Application of a General Material Balance for High-Pressure Gas Reservoirs

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Summary

This paper presents the derivation of a general gas material balance that has particular application to high-pressure gas reservoirs, [both normal pressured and overpressured (geopressured)]. Its main application is to calculate original gas in place and assist in calculating remaining recoverable reserves from pressure/production data.

The form of the material-balance equation is $(p/z)[1 - \bar{c}_e(p)(p_i - p)] = (p/z)_i(1 - G_{p}/G)$, which includes a pressure-dependent cumulative effective compressibility term $\bar{c}_e(p)$ that is defined in terms of the following reservoir parameters: pore compressibility, water compressibility, gas solubility, and total water associated with the gas reservoir volume. "Associated" water includes connate water, water within interbedded shales and nonpay reservoir rock, and any limited aquifer volume. \bar{c}_e physically represents the cumulative change in hydrocarbon pore volume (PV) caused by compressibility effects and encroaching water.

High pressure gas reservoirs typically have concave downward p/z vs. G_p plots which may result in serious overestimation of original gas in place and remaining recoverable reserves. The proposed form of the gas material balance equation provides a method to linearize the p/z vs. G_p plot, and thereby predict the true original gas in place. A method is suggested to determine initial gas in place by analyzing the behavior of cumulative effective compressibility backcalculated from pressure/production data. The $\bar{c}_e(p)$ function determined by this procedure, or estimated from logs and geological maps when sufficient production data is not available, is then used to forecast pressure/cumulative behavior. Two field examples are provided showing the application of the material balance equation to high pressure gas reservoirs.

Introduction

High pressure gas reservoirs experiencing depletion drive typically have downward curving p/z vs. G_p behavior. Incorrect extrapolation of early depletion data may result in serious overestimation of original gas in place and remaining reserves.

Bruns *et al.*¹ work in 1965 was a result of a field study conducted on a large moderately overpressured gas reservoir in the Texas Gulf Coast area. Investments were made, and never needed, based on linear extrapolation of the early field p/z vs. G_p performance to an apparent original gas in place that was later found to be overstated by about 200 Bscf. Fig. 5 in Ref. 1 (Run 20) shows the concave downward curvature typical for the pressure response of a limited external aquifer system that simulated the reservoir's response.

This type of "limited" aquifer behavior, where pressure in the reservoir and aquifer are virtually equal, led to the derivation of a general material balance for high pressure gas reservoirs (see Appendix, Ref. 2). The derivation includes pressure-dependent rock and water compressibility (with gas evolving from solution). All water and rock volumes associated with the reservoir and available for expansion, including a limited aquifer volume, were included in a cumulative effective compressibility term $\bar{c}_e(p)$. Rock and water compressibilities were defined to account for cumulative changes in volume to be multiplied by the cumulative pressure drop $(p_i - p)$; instantaneous compressibilities are not used at all. The final form of the material balance is similar to that published by Ramagost and Farshad,³ except that they considered \bar{c}_e as a con-

stant. The general gas material balance as presented in this paper defines a cumulative effective compressibility $\bar{c}_e(p)$ as a function of pressure.

Literature Review

Harville and Hawkins⁴ and Hammerlindl⁵ attribute the concave downward shape of p/z vs. G_p curves obtained in abnormally pressured gas reservoirs entirely to pore collapse and formation compaction. No definition of pore collapse is given in Ref. 4, but a plot of backcalculated PV change indicated a system compressibility change from 28×10^{-6} psi⁻¹ at initial pressure to about 6 $\times 10^{-6}$ psi⁻¹ at low pressures. This magnitude of PV change implies associated water volume. The decreasing "system" compressibility is expected for an overpressured reservoir with pressure-dependent PV compressibility, and based on results presented in this paper pore collapse is not a necessary condition for such behavior.

The Anderson "L" reservoir performance presented by Duggan⁶ shows curved p/z vs. G_p field behavior which was primarily attributed to shale water influx with no evidence of reservoir pore compaction. The water influx drive mechanism was supported by the fact that several wells watered out. Wallace⁷ also concluded that shale water influx is an important drive mechanism in abnormally pressured gas reservoirs. Bass⁸ discounts shale water influx, and attributes curved p/z vs. G_p behavior to peripheral water influx from a limited aquifer and formation compaction treated with a constant PV compressibility c_f . For a limited aquifer, Bass defines a term F_p as the ratio of peripheral water PV to the PV of gas-bearing rock.

Roach⁹ and Ramagost and Farshad³ both use the term $p/z[1 - c_e(p_i - p)]$ for geopressured and abnormally pressured gas reservoirs. Both authors consider c_e a constant and they consider the Anderson "L" example.

Bernard¹⁰ does not accept the rock collapse theory as the cause for overpressured p/z vs. G_p behavior, concluding that water influx is the basic drive mechanism. He also uses $p/z[1 - c(p_i - p)]$ where *c* is a "catch-all" term for treating the effects of rock and water compressibility, a small steady-state acting aquifer, and steady state shale water influx. He further states that the term *c* is almost impossible to quantify in terms of reservoir properties.

Begland and Whitehead,¹¹ Prasad and Rogers,¹² and Wang and Teasdale¹³ all present studies of overpressured gas reservoirs based on computer models. Refs. 11 and 12 treat c_f and c_w as functions of pressure, including the effect of solution gas in the water. External water sources are also included in Refs. 12 and 13. The differential forms of the material balance used in these references correctly apply instantaneous compressibility in a history-matching approach to determine initial gas in place. A direct plot of $(p/z)[1 - \bar{c}_e(p_i - p)]$ vs. G_p was not made because the \bar{c}_e term had not been defined.

Poston and Chen¹⁴ analyzed several abnormally pressured gas reservoirs, and recognized that calculated values of $c_e > 30 \times 10^{-6}$ psi⁻¹ required to linearize the material-balance plot reflected the influence of water influx.

Bourgoyne¹⁵ demonstrates that reasonable values of shale permeability and compressibilities treated as a function of pressure can be used to match abnormal gas reservoir performance behavior. He points out, however, that determining k and c_f of the shale necessary for modeling this behavior is practically impossible.

Ambastha¹⁶ uses Bourgoyne's general material-balance equation to develop a graphical matching technique based on a constant effective compressibility c_e . The example given in that paper shows a lack of uniqueness in determining initial gas in place.

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General Material Balance

The general form of the gas material balance is

$$\frac{p}{z} [1 - \bar{c}_e(p)(p_i - p)] = \left(\frac{p}{z}\right)_i - \frac{(p/z)_i}{G} \\ \left[G_p - G_{\text{inj}} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{\text{inj}} B_w - W_e)\right],$$
.....(1)

which reduces to

when water terms and gas injection are neglected. The cumulative effective compressibility term $\bar{c}_e(p)$ is pressure-dependent, consisting of a cumulative PV compressibility $\bar{c}_f(p)$, cumulative total water compressibility $\bar{c}_{tw}(p)$, and the total pore and water volumes associated (i.e., in pressure communication) with the gas reservoir,

$$\bar{c}_e(p) = \frac{S_{wi}\bar{c}_{tw}(p) + \bar{c}_f(p) + M[\bar{c}_{tw}(p) + \bar{c}_f(p)]}{1 - S_{wi}}.$$
 (3)

The formation and total water compressibility terms \bar{c}_f and \bar{c}_{tw} account for cumulative changes in volume from initial pressure to the current pressure.

