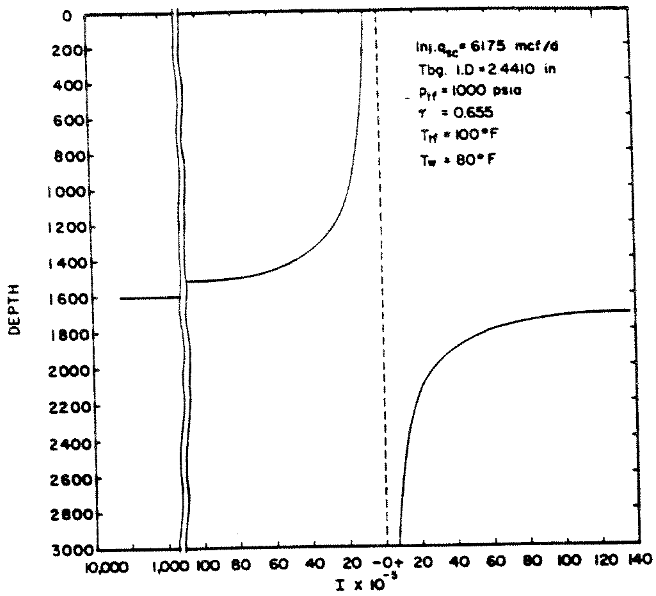

Fig. 6—Discontinuity for Injection Case.

CONCLUSIONS

1. An integration interval of 1,000 ft should be used to assure accurate trapezoidal integration of Eq. 2 or 3.
2. Simpson's rule should not be applied in an effort to correct for large trapezoidal integration intervals.
3. If flowing pressure at total depth is the desired quantity, change in kinetic energy may be ignored when depth is greater than 4,000 ft or wellhead flowing pressure is above 100 psia. If an accurate pressure traverse is desired, change in kinetic energy should be considered when wellhead flowing pressure is below 500 psia.
4. A discontinuity can develop when numerically integrating Eq. 2 or 3 for the injection case. When a discontinuity occurs, pressure change in that interval should be set equal to zero. Also, it should be noted that Simpson's rule cannot be applied in this situation.
5. Temperature and compressibility can be assumed constant at their average values for depths up to 8,000 ft. The average temperature and compressibility method, however, should not be applied unless change in kinetic energy is insignificant.

For all normal field situations, Eq. 3 should be used to calculate bottom-hole pressure in gas wells. When depth exceeds 8,000 ft, the calculation may be broken into two or more intervals. When unusual conditions, such as significant kinetic energy change, prohibit use of Eq. 3, then Eq. 2 should be used.

NOMENCLATURE

d = internal diameter of flow string, in.
e = 2.7183
D = vertical depth, ft
ΔD = trapezoidal integration interval, ft
I = integrand of Eq. 2
L = length of flow string, ft
p = pressure, psia
q_{sc} = flow rate, MMcf/D at 14.65 psia and 60F
$S = 0.0375 \dfrac{\gamma D}{\bar{T}\bar{z}}$
T = temperature, °R
$\dfrac{\Delta T}{D}$ = temperature gradient, °/ft
z = compressibility factor

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