The Material Balance as an Equation of a Straight Line—Part II, Field Cases

D. HAVLENA — HUDSON'S BAY OIL AND GAS CO., LTD., CALGARY, ALTA.
A. S. ODEH, MEMBER AIME — SOCONY MOBIL OIL CO., DALLAS, TEX.

ABSTRACT

The use of the straight-line method of solving the material balance equation is illustrated by means of six field cases. Also, the application of statistical criteria to arrive at the most probable answer is shown. The theory underlying the straight-line method of solution and the applicability of the statistical criteria was presented in a previous paper.[1]

The field cases include saturated and undersaturated oil reservoirs with and without water drive. The aquifers discussed are: limited radial, infinite radial, very small aquifer and infinite linear. The field cases also include a gas reservoir producing under water drive.

INTRODUCTION

In a previous paper,[1] the authors presented the theory underlying the solution of the material balance as an equation of a straight line. The appropriate equations for various material balance cases as well as the methods of analysis and interpretation with comments and discussion were also included.

To illustrate the various theoretical cases treated previously, five field cases are analyzed in this paper by employing the straight-line method of solving the material balance equation (MBE) and one example previously published is referred to. The use of statistical criteria to arrive at the most probable answer is also shown.

All the field examples presented, except Case 2, are excerpts from complete reservoir studies. To illustrate the method, only sections specifically dealing with the material balance principles are included. Additional geologic information and basic data are reported to better acquire an understanding of the cases and thus to better follow the reasoning that suggested the successful application of the straight-line method of solving the MBE. The six cases are: (1) saturated reservoir, small gas cap, limited aquifer; (2) saturated reservoir, very small gas cap, infinite aquifer; (3) undersaturated-saturated reservoir, very small aquifer; (4) highly undersaturated resetvoir, no water drive; (5) high undersaturated one-well reservoir, limited aquifer; and (6) gas reservoir, infinite linear aquifer.

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[1]References given at end of paper.

WATER DRIVE, A KNOWN GAS CAP

THE D_4 SAND, GUICO FIELD, VENEZUELA

The D_4 sand, which was discovered in 1943, is presently in a depleted state. Since its discovery it has produced under water drive, gas-cap-gas expansion, and solution gas drive. In Nov., 1947, water injection was initiated to arrest further pressure decline.

When discovered, the D_4 sand was a saturated reservoir with a gas cap/oil zone volume ratio m estimated volumetrically at 0.0731, an average permeability of 500 md, a porosity value of 25 per cent, and an oil viscosity at reservoir conditions of 0.3 cp. The volumetrically determined stock-tank oil initially in place was 23.1 million bbl. The volumetrically weighted physical data and production data available until Nov., 1953 are reported in Table 1.

In Ref. 1, the effects on the straight-line plot of various values of r_e/r_w for a constant Δt_D, or of various dimensionless times for a constant r_e/r_w, were theorized and were illustrated in Fig. 3A of that reference. In this field case, the previously theoretically predicted effects are established. Thus, the MBE calculations using Eq. 3c of Ref. 1 were performed for various r_e/r_w and dimensionless time values. Eq. 3c of Ref. 1 is:

$$\frac{F}{E_o + m\dfrac{B_{ti}}{B_{gi}}E_g} = N + C\,\frac{\Sigma \Delta p Q(\Delta t_D)}{E_o + m\dfrac{B_{ti}}{B_{gi}}E_g}$$

where F=net production in reservoir barrels, $E_o = B_t - B_{ti}$, and the other symbols conform to AIME standards.

In Fig. 1, three MBE plots are shown. The plot for

TABLE 1—PRESSURE-PRODUCTION-INJECTION HISTORY AND PVT DATA—THE D_4 SAND, GUICO FIELD, VENEZUELA

Date	Pressure (psig)	Cum. Oil Produced N_p (MM bbl)	Cum. GOR R_p (cu ft/bbl)	Cum. Water Produced W_p (MM bbl)	Cum. Water Injected W_i (MM bbl)	Total Formation Volume Factor B_t	Gas Formation Volume Factor $B_g \times 10^3$
10-7-43	2055	0	—	—	—	1.5166	1.2217
4-30-45	1964	1.383	970	—	—	1.5451	1.2835
9-30-45	1924	2.087	971	—	—	1.5623	1.3130
2-28-46	1897	2.861	966	—	—	1.5730	1.3337
5-31-46	1879	3.400	960	—	—	1.5808	1.3480
7-31-46	1846	3.770	952	0.001	—	1.5957	1.3745
4-30-47	1814	5.203	913	0.024	—	1.6107	1.4017
6-30-47	1799	5.494	909	0.028	—	1.6179	1.4143
9-30-47	1781	5.944	904	0.042	—	1.6270	1.4302
4-30-48	1778	7.967	916	0.013	0.478	1.6285	1.4330
5-31-49	1760	8.907	927	0.130	0.864	1.6376	1.4498
10-31-49	1750	9.555	939	0.222	1.124	1.6429	1.4590
6-30-50	1738	10.520	952	0.322	1.674	1.6491	1.4703
2-28-51	1736	11.655	956	0.442	2.238	1.6502	1.4723
6-30-51	1764	12.188	959	0.489	2.459	1.6355	1.4440
11-30-51	1734	12.790	963	0.557	2.752	1.6513	1.4742
1-31-52	1729	13.022	970	0.603	2.875	1.6541	1.4792
5-31-52	1704	13.463	984	0.717	3.159	1.6681	1.5040
11-30-52	1719	14.081	997	0.893	3.610	1.6597	1.4890
6-30-53	1747	14.651	1001	0.932	4.253	1.6446	1.4618
11-30-53	1722	15.092	1005	0.966	4.699	1.6580	1.4860

$r_e/r_w = 15$ and $t_D = 0.3t$ results in a line curving upward. This indicates that the latter values of the $\Sigma\Delta pQ(\Delta t_D)$ are too small relative to the early values. By examining the van Everdingen-Hurst $Q(t_D)$ function vs t_D for an $r_e/r_w = 15$ and for a closed exterior boundary, one notices that the maximum rate of increase in $Q(t_D)$ occurs for $100 \leq t_D \leq 500$. For $t_D > 500$ the $Q(t_D)$ vs t_D plot starts to level off and reaches its steady-state values at $t_D \simeq 1{,}000$. For the dimensionless time $t_D = 0.3t$ that was used, most of the points fell in the range $t_D > 500$. This resulted in a too slow rate of increase of $\Sigma\Delta pQ(\Delta t_D)$ as is evident in Fig. 1. To correct for this, one must decrease t_D. This was done and a $t_D = 0.078t$ resulted in a straight-line plot and an oil-in-place value of about 27 million STB.