The interbedded nonpay volume and limited aquifer contributions to pressure support are quantified in terms of the M ratio,

$$M = \frac{V_{pNNP} + V_{pAQ}}{V_{pR}}.$$
(4)

An important aspect of the material balance for high-pressure gas reservoirs is that the gas in solution in the connate and associated water provide both pressure support and additional gas available for production. The level of pressure support provided by the evolved solution gas depends on the level of depletion, and it is shown that this support is significant below about 1,500 psia. The solution gas available for production also depends on the level of depletion, i.e., how much of the original solution gas has evolved $[R_{sw}(p_i) - R_{sw}(p)]$ and the quantity of this gas that is mobile.

The term *G* is used for the initial free gas in place, and it is this quantity that will be determined from the material balance plot given by Eq. 2 when extrapolated to $(p/z)[1 - \bar{c}_e(p_i - p)] = 0$. This condition is reached at a pressure when $1 - \bar{c}_e(p)(p_i - p) = 0$, and not when p = 0, i.e., additional gas may be produced after G_p reaches original free gas in place *G*. At pressures where G_p exceeds *G* the corrected p/z term $(p/z)[1 - \bar{c}_e(p_i - p)]$ becomes negative. If reservoir pressure could be brought to standard conditions $(p = p_{sc})$ the total gas would be *G* plus the total solution gas in place G_s , $(G + G_s)$.

The effect of connate water saturation S_{wi} and M are important to the magnitude of \bar{c}_e . With typical values of $\bar{c}_f = c_{fi} = 4 \times 10^{-6}$ psi⁻¹ and $\bar{c}_{nv} = c_{wi} = 3 \times 10^{-6}$ psi⁻¹ for a high-pressured gulf coast sandstone reservoir, the cumulative effective compressibility is initially $\bar{c}_e = 7.5 \times 10^{-6}$ psi⁻¹ for $S_{wi} = 35\%$ and M = 0; and $\bar{c}_e = 15 \times 10^{-6}$ for $S_{wi} = 35\%$ and M = 1. Fig. 1 shows the percentage of true original free gas in place that would be overestimated by extrapolating early p/z vs. G_p data, indicating that the overestimation is greater for larger initial pressure of 10,000 psia and a $\bar{c}_e = 10 \times 10^{-6}$ psi⁻¹ the extrapolation of early data gives an estimate of G that is about 25% higher than the true original free gas in place. The sections below discuss the calculation of $\bar{c}_f(p)$ and $\bar{c}_{rw}(p)$ functions.

Cumulative PV Compressibility \bar{c}_f . The material balance presented in this paper uses a cumulative PV compressibility \bar{c}_f defined as

$$\bar{c}_f(p) = \frac{1}{V_{pi}} \left[\frac{V_{pi} - V_p(p)}{p_i - p} \right]. \tag{5}$$



Fig. 1 – Effect of p_i and \bar{c}_e on overestimating G.

The term in brackets is the slope of the chord from the initial condition (p_i, V_{pi}) to any lower pressure (p, V_p) , as shown in **Fig. 2**. This implies that \bar{c}_f is a function of both pressure and the initial condition. The instantaneous PV compressibility c_f is defined as

and is only a function of pressure. At initial pressure the two PV compressibilities are equal: $\bar{c}_f(p_i) = c_f(p_i)$. The instantaneous compressibility function $c_f(p)$ should be used in reservoir simulation and differential forms of the material balance, while the cumulative compressibility function $\bar{c}_f(p)$ must be used with forms of the material balance that apply the cumulative pressure drop $(p_i - p)$, i.e., p/z vs. G_p plots.

The pressure dependence of \bar{c}_f is best determined by special core analysis under appropriate reservoir conditions. **Table 1** summarizes the calculation of \bar{c}_f as a function of pressure using laboratory data for a gulf coast sandstone. **Fig. 3** shows how c_f and \bar{c}_f vary as a function of pressure for this overpressured reservoir rock.

In the absence of pore collapse \bar{c}_f is always greater than or equal to c_f . The cumulative PV compressibility remains higher than the instantaneous compressibility because of an averaging effect that reduces the pressure dependence of \bar{c}_f compared with c_f . An important consequence of this behavior is that a rock exhibiting large PV change because of a high level of overpressure will initially have and maintain a high cumulative compressibility \bar{c}_f as shown in Fig. 3.



Fig. 2-Cumulative PV compressibility as a chord slope.

TABLE 1-CALCULATION OF PORE VOLUME COMPRESSIBILITY FROM LABORATORY DATA

Reported Laboratory Data				Calculations for $p_i = 9,800$ psia				
p _o (psia)	V _ρ (cm ³)	V _b (cm ³)	ф (%)	C _f	p (psia)	$p_i - p$ (psi)	$V_{ ho i} - V_{ ho} \ (ext{cm}^3)$	Ē _f Eq. 5
200.0	3.420	20.530	16.70	16.50	9,800	0	0.000	16.50
1,000.0	3.379	20.489	16.49	13.70	9,000	800	0.041	14.99
2,000.0	3.337	20.447	16.32	11.40	8,000	1,800	0.083	13.48
3,000.0	3.303	20.413	16.18	9.10	7,000	2,800	0.117	12.22
4,000.0	3.276	20.386	16.07	6.90	6,000	3,800	0.144	11.08
5,000.0	3.257	20.367	15.99	5.00	5,000	4,800	0.163	9.93
6,000.0	3.243	20.353	15.93	3.80	4,000	5,800	0.177	8.92
7,000.0	3.230	20.340	15.88	4.10	3,000	6,800	0.190	8.17
8,000.0	3.213	20.323	15.81	7.30	2,000	7,800	0.207	7.76
9,000.0	3.177	20.287	15.70	16.80	1,000	8,800	0.243	8.07
9,500.0	3.144	20.254	15.50	25.80	500	9,300	0.276	8.68
All compressib	ilities in 10 ⁻⁶ psi ⁻¹							



Fig. 3—Cumulative and instantaneous c_f vs. p for a sandstone with pore collapse.

Pore collapse is defined as the condition when a rock's instantaneous PV compressibility starts to increase at decreasing reservoir pressure. Pore collapse provides greater pressure support when collapse occurs at a high pressure. However, pore collapse is not reflected by the $\bar{c}_{e}(p)$ function and will not therefore be seen on the p/z vs. G_p plot at the pressure when pore collapse occurs. In fact, pore collapse may not be identifiable at all on the cumulative compressibility term. For example, the gulf coast sandstone in Fig. 3 exhibits pore collapse at 4,000 psia (about 5,000 psi less than initial pressure p_i). Despite the increase in c_f from 4 to 25×10^{-6} psi^{-1} in the pressure range 4,000 to 1,000 psia, the change in \bar{c}_f over the same pressure range is almost insignificant. Fig. 4 shows a North Sea chalk sample from a reservoir with initial pressure of 7,000 psia exhibiting pore collapse at 6,000 psia. Here the effect pore collapse is greater, causing \bar{c}_f to increase from 6 to 20×10^{-6} psi⁻¹ in the pressure range from 6,000 to 2,000 psia. In general, however, pore collapse in and of itself does not have a significant effect on the p/z vs. G_p plot.

In the absence of laboratory data, PV compressibilities can be estimated from correlations presented by Hall¹⁷ and by Von Gonten and Choudhary.¹⁸ Hall's correlation (his Fig. 2) gives instantaneous PV compressibility as a function of porosity, i.e., there is no pressure dependence. Von Gonten develops correlations for instantaneous PV compressibility c_f as a function of net overburden



Fig. 4 – Cumulative and instantaneous c_f vs. p for a chalk with pore collapse.

pressure (p_o) , where p_o equals the overburden gradient times depth minus reservoir pressure.

Cumulative Total Water Compressibility \bar{c}_{tw} . The pressure support provided by water is made up of two components. First, the water expansion with decreasing pressure, and second, the release of solution gas and its expansion. The total or composite compressibility effect is expressed as

in terms of the total water formation volume factor B_{tw} ,

$$B_{tw}(p) = B_{w}(p) + \frac{[R_{swi} - R_{sw}(p)]B_{g}(p)}{5.615}.$$
 (8)

Fig. 5 shows typical behavior for B_w and B_{tw} as a function of pressure; the figure also shows the behavior of $\bar{c}_{tw}(p)$ where it is seen that little increase occurs before a pressure of about 1,500 psia, and that, at pressures below 1,000 psia, there is a significant increase in \bar{c}_{tw} with a limiting relationship $\bar{c}_{tw} \propto 1/p$ at low pressures,



Fig. 5–Cumulative total water compressibility, \bar{c}_{tw} , vs. p.

Specifically at standard conditions (p_{sc}) , \bar{c}_{tw} is given by

$$\bar{c}_{tw}(p_{sc}) = \left[\frac{1}{5.615} \frac{T}{T_{sc}} \frac{R_{swi}}{p_i B_{wti}} - \frac{1}{p_i}\right].$$
 (10)

To calculate \bar{c}_{tw} , values of B_w , R_{sw} , and B_g are tabulated with pressure as shown in **Table 2**. These properties can be obtained from correlations at pressures less than about 10,000 psia and 300°F. At more extreme conditions of pressure and temperature, and for gases with high concentrations of nonhydrocarbons CO₂, N₂, and H₂S, we have used the Peng-Robinson¹⁹ equation of state with volume translation and binary interaction coefficients that are dependent on both temperature and salinity.²⁰

Another approach for high pressures is simply to extrapolate B_w linearly and R_{sw} with a flattening curvature toward a constant value. Nonhydrocarbons can be treated by evaluating R_{sw} of each component separately at its partial pressure, and summing the values for all soluble components,

$$[R_{sw}(p)]_{\text{TOTAL}} = \sum_{j} [R_{sw}(y_{j}p)], \qquad (11)$$

where y_j is the reservoir gas mole fraction of Component *j*. Typically the only components with appreciable solubility are methane, CO₂, and H₂S.

TABLE 2—COMPARISON OF c, FOR NORMAL PRESSURE AND OVERPRESSURED CONDITIONS

Sample	Initial Porosity (%)	Normal Pressure c _{fi} (psi ⁻¹)	Over- Pressured c _{fi} (psi ⁻¹)	
Gulf coast sandstones				
Sample 1	13	4.8	6.4	
Sample 2	20	4.4	16.5	
North Sea chalk				
Sample 9 (pore collapse)	32	18.3	7.9	
Sample 10 (pore collapse)	30	20.1	7.4	
Von Gonten				
Sample 9A	11	3.0	6.0	
Sample 4A	22	4.6	9.2	
Sample 7A	26	5.9	7.2	
Sample 3A	28	8.6	10.6	
Sample 6A	25	7.8	8.6	
Normal Pressured is 0.5 nsi/ft \times Denth: Overpressured is 0.8 nsi/ft \times Denth. Denth				

Normal Pressured is 0.5 psi/ft \times Depth; Overpressured is 0.8 psi/ft \times Depth. Depth Used is 10,000 ft.

Associated Water Volume Ratio *M***.** The total compressibility effect on the gas material balance depends on the magnitudes of rock and total water compressibilities and on the total pore and water volumes in pressure communication with the gas reservoir (including connate water and the PV within the net pay).

Associated water and PVs external to the net pay include nonnet pay (NNP) such as interbedded shales and dirty sands, plus external water volume found in limited aquifers. Including these water volumes in reservoir simulation is referred to as using a "gross" model. In the proposed material balance equations this associated volume is expressed as a ratio relative to the PV of the net pay reservoir,

$$M = M_{NNP} + M_{AQ}, \qquad \dots \qquad (12)$$

where

and

$$M_{AQ} = \frac{V_{pAQ}}{V_{pR}}.$$
 (14)

In the simplest case when M = 0, there will be pressure support only from connate water and the net pay PV. This is equivalent in simulation to building a net model. The cumulative effective compressibility term \bar{c}_e will then be expected to have values ranging from 7 to 15×10^{-6} psi⁻¹ for normal-pressure reservoirs, where the larger values will generally result from high connate water saturation.

Net pay compressibility effects alone can cause noticeable curvature in the p/z vs. G_p plot with potential overestimation of initial free gas in place (G) (see Fig. 1).

 M_{NNP} . The nonnet pay water volume ratio M_{NNP} comprises interbedded reservoir PV, including shales and poor quality rock, that are assumed to be completely filled with water. With this definition M_{NNP} can be written in terms of the net to gross ratio R_{NG} defined as

$$R_{NG} = \frac{h_R}{h_R + h_{NNP}} = \frac{h_R}{h_t}.$$
 (15)

Accounting for different porosities in the net pay and nonnet pay M_{NNP} is given by

Properties and thicknesses of the net pay and nonnet pay are readily available from log analysis.

 M_{AQ} . Aquifers with sufficient permeability and limited areal extent can be treated as part of the total cumulative compressibility term. The water volume ratio of the aquifer M_{AQ} can be determined using geological maps and well control to define areal extent, and electric logs to define the gas/water contact. In general, M_{AQ} is defined as

$$M_{AQ} = \frac{(\phi hA)_{AQ}}{(\phi hA)_{R}}, \qquad (17)$$

and for a radial aquifer geometry quantified in terms of the aquifer to reservoir radius r_{AO}/r_R , the aquifer volume ratio can be expressed

Bruns *et al.*¹ show that limited aquifers with r_{AQ}/r_R ratios up to 5 have the same p/z vs. G_p behavior for permeabilities 100 md and higher. This implies that the transient effects in the aquifer have

negligible effect on reservoir performance and the aquifer can be treated as part of the cumulative effective compressibility term. Values of M_{AQ} used in the definition of \bar{c}_e may be as high as 25, $[M_{AQ} \approx (r_{AQ}/r_R)^2 - 1]$, in reservoirs with moderate permeability. With higher permeabilities, limited aquifers can include r_{AQ}/r_R ratios greater than 5 and still be treated as part of the cumulative effective compressibility term.

When the aquifer is sufficiently large and requires treatment with either superposition or the Schilthius infinite aquifer model, the \bar{c}_e term should still be used, but it will only contain the effect of net pay and nonnet pay volumes; i.e., $M = M_{NNP}$.

Cumulative Effective Compressibility \bar{c}_{e^*} . Total cumulative effective compressibility represents all available pressure support from rock and water. The equation for \bar{c}_{e} is

$$\bar{c}_e(p) = \frac{S_{wi}\bar{c}_{tw}(p) + \bar{c}_f(p) + M[\bar{c}_{tw}(p) + \bar{c}_f(p)]}{1 - S_{wi}}.$$
 (19)

For a specific reservoir a family of $\bar{c}_e(p)$ curves can be generated for several *M* values. These curves will have specific characteristics depending on the pressure dependence of rock and water compressibilities. The $\bar{c}_{tw}(p)$ curves are relatively constant at high pressure, increasing slightly as pressure decreases, then rising sharply at low pressure around 1,000 psia. Typically, a constant PV compressibility c_f can be assumed and the $\bar{c}_e(p)$ curves will then have the same character as the $\bar{c}_{tw}(p)$ curve. **Fig. 6** illustrates an example of $\bar{c}_e(p)$ curves at various *M* ratios for a typical gulf coast reservoir with $p_i = 9,000$ psia, $T = 200^{\circ}$ F, $\gamma_g = 0.7$ (air = 1), and a constant $\bar{c}_f = 3.2 \times 10^{-6}$ psi⁻¹.