In this case it was necessary to decrease t_D to correct for the upward bending. In other cases, depending on the shape of the $Q(t_D)$ vs t_D plot of interest, it may be necessary to increase t_D for the same condition. One must examine carefully the $Q(t_D)$ vs t_D plot of interest to determine if t_D should be increased or decreased to straighten out the MBE plot.

To show the effect of r_e/r_w for a constant t_D, several values of r_e/r_w were assumed. The calculations were performed for a $t_D = 0.3t$. The effect of increasing r_e/r_w is to increase the latter values of $\Sigma\Delta pQ(\Delta t_D)$ faster than the early values, which ultimately results in downward bending of the MBE plot, as in Fig. 1.

It must be noted that various combinations of r_e/r_w and t_D might satisfy the straight-line requirement imposed on the successful solution. However, to obtain the most probable value for N, the aquifer configuration, and t_D and r_e/r_w when applicable, one must resort to the statistical criteria advocated in Ref. 1. These criteria were not applied in this field case as they are illustrated fully in other cases and because the interest in this field case was mainly to show the effect of Δt_D and r_e/r_w values on the predicted straight-line plot.

A summary of the calculations is given in Table 2. The starting point of these calculations was April 30, 1947, when the reservoir pressure had declined by about 200 lb. However, the reference point for the water-influx calculations was the discovery date, Oct., 1943.

STURGEON LAKE SOUTH D-3 POOL, CANADA

This field example was reported in detail in the *Journal of Canadian Petroleum Technology*.[2] In the study, complete data both in numerical and graphical form are presented. The material balance as an equation of a straight

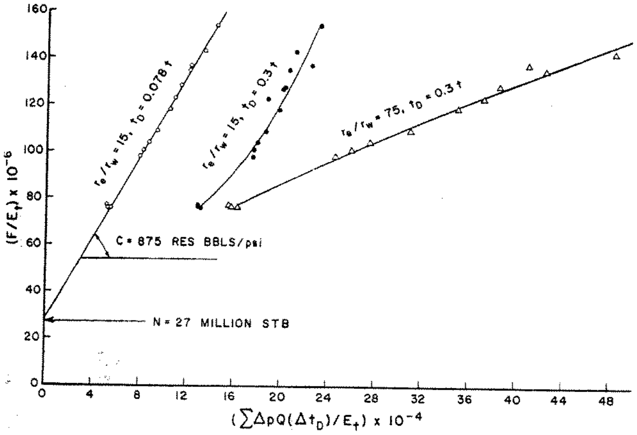

FIG. 1—D_4 SAND, GUICO FIELD.

line is applied, and the use of the consistency test and the determination of the confidence band for a pre-assigned degree of probability are fully illustrated.

WATER DRIVE, VERY SMALL AQUIFER, THE L-2b RESERVOIR, NORTH OSCUROTE, VENEZUELA

GENERAL DESCRIPTION

This dipping (3 to 5°) sand reservoir is limited at its updip side by an extensive fault of some 300 ft displacement and at both edges by minor faults which are more or less perpendicular to the main fault. The sand is fairly silty, and rather poorly sorted with numerous discontinuous shale breaks. It is composed of several lenticular bodies, a few of which are continuous through the entire investigated area. The reservoir was discovered in 1953, and in 1958 it was exploited by a total of 24 successful producers. The reservoir thickness ranged between 15 and 25 ft, and from numerous core analyses the following average properties were established: porosity = 18 per cent, connate-water saturation = 24 per cent, permeability = 580 md, and the stock-tank oil initially in place = 747 bbl/acre-ft. The volumetrically calculated stock-tank oil initially in place ranged between 30.6 and 37.2 million bbl depending on the location of the original oil-water contact, which was estimated to be between 9,050 and 9,100 ft subsea.

By June, 1960, cumulative oil production amounted to 5.54 million STB. The maximum number of wells producing at any particular month was 15, which was attained in 1956. Since that time, the number of producers diminished as additional wet wells were shut in. Thus, the instantaneous monthly water production was maintained at less than 10 per cent while the cumulative water cut reached 6.5 per cent in 1960. The cumulative gas-oil ratio increased slowly and surpassed the solution gas-oil ratio of 705 by only 60 cu ft/bbl.

Due to the advancing water table, a variable pressure datum corresponding to the volumetric midpoint of the oil leg was used. This procedure resulted in a 120-ft upward change in the reference pressure datum during the productive life of the field. Average reservoir pressures were always referred to the proper datum. The original pressure at the oil-water contact was evaluated from data reported on low structural wells. The original pressures used in this study were 3,909 and 3,985 psig for the oil reservoir and the oil-water contact, respectively. The bubble-point pressure was 3,765 psig at the original datum of 8,975 ft subsea. B_t was equal to

TABLE 2—MBE CALCULATIONS, THE D_4 SAND, GUICO FIELD, VENEZUELA

Pressure (psig)	E_t*	$F = (N_p$** $+ W_p - W_i)$ (MM bbl)	$(F/E_t)10^{-6}$	$[\Sigma\Delta pQ(\Delta t_D)/E_t] \times 10^{-4}$ $r_e/r_w = 15$ $t_D = 0.078t$	$[\Sigma\Delta pQ(\Delta t_D)/E_t] \times 10^{-4}$ $r_e/r_w = 15$ $t_D = 0.3t$	$[\Sigma\Delta pQ(\Delta t_D)/E_t] \times 10^{-4}$ $r_e/r_w = 75$ $t_D = 0.3t$
1814	0.1104	8.499	76.98	5.17	12.88	15.48
1799	0.1188	8.987	75.65	5.24	12.91	15.71
1781	0.1293	9.747	75.38	5.42	13.16	16.34
1778	0.1311	12.782	97.50	7.86	17.53	24.41
1760	0.1417	14.200	100.21	8.22	17.56	25.85
1750	0.1478	15.340	103.79	8.66	17.96	27.55
1738	0.1551	16.801	108.32	9.41	18.59	30.48
1736	0.1563	18.397	117.70	10.42	19.64	34.44
1764	0.1391	19.002	136.61	12.22	22.41	40.82
1734	0.1576	20.113	127.62	11.34	20.14	38.42
1729	0.1609	20.615	128.12	11.37	19.98	38.87
1704	0.1771	21.716	122.62	10.81	18.65	37.14
1719	0.1674	22.573	134.84	12.15	20.44	42.41
1747	0.1498	22.937	153.12	14.27	23.10	50.88
1722	0.1654	23.644	142.95	13.37	21.06	48.37

$$*E_t = \frac{mB_{ti}(B_g - B_{gi})}{B_{gi}} + (B_t - B_{ti}),\ m = 0.0731$$

$**B_t + (R_p - R_{si})B_g \quad R_{si} = 900$ cu ft/bbl

$$1.42\left[\frac{3{,}779 - p}{\alpha p + \beta p^2 + \gamma p^3} + 1\right]$$

where $\alpha = 2.34212$, $\beta = 0.25542 \times 10^{-3}$ and $\gamma = 0.05047 \times 10^{-6}$.