For overpressured reservoirs exhibiting a pressure-dependent $\bar{c}_f(p)$, the family of $\bar{c}_e(p)$ curves at high pressures will tend to decrease with depletion. In the absence of pore collapse $\bar{c}_f(p)$ decreases to a constant value at lower pressure and the $\bar{c}_e(p)$ curves at lower pressure are dominated by the increasing $\bar{c}_{tw}(p)$ function. If pore collapse occurs, but not early in depletion, the pore collapse is almost insignificant because the $\bar{c}_f(p)$ function does not start increasing until low pressures because it represents a cumulative PV change, and, when the $\bar{c}_f(p)$ function finally starts to increase it will be masked by the $\bar{c}_{tw}(p)$ function which is increasing as 1/p. **Fig. 7** illustrates this point for a gulf coast overpressured reservoir with $p_i = 9,000$ psia, $T = 300^{\circ}$ F, and $\gamma_g = 0.71$ (air = 1). Although pore collapse occurs at approximately 3,500 psia (Fig. 3), \bar{c}_f does not start increasing until 2,000 psia. The increase is insignificant relative to the increase in $\bar{c}_{tw}(p)$ at lower pressures.

The next example is a North Sea chalk (Fig. 4) that shows pore collapse at a pressure only 1,000 psi below initial pressure of 7,000 psia. The $\bar{c}_f(p)$ function increases almost simultaneously with instantaneous c_f and the effect of $\bar{c}_f(p)$ on $\bar{c}_e(p)$ is shown in **Fig. 8**. Although $\bar{c}_e(p)$ has an impact on $\bar{c}_e(p)$ at moderate and high



Fig. 6 – Cumulative effective compressibility vs. *p* at various *M* ratios.



Fig. 7—Cumulative effective compressibility vs. p for a sandstone w/pore collapse.



Fig. 8—Cumulative effective compressibility vs. p for a chalk w/pore collapse.

pressures for this example, the $\bar{c}_{rw}(p)$ function still dominates the behavior of $\bar{c}_e(p)$ at pressures less than 1,500 psia.

Estimating Gas-in-Place. A method is proposed for estimating the initial (free) gas in place *G* based on historical pressure/ cumulative data. The procedure also determines the water volume ratio *M* and the $\bar{c}_e(p)$ function. First, a plot of p/z versus cumulative gas production G_p should have the characteristic concave downward shape of a high-pressure reservoir influenced by associated water and PV compressibility.

A range of values for G should then be assumed, with the largest value based on an extrapolation of the early depletion data, and the lowest value being somewhat larger than the current G_p . For an assumed value of G, calculate for each measured p/z and G_p data the \bar{c}_e value from the rearranged material balance, Eq. 2,

$$(\bar{c}_e)_{\text{backcalculated}} = \left[1 - \frac{(p/z)_i}{(p/z)} \left(1 - \frac{G_p}{G}\right)\right] \frac{1}{(p_i - p)}.$$
 (20)

At this point, a plot can be made of backcalculated \bar{c}_e as a function of pressure given the assumed *G*. Using reservoir rock and water properties, a family of $\bar{c}_e(p)$ curves at various *M* values can be generated independently to match against the backcalculated \bar{c}_e values. The data should honor the shape and magnitude of one $\bar{c}_e(p)$ curve, where this match yields *G*, the *M* value, and a $\bar{c}_e(p)$ function that can be used to forecast future p/z vs. G_p behavior. This procedure gives a sound physical significance to the estimation of *G* as opposed to a pure statistical best fit that may lead to unrealistic

solutions. The Field Examples section discusses criteria for matching field data, and the expected behavior of $\bar{c}_e(p)$.

Characteristics of p/z vs. G_p Plots for High-Pressure Reservoirs

PV reduction, water expansion, and solution gas evolution, expressed in terms of $\bar{c}_e(p)$ in the general material balance equation, provide pressure support for all reservoirs during depletion. The reservoir does not have to be overpressured or geopressured. The term $\bar{c}_e(p)(p_i - p)$ determines whether the conventional p/z vs. G_p plot yields a straight line. For most low-pressure reservoirs this term is small and is often neglected because a straight-line p/z vs. G_p plot is obtained. Reservoirs undergoing depletion with initial pressure exceeding 5,000 psia are automatically candidates for being treated with the complete material balance equation.

Fig. 9 presents three generated p/z vs. G_p curves for a gulf coast overpressured sandstone reservoir using M = 0 (i.e., $\bar{c}_e(p) = [\bar{c}_f(p)]$ + $S_{wi}\bar{c}_{tw}(p)]/(1 - S_{wi})$. Curve A accounts for PV reduction, including pore collapse at about 4,000 psia. Curve B uses the same $\bar{c}_{\ell}(p)$ function as Curve A down to 4,000 psia (where pore collapse occurs) and thereafter uses a constant instantaneous compressibility of 4 \times 10⁻⁶ psi⁻¹. Plots of p/z vs. G_p for A and B are almost identical, showing only a slight separation at pressures less than 3,500 psia. This clearly shows the limited effect of pore collapse on the p/z vs. G_p plot when collapse occurs late in depletion. Curve C assumes that the initial PV compressibility of 13×10^{-6} psi⁻¹ remains constant throughout depletion. The difference between the two p/z vs. G_p Curves A and C is a result of the actual decrease in PV compressibility. Including an external water volume quantified with M = 2 produces more curvature in the p/z vs. G_p plots, but the separation between curves with and without pore collapse is still very small (not shown).

Another example relates to a North Sea chalk reservoir where pore collapse occurs just below initial pressure. **Fig. 10** presents generated p/z vs. G_p plots for M = 0 with pore collapse (Curve A) and with no pore collapse (Curve B). The effect of pore collapse is more significant than in the previous example because it occurs at a relatively high pressure.

Field Examples

Ellenburger Gas Reservoir. This field example is for a normal pressured (0.5 psi/ft) 1,600-ft-thick, dry gas reservoir with initial reservoir pressure of 6,675 psia at 200°F. Average porosity is about 5% with connate water saturation in the pay of about 35%. Permeability is high because of an extensive microfracture system that results in a high degree of interwell pressure communication and almost instantaneous pressure buildup to static conditions. Initial CO₂ concentration was about 28 mol%, and a gradual increase in CO₂ concentration to 31 mol% at the present time has been



Fig. 9 – Effect on p/z vs. G_p with and without pore collapse, sandstone.



Fig. 10 – Effect on p/z vs. G_p with and without pore collapse, chalk.

observed. The reservoir has produced about 3.1 Tscf, and currently has an average fieldwide bottomhole pressure of approximately 1,000 psia. The p/z vs. G_p plot shows a characteristic concave downward behavior, with an initial gas in place estimate of more than 4.4 Tscf using early data (**Fig. 11**). The p/z vs. G_p data at low pressures has started flattening.

The procedure outlined earlier for determining initial free gas in place *G* was used for this reservoir. **Fig. 12** shows a plot of backcalculated \bar{c}_e vs. pressure for a range of *G* from 3.0 Tscf to 3.6 Tscf. Another plot of $\bar{c}_e(p)$ was generated independently from rock and fluid properties by use of an equation of state for several values of *M* with $S_{wi} = 0.35$, $\bar{c}_f = 6.5 \times 10^{-6}$ psi⁻¹ (from Hall¹⁷), and $\bar{c}_{rw}(p)$. **Fig. 13** shows the best-fit of data on the $\bar{c}_e(p)$ curve for M = 3.3, corresponding to an initial free gas in place G = 3.15 Tscf.

The total water volume including connate and associated waters is given by

$$W = \frac{1}{5.615} \frac{GB_{gi}}{B_{rwi}} \frac{(S_{wi} + M)}{(1 - S_{wi})}, \qquad (21)$$

which yields 8.45(10⁹) STB. The initial solution gas in place G_s is equal to W times the initial solution gas/water ratio R_{swi} ,

$$G_s = WR_{swi}. \qquad (22)$$

Because of the high CO₂ concentration in this reservoir, the solution gas/water ratio ($R_{swi} = 67.5 \text{ scf/STB}$) is about three times larger than for hydrocarbon gas systems. This yields a solution gas in place of $G_s = 0.55$ Tscf and a total initial gas in place of $G + G_s = 3.70$ Tscf. Fig. 11 shows the p/z vs. G_p forecast using the *M* value determined from the match to calculate the $\bar{c}_e(p)$ function from S_{wi} , M, \bar{c}_f and $\bar{c}_{tw}(p)$. Also shown on this figure is the plot of $(p/z)[1 - C_{tw}(p)]$



Fig. 11-Pressure vs. cumulative production, Ellenburger gas reservoir.



Fig. 12—Backcalculated \bar{c}_e vs. p at various original gas in place (OGIP) values, Ellenburger gas reservoir.



Fig. 13—Matching backcalculated \bar{c}_e to generated \bar{c}_e curves, Ellenburger gas reservoir.

 $\bar{c}_e(p)(p_i - p)$] vs. G_p for historical performance data and for the forecast, where it is seen that the current cumulative gas produced equals the original free gas in place.

The associated water volume given by M = 3.3 consists of nonnet pay and an external limited aquifer. Log analysis indicates a net-to-gross ratio $R_{NG} = 0.5$, $\phi_R = 0.05$, and $\phi_{NNP} = 0.03$, yielding $M_{NNP} = 0.6$. External water is known to exist but has not been mapped because of lack of well control. The calculated aquifer water volume ratio $M_{AQ} = 2.7$ (3.3 – 0.6), or an equivalent r_{AQ}/r_R = 1.9, seems reasonable for a limited aquifer.

Anderson "L". This reservoir has been studied by several authors and it is perhaps the best recognized example of a high-pressure gas reservoir with concave downward $p/z - G_p$ behavior (Fig. 14). The reservoir was abandoned after producing 55 Bscf, but pressure tests of public record were discontinued after 40 Bscf had been produced.

Different analyses by other authors have indicated original free gas in place between 65 to 75 Bscf. **Fig. 15** shows backcalculated \bar{c}_e vs. pressure for values of *G* equal to 65, 72, and 90 Bscf. The 72 Bscf volume is chosen based on a best-fit match with the $\bar{c}_e(p)$ function calculated using M = 2.25, $S_{wi} = 0.35$, $\bar{c}_f = 3.2 \times 10^{-6}$ psi⁻¹, and a $\bar{c}_{rw}(p)$ function from equation of state results. Although the first four data do not fall on the slightly increasing $\bar{c}_e(p)$ curve, data at pressures below this value do follow the trend down to the last pressure datum near 3,000 psia.

The 90 Bscf estimate produces unrealistically low \bar{c}_e values, lower than would be calculated using the net reservoir PV and



Fig. 14 -p/z vs. cumulative production, Anderson "L" reservoir.



Fig. 15—Backcalculated \bar{c}_e vs. p at various OGIP, Anderson "L" reservoir.

connate water compressibilities. The lowest estimate of 65 Bscf gives a shape for $\bar{c}_e(p)$ that cannot be accounted for using normal $\bar{c}_t(p)$ and $\bar{c}_{tw}(p)$ functions.

The forecasted p/z vs. G_p performance (Fig. 14) is calculated with the match determined above. Total gas in place of is 76 Bscf, which includes 72 Bscf of original free gas plus 4 Bscf of solution gas.

Conclusions

1. A general form of the material balance equation for gas reservoirs has been presented. This equation has particular application to high-pressure reservoirs. A cumulative effective compressibility term $\bar{c}_e(p)$ has been defined in terms of pressure-dependent PV and total water cumulative compressibilities, $\bar{c}_f(p)$ and $\bar{c}_{tw}(p)$, and the total volume of water associated with the net pay reservoir expressed as a ratio M.

2. The general material balance equation applies to all highpressure reservoirs, both normal pressured and abnormally pressured (overpressured and geopressured).

3. The effect of a limited aquifer can be included as part of the M term for most depletion-type reservoirs. Using the water volume ratio M in the cumulative effective compressibility term, together with normal values of \bar{c}_f and \bar{c}_{nv} , explains the "large" \bar{c}_e values commonly reported for high-pressure gas reservoirs when linearizing the material balance equation. In fact, large values of \bar{c}_e backcalculated from field performance data indicate that associated water influx is a dominant drive mechanism.

4. Only cumulative compressibilities (\bar{c}_f and \bar{c}_{rw}) can be used in the general gas material balance equation because they are applied

against the cumulative pressure drop $(p_i - p)$ in p/z vs. G_p plots. A method is given for calculating cumulative total water and PV compressibility $\bar{c}_{tw}(p)$ and $\bar{c}_t(p)$.

5. A method is proposed for estimating the original free gas in place from production data. This method uses backcalculated cumulative effective compressibility \bar{c}_e which is plotted against pressure and compared with expected $\bar{c}_{e}(p)$ behavior calculated solely from rock and water properties for a range of values of the associated water volume ratio M.

6. Pore collapse, in and of itself, does not contribute significantly to pressure support in overpressured gas reservoirs. In fact, pore collapse has little effect unless it occurs early in depletion at a relatively high pressure. The effect of pore collapse, if present, is a positive effect and tends to flatten the p/z vs. G_p curve, not bending the curve downward as has been implied by others.

7. Gas found initially in solution in the connate and associated water is an important component of pressure support late in depletion (below 1500 psia) and may contribute additional producible volumes of gas. Typically, the solution gas in place G_s represents 2 to 10% of the original free gas in place, the value depending primarily on total water volume $(M + S_{wi})/(1 - S_{wi})$ and the initial solution gas/water ratio R_{swi} . Gas reservoirs with high CO₂ concentration (>20 mol%) can have even higher solution gas in place, G_{s} .