MATERIAL BALANCE CALCULATIONS

From pressure vs production plot, cumulative production before reaching the bubble-point pressure was estimated. This amount in reservoir barrels was subtracted as a constant from the cumulative net production F in all subsequent MBE calculations which were referred to the bubble-point pressure.

Fig. 2 gives the plot of a depletion-type MBE, $F = NE_o$, as shown in Eq. 1, Ref. 1. The early part of the plot, up to Point 5 corresponding to June, 1956, results in a straight line going through the origin. Beyond that date the points deviate from the straight line.

This behavior is easily explained if a very small aquifer exists. In this case[1]

$$F - \text{const} = NE_o + W_v E_{w,f}$$

where W_v is the aquifer water volume in reservoir barrels, and $E_{w,f}$ is the total water and rock expansion. The PVT properties for this reservoir show that E_o approximately varies linearly with p for $3{,}380 \leq p \leq 3{,}765$. The pressure in June, 1956, was 3,360. Thus, for this range of pressure E_o per unit pressure change is nearly constant. Since $E_{w,f}$ is also constant for all pressure ranges, then up to June, 1956, the MBE can be written as

$$F - \text{const} = (N + aW_v)E_o$$

where $a = \frac{E_{w,f}}{E_o}$. A plot of $(F - \text{const})$ vs E_o should result in a straight line with a slope equal to $(N + aW_v)$.

For $p < 3{,}380$, the relation between E_o and pressure begins to deviate considerably from linear and the above MBE equation does not hold. Thus, the points will deviate from the straight line, as Fig. 2 shows.

Because of the above behavior and because of the steady and rather large decline in pressure, the presence of a very small aquifer was suspected. Consequently Eq. 3b of Ref. 1 was used. [$F/E_o = N + C'(\Delta p'/E_o)$, where $\Delta p' = p_i - p$ and $C = Wc_{w,f}$]. It resulted in Fig. 3. As predicted by this equation the plot resulted in a straight line moving in a backward sequence with time. Point 1 corresponds to June, 1955, and Point 20 corresponds to June, 1960. The intersection of the straight line with the ordinate gave an original oil-in-place value of 32.6 million STB.

CALCULATION OF AQUIFER SIZE

A depletion-type MBE of the following form was employed: $F - \text{const} = N(E_o + nB_w E_{w,f})$ where $n = W/N$, B_w is the initial water formation volume factor, and $E_{w,f}$, the total water and rock expansion, is equal to $c_{w,f}\Delta p'$. W is the aquifer size in stock-tank barrels.

A plot of $(E_o + nB_w E_{w,f})$ vs $(F - \text{const})$ should result in a straight line going through the origin if the correct value for n is assumed. Such a straight line was obtained for a value of n of 14.2 and is shown in Fig. 4. Thus, the aquifer contained about 0.5 billion bbl of water.

The aquifer size could also be calculated from the slope of the straight-line plot of Fig. 3, which is equal to $Wc_{w,f}$. This was done and the value of 0.5 billion bbl of water was verified.

APPLICATION OF STATISTICAL CRITERIA

To check the above solutions, the standard deviation and consistency tests were applied. These tests are illustrated fully in Sturgeon Lake South D-3 reservoir, referred to previously. Therefore they will not be discussed in detail here. However, the results of the statistical investigation showed that a bubble-point pressure of 3,760 would have been a better choice than 3,765. The new bubble point pressure (3,760) resulted in an initial oil-in-place of 32.8 million STB. The standard deviation was 0.06 million STB, and the slope of the straight line of the consistency test plot for a four-year period (July 1, 1956, to June 30, 1960) was 2,560 STB/month. The confidence band for a probability range of 75 to 90 per cent was ± 1.2 million STB, and for a probability range of 95 to 100 per cent was ± 1.7 million STB.

DETERMINATION OF THE ORIGINAL OIL-WATER CONTACT

Three positions for the original oil-water contact were assumed. These were 9,100, 9,072 and 9,050 ft sub-

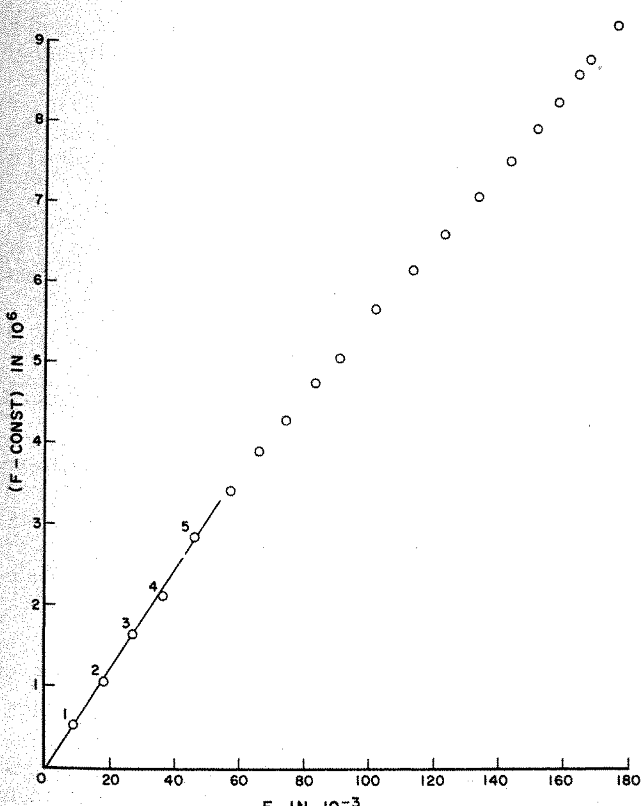

FIG. 2—L-2b RESERVOIR, OIL EXPANSION VS NET PRODUCTION.