Nomenclature

- $A = \text{area, ft}^2 [\text{m}^2]$
- B = formation volume factor, reservoir per standard volume
- c = instantaneous compressibility, 1/psi [1/kPa]
- \bar{c} = cumulative compressibility, 1/psi [1/kPa]
- G = original free gas-in-place, Bscf [std m³]
- G_p = cumulative gas production, Bscf [std m³]
- $\dot{G_s}$ = initial solution gas in place, Bscf [std m³]
- G_x = early overestimate of G, Bscf [std m³]
- $G_{\rm inj}$ = cumulative gas injection, Bscf [std³]
- $\ddot{h} =$ thickness, ft [m]
- M = volume ratio, dimensionless
- R_{NG} = net to gross ratio, dimensionless
 - p = reservoir pressure, psia [kPa]
 - p_i = initial reservoir pressure, psia [kPa]
- $p_o =$ net overburden pressure, psia [kPa]
- r_R = radius of reservoir, ft [m]
- r_{AQ} = radius of aquifer, ft [m]
- solution gas water ratio, scf/STB [std m³/m³] R_{sw}
- S_{wi} = initial water saturation, fraction T = reservoir temperature, °R [K]
- $V = \text{volume, ft}^3 [\text{m}^3]$
- $V_p = PV, cm^3 and ft^3 [m^3]$
- V_b^r = bulk volume, cm³ [m³]
- W = total water in place, bbl [m³]
- W_e = cumulative water influx, bbl [m³]
- $W_{\rm inj}$ = cumulative water injection, bbl [m³]
- W_p = cumulative water production, bbl [m³]
- z = gas compressibility factor, dimensionless
- ϕ = porosity, fraction

Subscripts

- A = associated water
- AQ = limited aquifer
- e = effective
- f = PV ("formation")
- g = gas
- t = gross interval thickness
- i = initial
- inj = injection
- NNP = nonnet payR = reservoir
 - sc = standard conditions
 - tw = total water
 - w = water

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Appendix A-Derivation of General Gas Material Balance

The derivation that follows is based on the following assumptions: 1. Any pressure change caused by production or injection into the reservoir will be felt immediately throughout the total system including (a) *net pay reservoir* (R); (b) *nonnet pay* (NNP), including interbedded shales and poor quality rock assumed to be 100% water-saturated; and (c) *limited aquifer* (AQ), when present, also assumed to be water-saturated. The nonnet pay and aquifer volumes are referred to as "associated" water volumes and both contribute to water influx during depletion.

2. Simple modifications to the material balance equations can be made to generalize for nonnet pay that has an initial free gas saturation.

3. All water in the system is initially saturated with solution gas.

Practically, the assumption of equal pressure throughout the system is reasonable, and any transient effects caused by a large aquifer may be treated by a conventional water influx term (W_e) as shown below.

For the sake of brevity we have chosen to omit explicit reference to pressure dependence—i.e., \bar{c}_e , \bar{c}_{fi} and \bar{c}_{tw} should actually read $\bar{c}_e(p)$, $\bar{c}_f(p)$, and $\bar{c}_{tw}(p)$.

Derivation. The volumetric balance at any pressure states that the total PV $(V_{pR} + V_{pA})$ equals the net reservoir PV occupied by gas and water $(V_{gR} + V_{wR})$ plus the associated (nonnet pay and aquifer) PV which also is occupied by gas and water $(V_{gA} + V_{wA})$:

$$(V_{pR} + V_{pA}) = (V_{gR} + V_{wR}) + (V_{gA} + V_{wA}).$$
 (A-1)

The net-pay reservoir PV V_{pR} is given by the initial volume V_{pRi} less the change in PV ΔV_{pR} ,

$$V_{pR} = V_{pRi} - \Delta V_{pR}, \qquad (A-2)$$
$$V_{pRi} = V_{gRi} + V_{wRi}, \qquad (A-3)$$

$$V_{pRi} = GB_{gi} + \frac{GB_{gi}}{1 - S_{wi}} S_{wi},$$

and

yielding

$$V_{pR} = GB_{gi} + \frac{GB_{gi}}{1 - S_{wi}} S_{wi} - \frac{GB_{gi}}{1 - S_{wi}} \bar{c}_f(p_i - p). \quad \dots \dots (A-5)$$

PV of the associated rock is given by the initial PV less the change in PV, i.e.,

$$V_{pA} = \frac{GB_{gi}}{1 - S_{wi}} M - \frac{GB_{gi}}{1 - S_{wi}} M \bar{c}_f(p_i - p). \quad \dots \dots \dots \dots \dots (A-6)$$

The net reservoir gas volume is given by the sum of unproduced free gas, gas released from solution, and any injected gas,

$$V_{gR} = (V_{gR})_{\text{Free Gas}}^{\text{Unproduced}} + (V_{gR})_{\text{From Solution}}^{\text{Released}} + (V_{gR})_{\text{Injected}}, \quad \dots \quad (A-7)$$

resulting in

$$V_{gR} = [G - (G_p - W_p R_{sw})]B_g$$

+ $\frac{GB_{gi}}{1 - S_{wi}} \frac{S_{wi}}{B_{wi}} (R_{swi} - R_{sw}) \frac{B_g}{5.615} + G_{inj}B_g.$ (A-8)

pressure/volume/temperature properties B_g and R_{sw} are evaluated at current reservoir pressure. Value G_p for a gas condensate is the wet gas volume calculated by adding separator gas to liquid condensate converted to an equivalent surface gas volume. Also, the two-phase Z-factor must be used to calculated B_g for gas condensate reservoirs. Strictly speaking the cumulative water production term W_p represents "free" water production and not the water condensed out of solution from the produced gas wellstream.

The gas volume in the associated PV is a function of the amount of gas that has come out of solution,

$$V_{gA} = \frac{GB_{gi}}{1 - S_{wi}} M \frac{1}{B_{wi}} (R_{swi} - R_{sw}) B_g \frac{1}{5.615}.$$
 (A-9)

The water volume in the net-pay reservoir equals the unproduced initial water plus injected water plus water encroachment from an external aquifer,

$$V_{wR} = (V_{wR})_{\text{Unproduced}} + (V_{wR})_{\text{Injected}} + [(V_{wR})_{\text{Encroachment}}],$$
.....(A-10)

yielding

$$V_{wR} = \left(\frac{GB_{gi}}{1 - S_{wi}}\frac{S_{wi}}{B_{wi}}B_w - \frac{W_p B_w}{5.615}\right) + 5.615W_{\text{inj}}B_w + 5.615W.$$
(A-11)

The aquifer encroachment term W_e represents any external water volume that is not already included in the M term. Later in the derivation, we show the conditions required so that water encroachment (treated rigorously by the method of superposition) can be included as part of the M term used in the cumulative effective compressibility \bar{c}_e .

The water volume in the associated PV is given by simple expansion,

$$V_{wA} = \frac{GB_{gi}}{1 - S_{wi}} M \frac{1}{B_{wi}} B_w.$$
 (A-12)

Inserting the appropriate equations above in Eq. A-1, rearranging, and grouping terms yields,

$$\begin{split} G(B_g - B_{gi}) &+ \frac{GB_{gi}}{1 - S_{wi}} \bigg\{ S_{wi} \bigg[\frac{(B_w + ((R_{swi} - R_{sw})B_g)/5.615)}{B_{wi}} - \frac{B_{wi}}{B_{wi}} \bigg] \\ &+ \bar{c}_f(p_i - p) + M \bigg[\frac{(B_w + ((R_{swi} - R_{sw})B_g)/5.615)}{B_{wi}} - \frac{B_{wi}}{B_{wi}} \bigg] \\ &+ M \bar{c}_f(p_i - p) \bigg\} \end{split}$$

$$= (G_p - W_p R_{sw} - G_{inj}) B_g + 5.615 \left(W_p - W_{inj} - \frac{W_e}{B_w} \right) B_w.$$
.....(A-13)

Defining the total water/gas formation volume factor B_{tw} ,

$$B_{tw} \equiv B_w + \frac{(R_{swi} - R_{sw})B_g}{5.615}.$$
 (A-14)