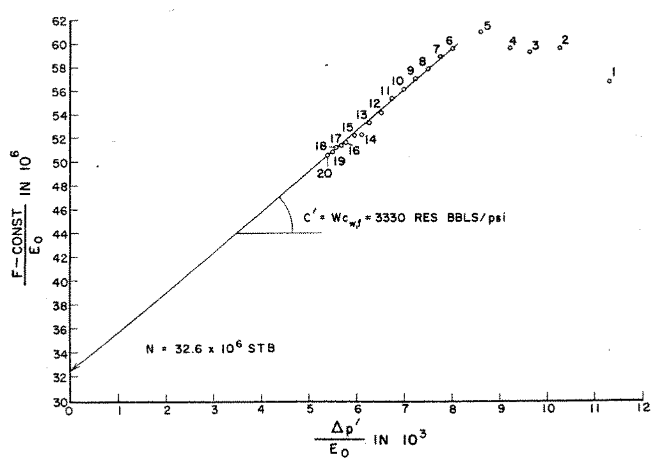

FIG. 3—L-2b RESERVOIR, SMALL AQUIFER PLOT.

sea. The position of the oil-water contact as a function of time was determined from production data by assuming that in a well the instantaneous produced per cent water in total fluid is equal to the flooded-out productive interval divided by the total productive interval expressed in per cent. This assumption clearly neglects coning.

Having determined the position of the oil-water contact with time, the flooded-out volume as a function of time for the three assumed values of the original position of the oil-water contact were calculated and plotted vs the net cumulative water influx, $We - Wp$, which was obtained from the MBE. The original oil in place was taken as 32.8×10^6 STB. This plot is shown in Fig. 5.

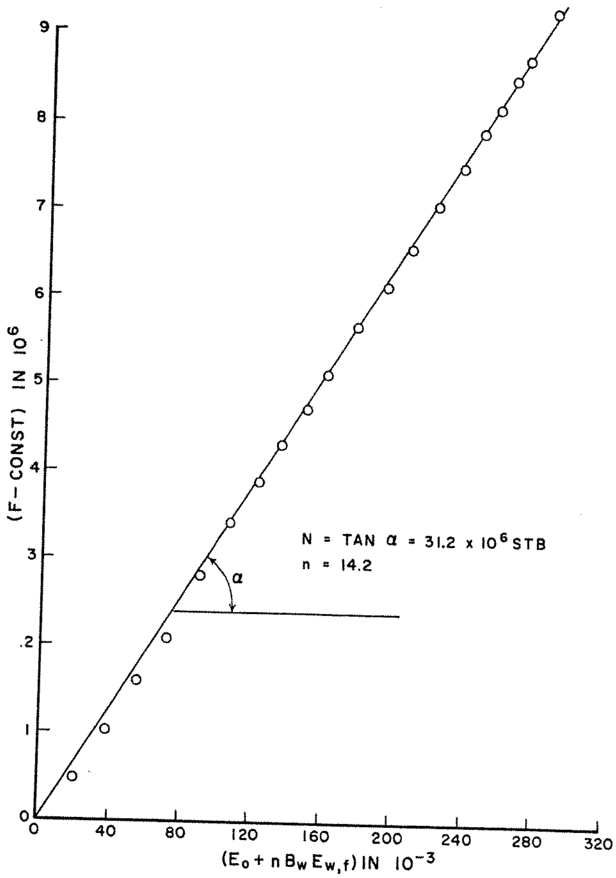

Fig. 4—L-2b Reservoir, North Oscurote, Determination of N and Aquifier Size.

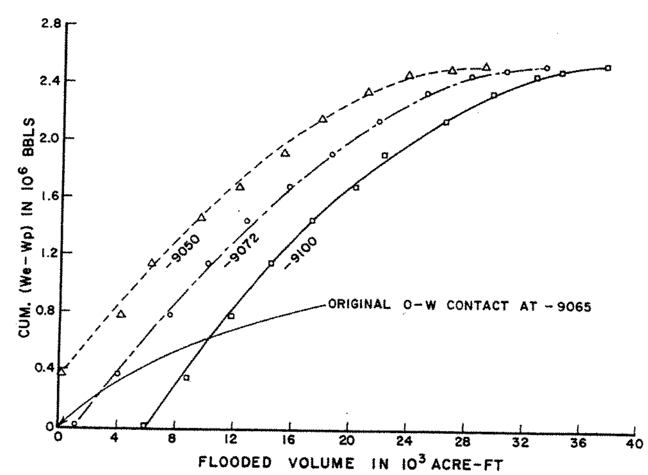

Fig. 5—L-2b Reservoir, Determination of Original Position of Oil-Water Contact.

If the correct position of the original oil-water contact is assumed, then the plot should show zero flooded-out volume for zero net water influx. The plot shows that this obtains for oil-water contact of 9,065 ft subsea. Thus, 9,065 ft subsea was taken as the original position of the oil-water contact. The correspondingly volumetrically determined original oil in place was 32.9 million STB, which is 0.3 per cent from the N calculated by the MBE.

UNDERSATURATED RESERVOIR, NO WATER DRIVE — ONE EXPLICIT UNKNOWN

The Virginia Hills Beaverhill Lake reservoir, located some 120 miles northwest of Edmonton in Alberta, was discovered in March, 1957, and at the end of 1961 it had been developed by 97 wells drilled on 160-acre spacing. At the present time there are about 102 producers within the field limits. The daily production rate amounts to 7,000 to 8,000 BOPD with 400 to 480 scf/bbl gas-oil ratio. By the end of Dec., 1961, the cumulative production amounted to 3.56×10^6 STB of 39° API oil and virtually no water. Tables 3 and 4, which present the solution of the MBE's, summarize also the production performance of this pool.

Detailed, foot-by-foot, petrophysical and geological evaluations on each well were made. During the subsequent well-to-well correlations of the numerous individual streaks which form the effective net pay, it was noted that, vertically, the porosity development is divisible into two units separated by a dense shaly carbonate interval varying in thickness from 2 to 10 ft. The upper zone was termed Hope Creek while the lower, thicker, porous unit was named the Main Zone.. Although both zones are being exploited as one reservoir, it was thought that for the purpose of the basic reservoir evaluation it may be advantageous to evaluate each of them separately. The reason for this approach was to avoid any eventual errors in incorrect weighting of the "average" parameters, mainly the volumetric reservoir properties and PVT's. The initial volumetric active oil in place flashed through 40 psig separator was calculated to be 74.3 and 272 million STB for the Hope Creek and Main Zone, respectively. Thus, the total Virginia Hills reservoir contained 346.3 million STB of oil.

In making the volumetric estimates of the active original oil in place only permeable intervals with connate-water saturation less than 60 per cent were considered as net pay.

The straight-line method of solving the MBE was used to answer the following questions.

1. Was the 60 per cent connate-water cut-off appropriate in defining active oil-in-place?

2. After correcting for the man-created communications (four wells were perforated through) are the two zones actually physically separated?

3. Are the two aquifers associated with the "two zones" active, and, if so, are they interconnected? Only the Main Zone is believed to be underlain by water, and Hope Creek probably has edge water.

PRESSURES AND PVT DATA

Two separate pressure datums were determined, 5,587 ft subsea for the Hope Creek and 5,617 ft subsea for the Main Zone, respectively. The individual pressures, appropriately corrected, were averaged volumetrically for each of the zones. The p_i's determined from early pressure measurements were 3,685 psig for the Main and 3,654

TABLE 3—VIRGINIA HILLS RESERVOIR, MAIN ZONE DATA

Date	No. of Producing Wells	Average* Reservoir Pressure (psig)	Estimated N_p (in 10^3)	Estimated W_p (in 10^3)	B_o (vol/vol)	$F = N_pB_o + W_p$ (in 10^3)	c_o vol/vol/psi (in 10^{-6})	$\frac{S_oc_o + S_wc_w + c_f}{1-S_w}$** (in 10^{-6})	$\Delta p'$ (in psi)	E_t*** (in 10^{-6})
10-1-57	1	3685	0.342		1.3102	0.448	11.01	18.674	0	
1-1-58	1	3685	0.342		1.3102	0.448	11.01	18.674	0	
4-1-58	2	3680	20.481		1.3104	26 838	11.02	18.685	5	93
7-1-58	2	3680	20.481		1.3104	26.838	11.02	18.685	5	93
10-1-58	2	3680	20.481		1.3104	26.838	11.02	18.685	5	93
1-1-59	2	3676	34.750		1.3104	45.536	11.03	18.694	9	168
4-1-59	3	3667	78.557		1.3105	102.949	11.04	18.704	18	337
7-1-59	3	3667	78.557		1.3105	102.949	11.04	18.704	18	337
10-1-59	3	3667	78.557		1.3105	102.949	11.04	18.704	18	337
1-1-60	4	3664	101.846		1.3105	133.469	11.05	18.715	21	393
4-1-60	19	3640	215.681		1.3109	282.736	11.08	18.745	45	844
7-1-60	25	3605	364.613		1.3116	478.226	11.13	18.795	80	1504
10-1-60	36	3567	542.985	0.159	1.3122	712.664	11.18	18.844	118	2224
1-1-61	48	3515	841.591	0.805	1.3128	1105.646	11.26	18.924	170	3217
4-1-61	59	3448	1273.530	2.579	1.3130	1674.723	11.35	19.015	237	4506
7-1-61	59	3360	1691.887	5.008	1.3150	2229.839	11.48	19.144	325	6228
10-1-61	61	3275	2127.077	6.500	1.3160	2805.733	11.60	19.264	410	7898
1-1-62	61	3188	2575.330	8.000	1.3170	3399.709	11.86	19.524	497	9703

*$p_i = 3{,}685$
**$S_wc_w = 0.868 \times 10^{-6}$, $c_f = 4.95 \times 10^{-6}$
***$E_t = \Delta p' \dfrac{S_oc_o + S_wc_w + c_f}{1 - S_w}$

TABLE 4—VIRGINIA HILLS RESERVOIR, HOPE CREEK ZONE DATA

Date	No. of Producing Wells	Average* Reservoir Pressure (psig)	Estimated N_p (in 10^3)	B_o (vol/vol)	$F =$ ** $N_pB_o + W_p$ (in 10^3)	c_o (vol/vol/psi) (in 10^{-6})	$\frac{S_oc_o + S_wc_w + c_f}{1 - S_w}$*** (in 10^{-6})	$\Delta p'$ (psi)	$E_t = \Delta p' \frac{S_oc_o + S_wc_w + c_f}{1 - S_w}$ (in 10^{-6})
4-1-59	1	3654	9.269	1.354	12.550	11.96	20.238	0	
7-1-59	1	3654	9.269	1.354	12.550	11.96	20.238	0	
10-1-59	1	3645	15.889	1.355	21.530	11.98	20.258	9	182
1-1-60	1	3639	22.673	1.355	30.722	11.99	20.268	15	304
4-1-60	4	3620	39.562	1.355	53.606	12.02	20.298	34	690
7-1-60	6	3580	86.100	1.356	116.666	12.07	20.348	74	1505
10-1-60	10	3533	144.804	1.356	196.354	12.10	20.378	121	2465
1-1-61	18	3470	250.436	1.357	339.842	12.22	20.498	184	37771
4-1-61	25	3381	401.617	1.358	545.396	12.37	20.648	273	5637
7-1-61	25	3267	563.481	1.360	766.334	12.55	20.828	387	8060
10-1-61	32	3140	767.155	1.363	1045.632	12.74	21.019	514	10804
1-1-62	36	3008	985.403	1.365	1345.075	12.95	21.228	646	13713

*$p_i = 3{,}654$
**$W_p = 0$
***$S_wc_w = 0.842 \times 10^{-6}$, $c_f = 5.5 \times 10^{-6}$

psig for the Hope Creek, respectively. Average reservoir pressures at intermediate time intervals were obtained from plots of pressures vs the respective cumulative oil production.

Two subsurface Hope Creek samples and one subsurface Main Zone sample indicated that both crudes were highly undersaturated at the time of discovery, with bubble-point pressures of 1,960 and 1,792 psig, respectively. The pertinent PVT data as used in the solution of the MBE's are reported in Tables 3 and 4.

The expansion factor E_t is defined by the right-hand-side variable of Eq. 5 of Ref. 1, which is:

$$N_pB_o = NB_{oi} \frac{(S_oc_o + S_wc_w + c_f)\Delta p'}{1 - S_w}.$$

In the computations of the expansion factor the compressibility of the connate water was taken as 3.6×10^{-6} vol/vol/psi and the appropriate compressibilities of the rocks were obtained from the tables of Hall.[3] The average porosities and connate-water saturations for the Hope Creek were 7.58 and 23.4, and for the Main Zone were 9.25 and 24.1 per cent, respectively.

MBE CALCULATIONS

Eq. 5 of Ref. 1 was used with the appropriate production, pressure and PVT data discussed above. The computations are shown in Tables 3 and 4 and the results are presented graphically in Fig. 6.

Since all the points plotted in two separate straight lines going through the origin, it was concluded that the reservoirs were not in communication except through perforations, as indicated above. This must be true, since if there were cross flow between the two zones the points would not plot in straight lines but, instead, would bend.

From the slopes of the two straight lines the active oil in place in millions of stock-tank barrels was calculated to be 72.6 for the Hope Creek, 270.6 for the Main Zone, with 343.2 for the Virginia Hills reservoir. This compares with volumetrically determined values of 74.3, 272 and 346.3 million STB for the Hope Creek, the Main Zone and total Virginia Hills reservoir, respectively. This close agreement between the MBE results and the volumetrically determined values indicated that the 60 per cent connate-water cut-off was appropriate in defining the active oil in place. Moreover, this close agreement coupled with the fact that the points as calculated by Eq. 5 of Ref. 1 plotted in two straight lines going through the origin indicated that the reservoirs up to the end of 1961 were not producing under water drive. Thus, since the aquifers were not active, it is irrelevant as to whether they are or are not interconnected.