Noting that $B_{iwi} = B_{wi}$, and defining the cumulative total water/gas compressibility \bar{c}_{iw} ,

Now, defining a cumulative effective compressibility \bar{c}_e ,

gives

$$G(B_{g} - B_{gi}) + GB_{gi}[\bar{c}_{e}(p_{i} - p)]$$

= $B_{g}\left[G_{p} - G_{inj} + W_{p}R_{sw} + \frac{5.615}{B_{g}}(W_{p}B_{w} - W_{inj}B_{w} - W_{e})\right].$
.....(A-17)

Dividing through by GB_{gi} and expressing $B_g = (p_{sc}/T_{sc})(zT/p)$ gives the final form of the material balance

$$\frac{p}{z} [1 - \bar{c}_e(p_i - p)] = \left(\frac{p}{z}\right)_i \left\{ 1 - \frac{1}{G} \left[G_p - G_{\text{inj}} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{\text{inj}} B_w - W_e) \right] \right\}.$$
(A-18)

The p/z vs. cumulative plot, including all terms, would consider $(p/z)[1 - \bar{c}_e(p_i - p)]$ vs. the entire production/injection/encroachment term Q

with

$$Q = G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e),$$
.....(A-20)

where the intercept is given by $(p/z)_i$ and the slope equals $(p/z)_i/G$. Setting $G_{inj} = W_{inj} = W_p = W_e = 0$ gives the common form of the gas material balance,

Treating Limited Aquifers in \bar{c}_e **Term.** The material balance thus far has considered any associated water volume expressed in terms of the *M* parameter. In fact *M* may include a limited aquifer with up to 25 times the reservoir PV for a system permeability greater than about 100 md, and even larger aquifer volumes for higher permeabilities. The condition that determines when a limited aquifer can be treated as part of the \bar{c}_e term is outlined below. We start with the general material-balance equation including a water encroachment term W_e and a \bar{c}_e term that considers only nonnet pay.

$$\frac{p}{z}[1 - \bar{c}_e(p_i - p)] = \left(\frac{p}{z}\right)_i \left(1 - \frac{G_p}{G} + 5.615 \frac{W_e}{GB_g}\right) \dots (A-22)_i$$

and

$$\bar{c}_e = \frac{S_{wi}\bar{c}_{tw} + \bar{c}_f + (V_{pNNP}/V_{pR})(\bar{c}_{tw} + \bar{c}_f)}{1 - S_{wi}}.$$
 (A-23)

The water encroachment term calculated by superposition is expressed,

$$W_e = B \sum_j Q_D(\Delta t_j)_D \Delta p_j, \quad \dots \quad (A-24)$$

where $Q_D(t_D)$ is the dimensionless cumulative influx given as a function of dimensionless time t_D and aquifer to reservoir radius $r_D = r_{AQ}/r_R$. Value Δp_j is given by $p_j - p_{j-1}$ (in the limit for small time steps), and $\Delta t_j = t - t_{j-1}$. Assuming that permeability is reasonably high and the ratio r_{AQ}/r_R is not too large, Q_D for the smallest time step will approach the limiting value Q_D^{∞} , and the

summation can be closely approximated by

$$\sum_{j} Q_D(\Delta t_j)_D \Delta p_j \approx Q_D^{\infty}(p_i - p), \quad \dots \quad \dots \quad (A-25)$$

giving a simple expression for W_e that is independent of time and only dependent on reservoir pressure,

$$W_e = BQ_D^{\infty}(p_i - p); \quad W_e(\text{bbl}) \dots \dots \dots \dots \dots (A-26)$$

$$B = \frac{2\pi}{5.615} \phi r_{R}^{2} h(\bar{c}_{tw} + \bar{c}_{f}),$$

$$Q_{D}^{\infty} = \frac{1}{2} \left[\left(\frac{r_{AQ}}{r_{R}} \right)^{2} - 1 \right]. \qquad (A-27)$$

Expressing W_e in terms of aquifer PV V_{pAQ} ,

$$W_{e} = \pi (r_{AQ}^{2} - r_{R}^{2})\phi h(\bar{c}_{tw} + \bar{c}_{f})(p_{i} - p);$$

and

$$W_e(\mathrm{ft}^3) = V_{pAQ}(\bar{c}_{tw} + \bar{c}_f)(p_i - p). \quad \dots \quad \dots \quad (A-28)$$

The material-balance equation can then be written

$$\frac{p}{z}[1-\bar{c}_e(p_i-p)] = \left(\frac{p}{z}\right)_i \left(1-\frac{G_p}{G}\right) + \left(\frac{p}{z}\right)_i \frac{W_e}{GB_g} 5.615$$
....(A-29)

and simplified in a form where the \bar{c}_e term includes the aquifer contribution to pressure support,

Rearranging, we arrive at the general form of the material balance (without water production and gas/water injection terms):

where

$$\bar{c}_{e} = \frac{S_{wi}\bar{c}_{w} + \bar{c}_{f} + ((V_{pNNP}/V_{pR}) + (V_{pAQ}/V_{pR}))(\bar{c}_{w} + \bar{c}_{f})}{1 - S_{wi}}$$
.....(A-32)

and

$$\bar{c}_e = \frac{S_{wi}\bar{c}_{rw} + \bar{c}_f + M(\bar{c}_{rw} + \bar{c}_f)}{(1 - S_{wi})}.$$
 (A-34)

SI Metric Conversion Factors

°F (°F-32)/1.8	= °C	
$in.^{3} \times 1.638~706$	$E + 01 = cm^3$	
$ft^3 \times 2.831685$	$E - 02 = m^3$	
psi × 6.894 757	E + 00 = kPa	
1		SPEJ

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Reese



Fetkovich

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Discussion of Application of Material Balance for High-Pressure Gas Reservoirs

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Introduction

I enjoyed reading "Application of a General Material Balance for High-Pressure Gas Reservoirs" by Fetkovich *et al.*¹ They did an excellent job of establishing the limitations of conventional isothermal compressibilities in material-balance calculations and the need to account for the pressure-dependent effects of rock and water compressibilities in high-pressure gas reservoirs. They addressed this limitation by introducing a new, pressure-dependent parameter, namely, the cumulative compressibility. They ultimately developed a viable method, centered around the new cumulative compressibility, to estimate effectively the original gas in place (OGIP) in gas reservoirs where the effects of rock and water compressibility are obviously important.

I compliment the authors on their interest in this problem and their ingenuity. My associates and I, too, have examined this problem; however, we routinely use a slightly different solution technique. Our method is analogous to the popular work of Havlena and Odeh² and, attractively, does not require the use of any isothermal compressibilities. Those familiar with the work of Havlena and Odeh will find our development straightforward, tractable, easy to implement, and possibly preferable.

Mathematical Development

Our approach is based on the linearized form of the applicable material-balance equation,

$$F = G_{fgi}E_g + WE_w + V_pE_f + W_e. \qquad (D-1)$$

Our nomenclature purposely follows Havlena and Odeh's² to minimize the introduction of new variables and terminology. The rock expansivity, E_f , is expressed in units of pore volume (PV) change per unit PV. For the case of a strictly undersaturated gas in a system containing gas-saturated water, F is given by

The expansivities are measured directly from laboratory expansion tests or can be evaluated from the following expressions:

 $E_g = B_g - B_{gi}, \qquad (D-3)$

$$E_w = B_{tw} - B_{twi}, \qquad \dots \qquad (D-4)$$

and
$$E_f = \frac{V_{pi} - V_p}{V_{ni}}$$
,(D-5)

where Subscript *i* denotes the initial condition. The expansivities are, of course, functions of pressure. More general expressions of Eqs. D-1 through D-3 can be found elsewhere.³⁻⁷

 B_{tw} in Eq. D-4 is the two-phase water/gas formation volume factor (FVF) and is related to B_w and R_{sw} by

$$B_{tw} = B_w + B_g(R_{swi} - R_{sw}). \qquad \dots \qquad (D-6)$$

Eqs. D-4 and D-6 provide a convenient and reliable means to estimate E_w and B_{nw} without direct measurement because B_w and R_{sw} can be reliably obtained from correlation and B_g is invariably known.