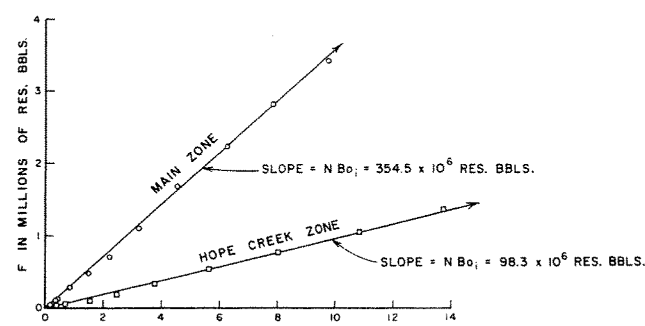

FIG. 6—VIRGINIA HILLS, BEAVERHILL LAKE RESERVOIR, DETERMINATION OF ORIGINAL OIL IN PLACE.

SPECIAL FIELD CASE RESERVOIR X

Production from this one-well reservoir is obtained from about 15 ft of net pay which is underlain by a water table. The areal extent of this reservoir, which fringes around a granite knob of the pre-Cambrian basement, is completely unknown. The well was brought in with an initial production rate of 210 BOPD, which later increased to about 1,000 BOPD. Because of these encouraging results, several additional wells were drilled as close offsets, but despite these extensive exploration efforts, no additional producer was completed. To assist in the geological interpretation and to determine the size of this reservoir, which was impossible to estimate by volumetric methods, comprehensive reservoir and production data were collected during six years of production.

PRODUCTION, PRESSURE AND PVT DATA

Fig. 7 presents in a graphical form the six years' production-pressure performance of this interesting, but rather small, reservoir. It may be noted that the well initially produced with a 30 per cent water cut, which decreased to about 6 per cent after a cumulative oil production of about 19,000 bbl and a prolonged shut-in time of about 50 days. Afterwards, the water cut remained essentially unchanged, varying between 4 and 9 per cent. Moreover, on the basis of numerous production tests, it appears that the water cut over a wide range of production rates is rather insensitive to the rate of fluid withdrawals. Similar characteristics as discussed for the water production are exhibited also by the GOR curve (Fig. 7).

Considerable subsurface pressure measurements, at least five, of a prolonged shut-in time duration were obtained on this well. A pressure build-up test taken during the initial production test indicated that the initial reservoir pressure was 2,913 psig.

The early surface-recombined PVT sample suggested that this oil was highly undersaturated ($p_b = 2,297$ psig). The combined oil, rock and connate-water compressibility was calculated to be $20 \times 10^{-6} \times 1.28$, or 25.6×10^{-6} vol/vol/psi referred to stock-tank conditions through a 40 psig separator. B_{oi} was 1.28.

ORIGINAL OIL IN PLACE

By MBE

Because of the presence of the free water table and because of the repressuring of the oil reservoir by a slow water influx, as will be discussed in the following subsections on the pressure build-ups, the MBE was applied in the form of Eq. 6 of Ref. 1, which is

$$\frac{N_pB_o + W_p - W_i}{\dfrac{B_{oi}\Delta p'}{1 - S_w}\ (S_oc_o + S_wc_w + c_f)} = N + C\frac{\Sigma \Delta p Q(\Delta t_p)}{\dfrac{B_{oi}\Delta p'}{1 - S_w}\ (S_oc_o + S_wc_w + c_f)}.$$

TABLE 5

Effective aquifer radius — r_e/r_w	= 8 reservoir radii
Dimensionless time — t_D	= 0.22/month
Original oil-in-place — N	= 2.15×10^6 STB
Aquifer constant — C	= 50 res. bbl/psi
Min. standard deviation — σ_{min}	= 148×10^3 STB

Furthermore, because a limited aquifer was suspected as suggested by numerous close offsets (dry holes), several combinations of r_e/r_w and t_D were used. A plot of the calculations, all carried out on a digital computer, was made for each combination of r_e/r_w and t_D. An example is shown in Fig. 8. The most probable values corresponding to the minimum standard deviation and as determined by the consistency test (for details of which refer to the Sturgeon Lake South D-3 study) are shown in Table 5.

Using statistical methods, the confidence band for a probability of 89 to 95 per cent was calculated to be $\pm 0.06 \times 10^6$ STB.

If the pay thickness of 15 ft, as found in the well, were uniform, the 2.15 million STB would extend over about 400 acres. The reservoir has a weak water drive from an aquifer which apparently extends out about 8 field radii. From the constants C and t_D, and speculating on the basis of seismic and geological information that the aquifer thickness h might be about 30 ft and that the water influx is effected over π radians, the permeability k of the aquifer would be about 1.5 md. This deduction is substantiated by solution of the radial flow formula which suggested that the aquifer permeability might be about 3 md. This small permeability of the aquifer was further confirmed by core analysis made on samples obtained from the aquifer zone of the well and from offsetting dry holes. The results gave an average aquifer permeability of about 1 md.

By Pressure Build-Ups

A plot of a typical two-month pressure build-up is presented in Fig. 9. Because the production rates were usual-

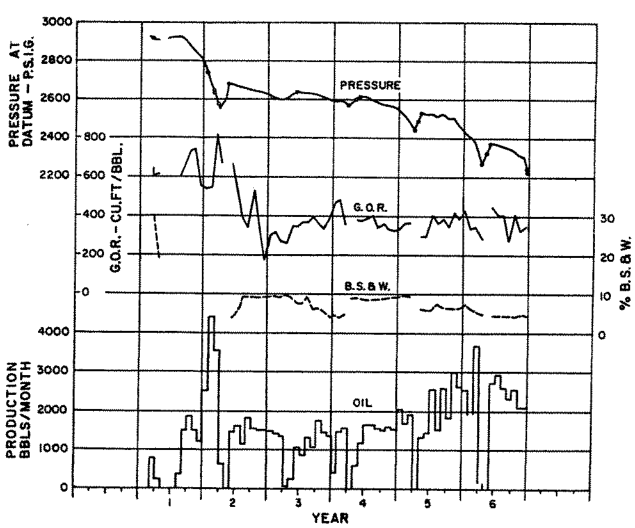

FIG. 7—RESERVOIR X, PRODUCTION PERFORMANCE VS TIME.

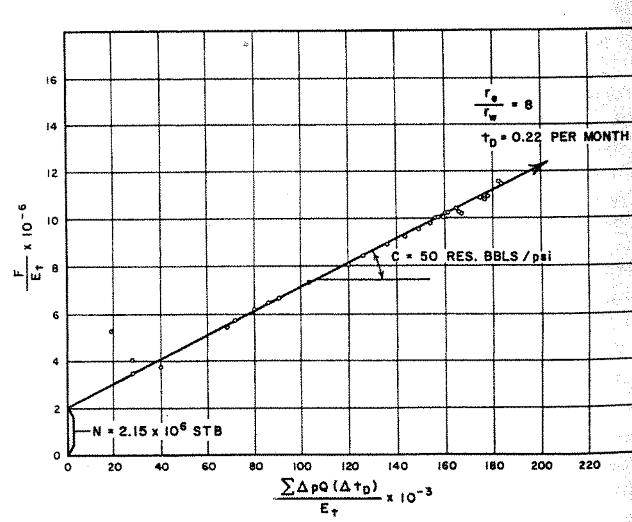

FIG. 8—RESERVOIR X, DETERMINATION OF N.

ly changed many times prior to shutting in the well, Horner's superposition approach was applied. The shut-in time in hours is given at each calculated pressure point.

Using Slope 1, which extends from 0.75 to 4 hours, of Fig. 9, a kh of 14,000 md-ft corresponding to a k of about 1 darcy was calculated. The second slope, which persisted from 4 to about 61 hours shut-in time, yields a kh of 2,400 md-ft corresponding to a k of about 160 md. Thus, the formation in the vicinity of the wellbore was more permeable than the formation away from it. The increased conductivity kh in the vicinity of the wellbore was probably caused by the treatment with 18 bbl (1.2 bbl/ft) of 30 per cent hydrochloric acid which was given to this well in July of its first year. The steep increase in the rate of pressure build-up, noticeable at prolonged shut-in time, probably is caused by water influx into the oil reservoir. This slow action of water drive is undoubtedly caused by the low permeability of the aquifer, as discussed in the previous subsection.

Pressures obtained from four pressure build-ups were plotted vs shut-in time on regular coordinate paper. A typical plot is given in Fig. 10. The rate of pressure increase due to water influx was constant. From the solution of the MBE as given by Fig. 8, the necessary parameters to calculate the rate of water influx were obtained. Thus, the rates of water influx during the shut-in periods and for constant rates of pressure increase were calculated and used in the following equation to calculate N:

$$N = \frac{\text{Rate of water influx}}{\text{Rate of pressure increase} \times \text{compressibility} \times B_{oi}}$$

Table 6 summarizes the results of the calculations. It shows that the arithmetically determined average for N is 2.06×10^6 STB.

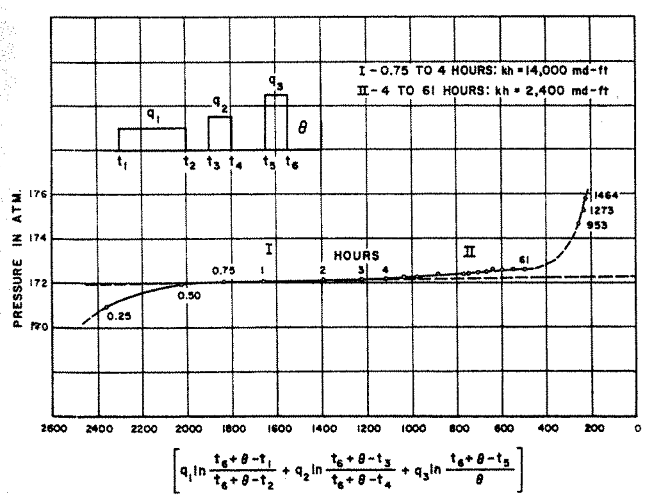

FIG. 9—RESERVOIR X, PRESSURE BUILD-UP.

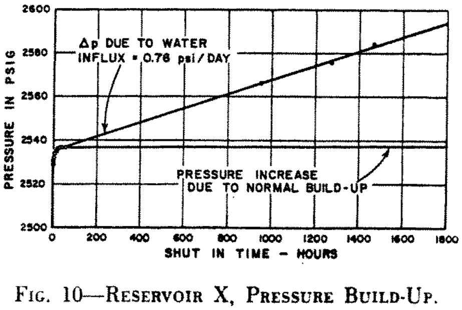

FIG. 10—RESERVOIR X, PRESSURE BUILD-UP.

TABLE 6—RESERVOIR X, DETERMINATION OF N FROM PRESSURE BUILD-UPS

Year of Survey	Rate of Water Influx (B/D) from MBE	Rate of Pressure Rise from Pressure Build-Ups psi/day		N (in 10^6 STB)
3	36.5	0.64		2.22
4	43.5	0.76	(Figure 10)	2.24
5	75.	1.70		1.72
6	105	2.00		2.05
			average	2.06

By Park Jones' Approximation

Twice during the life of the well sufficient data were obtained to attempt application of Park Jones' reservoir limit test.[4] Typical plots of pressure vs flow time are presented in Figs. 11 and 12. It was concluded that semi-steady-state conditions did not obtain at the end of the test. At that time the pressure decline was 30 psi/day. By applying Park Jones' approximation for unsteady-state flow:

$$N = \frac{2.5\, q}{c(-dp/dt)} .$$

A value of 2.1×10^6 STB was obtained for the oil associated with an unsteady-state flow test of 18 hours duration.

Thus, the original oil in place as determined by MBE, from pressure build-up, and by Park Jones approximation is, respectively, $(2.15 \pm 0.06) \times 10^6$, 2.06×10^6 and 2.1×10^6 STB.

SUMMARY

By using three different methods of determining N, a considerable amount of information was gained on this

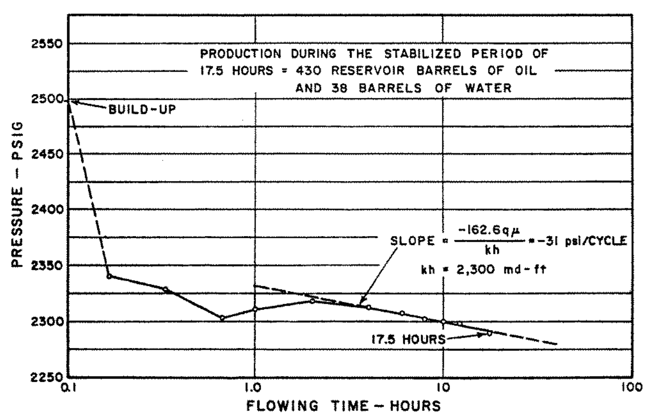

FIG. 11—RESERVOIR X, FLOW TEST.

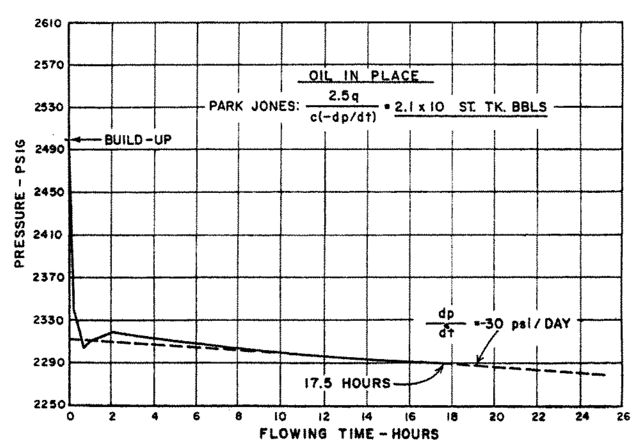

FIG. 12—DETERMINATION OF ORIGINAL OIL IN PLACE.

TABLE 7—RESERVOIR Y DATA

Time (Months)	Average Reservoir Pressure (psig)	$E_g = (B_g - B_{gi})$ (in 10^{-6} res cu ft/scf	$F = G_pB_g$ (in 10^6 res cu ft)	$\frac{\Sigma \Delta p_n \sqrt{t - t_n}}{E_g}$ (in 10^6)	F/E_g (in 10^{12})
0	2883	0.0	—		
2	2881	4.0	5.5340	0.3536	1.3835
4	2874	18.0	24.5967	0.4647	1.3665
6	2866	34.0	51.1776	0.6487	1.5052
8	2857	52.0	76.9246	0.7860	1.4793
10	2849	68.0	103.3184	0.9306	1.5194
12	2841	85.0	131.5371	1.0358	1.5475
14	2826	116.5	180.0178	1.0315	1.5452
16	2808	154.5	240.7764	1.0594	1.5584
18	2794	185.5	291.3014	1.1485	1.5703
20	2782	212.0	336.6281	1.2426	1.5879
22	2767	246.0	392.8592	1.2905	1.5970
24	2755	273.5	441.3134	1.3702	1.6136
26	2741	305.5	497.2907	1.4219	1.6278
28	2726	340.0	556.1110	1.4672	1.6356
30	2712	373.5	613.6513	1.5174	1.6430
32	2699	405.0	672.5969	1.5714	1.6607
34	2688	432.5	723.0868	1.6332	1.6719
36	2667	455.5	771.4902	1.7016	1.6937

one-well reservoir. Two of these methods, the pressure build-up and Park Jones', may not always apply. Thus, they do not have the general applicability of the MBE. However, in the case of this reservoir they resulted in satisfactory answers which may be due to the high permeability of the reservoir, to its size, and to the fact that the oil was undersaturated during the six producing years.

GAS RESERVOIR WITH WATER DRIVE RESERVOIR Y

GENERAL DESCRIPTION

This dry-gas reservoir was discovered in the late forties, and at the present time it is being exploited by about 10 wells. The reservoir is about 11 miles long and 1 to 1.5 miles wide. The productive structure is found at a depth of about 5,900 ft subsea and attains a maximum pay thickness of 440 ft. Its original gas-water contact, established by logs and tests of several wells, is placed at 6,340 ft subsea. The areal extent of the original gas-water contact covers some 16 sq. miles. The volumetric estimates of the original dry gas in place varies from 1.3 to 1.65 Tscf, depending mainly on the structural interpretation and estimates of percentage "net" hydrocarbon volume. Other minor differences in interpretation and averaging of the basic data also contribute to the above discrepancy of 27 per cent in the original gas in place.

Production, pressures and the pertinent expansion factors are presented in Table 7. For convenience, the original basic data were converted from centimeters-grams-seconds to standard U. S. units. Cumulative production is expressed in reservoir cubic feet. Since no pressures at the original oil-water table were available, the average reservoir pressures were used for the evaluation of the gas expansion factor E_g and also for calculation of the effective pressure drops which govern the calculations of the "water influx". Any error caused by this simplification should be relatively small since only the "changes" in the pressure drops are involved and the pressure equalizes relatively fast within the gas reservoir.

MATERIAL BALANCE CALCULATIONS

A summary of data and calculations is presented in Table 7. The depletion-type MBE ($G_pB_g = GE_g$, as shown in Eq. 7 of Ref. 1) was tried first. F, i.e. G_pB_g, was plotted vs E_g on cartesian coordinate paper. The line represented by these points curved upwards and thus did not satisfy the necessary straight-line relation.

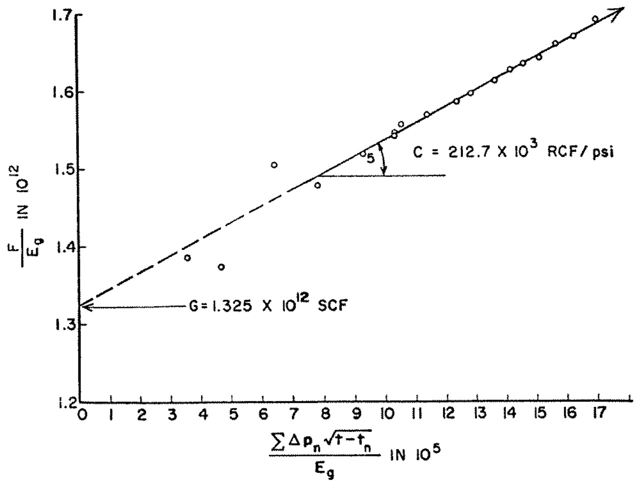

FIG. 13—RESERVOIR Y, DETERMINATION OF ORIGINAL GAS IN PLACE.

Because of this condition, the MBE with water drive was next tried. Since an infinite linear case does not require the estimation of dimensionless time and is easy to perform, it was tried first. The results are shown in Table 7, and are illustrated in Fig. 13. The necessary straight-line relationship was evident and the solution was regarded as satisfactory. The best straight line through Points 5 to 18 was drawn by means of the least-squares method. The original gas in place was 1.325 Tscf and the standard deviation was 0.0035 Tscf. The confidence band for a probability range of 75 to 90 per cent was ± 1.4 Bscf and for a probability range of 95 to 100 per cent was ± 2.9 Bscf. The consistency-test straight-line plot resulted in a slope equal to 28.8 MMscf for two months. This very small slope of the consistency-test straight line indicated a high degree of consistency with time. Because of this, the infinite linear aquifer case was accepted and no further calculations were deemed necessary.

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