If laboratory rock compressibility tests are conducted, E_f is evaluated from Eq. D-5. The quantity $(V_{pi} - V_p)/V_{pi}$ is invariably available from such tests, as illustrated by Fetkovich *et al.*¹ As a

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last resort, E_f can be approximated from

$$E_f = c_f(p_i - p), \quad \dots \quad (D-7)$$

where c_j is the rock compressibility that is obtained from correlation and is treated as a constant. Application of Eq. D-7 is not preferable, as Fetkovich *et al.* point out, inasmuch as it ignores the pressure dependence of the rock isothermal compressibility.

Notice that no isothermal compressibilities—neither instantaneous nor cumulative—are needed in our development. Instantaneous compressibilities are avoided because they limit the generality. Cumulative compressibilities are not needed because the more popular two-phase FVF or expansivity can be used alternatively.

Eq. D-1 requires slight manipulation before it can be cast into its final form. The system PV is

$$V_p = G_{fgi}B_{gi} + WB_{wi}.$$
 (D-8)

W is related to G_{fgi} and M by

where *M* is V_{pA}/V_{pR} . Eq. D-9 assumes the system is divided into reservoir and associated segments, the associated PV is saturated with water, and the reservoir PV contains gas and water at an average water saturation of S_{wi} . Substituting Eqs. D-8 and D-9 into Eq. D-1 yields

$$F = G_{fei}E_t + W_e, \qquad (D-10)$$

where E_t is the total expansivity defined by

$$E_{t} = E_{g} + E_{w} \left[\frac{B_{gi}(S_{wi} + M)}{B_{wi}(1 - S_{wi})} \right] + E_{f} \left[\frac{B_{gi}(1 + M)}{(1 - S_{wi})} \right]. \quad \dots \dots (D-11)$$

Eq. D-10 agrees with Eq. A-13 in the authors' paper if Eqs. D-3, D-6, and D-7 apply and c_f is replaced by \bar{c}_f . In the absence of water encroachment from outside the system, Eq. D-10 reveals that a plot of *F* vs. E_t yields a straight line that emanates from the origin and whose slope is G_{fgi} . The *F*-vs.- E_t plot is analogous to the *F*-vs.- (mE_gB_{if}/B_{gi}) plot described by Havlena and Odeh.² Once G_{fgi} is determined, *W* is given by Eq. D-9 and the total OGIP is

$$G = G_{fgi} + WR_{swi}, \qquad (D-12)$$

where the product WR_{swi} represents the solution gas, G_s . [Note that we use G and G_{fgi} to denote the OGIP and original free gas in place (GIP), respectively, whereas Fetkovich *et al.* use G to denote the original free GIP.]

Because material balance demands a straight line, this requirement yields a very simple criterion to determine M; namely, M = the value needed to obtain a straight line. This solution procedure is completely analogous to the method used by Havlena and Odeh to determine the gas-cap size in initially saturated oil reservoirs. M affects the shape of the F-vs.- E_t plot because E_t is a weak function of M (see Eq. D-11). Fig. 1 schematically shows how M affects the shape of a F-vs.- E_t plot. If M is too low, the data curve upward; if M is too high, the data curve downward. The accuracy of the routine can be enhanced by measuring the departure from linearity in terms of standard linear regression quantities, such as the correlation coefficient, standard error, or root-mean-square error. Our experience with this routine reveals it to be fast, reliable, and unambiguous.

Incidentally, it may appear that there is an even better way to solve this problem upon close inspection of Eq. D-1 and the



Fig. D-1-Effect of *M* on shape of *F*-vs.-*E*_t plot.

supplemental equations. Substituting Eq. D-8 into Eq. D-1, assuming no water encroachment, and rearranging yields

$$\frac{F}{(E_w + B_{wi}E_f)} = G_{fgi}\frac{(E_g + B_{gi}E_f)}{(E_w + B_{wi}E_f)} + W.$$
 (D-13)

This equation reveals that a plot of $F/(E_w + B_{wi}E_f)$ vs. $(E_g + B_{gi}E_f)/(E_w + B_{wi}E_f)$ yields a straight line whose slope is G_{fgi} and y intercept is W. This is obviously a superior approach because it solves for G_{fgi} and W simultaneously. This approach is analogous to Havlena and Odeh's plot of F/E_o vs. E_g/E_o to determine OGIP and original oil in place in a gas-cap reservoir simultaneously. This was how we initially approached this problem. Unfortunately, because the net expansion of the initial free gas phase, this method is not always reliable.

We have also successfully solved Eq. D-10 directly with multiple regression analysis. This approach is equivalent to finding the best plane through the data points in three-dimensional space, where the axes (x, y, z) are defined by $\{E_g + E_w B_{gl} S_{wl}/[B_{wl}(1 - S_{wl})] + E_{l} B_{gl}/(1 - S_{wl})\}, \{E_w B_{gl}/[B_{wl}(1 - S_{wl})] + E_{l} B_{gl}/(1 - S_{wl})\}, and F, respectively. This more sophisticated approach allows us to solve for$ *M* $and <math>G_{fgl}$ simultaneously. Indeed, this approach is mathematically preferable; however, it departs from the popular straight-line techniques of Havlena and Odeh. Also, it is more complicated, requires a more lengthy mathematical development, and does not appreciably improve the accuracy of the OGIP estimates. Accordingly, its presentation is purposely omitted.

Example

To illustrate our method and to make direct comparison with the authors' work, we purposely consider the Anderson "L" reservoir as an example. This example was studied by the authors. Fig. 2 shows the plot of F vs. E_t . E_g and E_w were evaluated from B_g , B_w , and R_{sw} data given by Fetkovich *et al.*⁸ E_f was computed with Eq. D-7 with $c_f = 3.2 \times 10^{-6} \text{ psi}^{-1}$, as assumed by the authors. We purposely used the same data as the authors so that a direct comparison of the two methods could be made. The straightest line was obtained with M = 0.51. A least-squares fit of the data yielded a line with a slope of 73.1 Bcf. The least-squares line is included in Fig. 2. **Table 1** summarizes the results. The comparison in Table 1 reveals that both methods yield very nearly identical results, except our method estimates less associated water than the authors'. Most importantly, the OGIP estimates are very close (74.6 vs. 76 Bcf). We suspect our method may be slightly more accurate because it is easier to find the straightest line while varying only M than to find the best match of two \bar{c}_e -p curves—one of which is a function of M and the other of which is a function of G—while varying both M and G simultaneously. Simply put, the authors'



Fig. D-2-F-vs.- E_t plot for Anderson L reservoir.

TABLE D-1-COMPARISON OF RESULTS				
	Walsh	Fetkovich, <i>et al.</i>		
G _{fgi} , Bcf	73.1	72		
G _s , Bcf	1.5	4		
G, Bcf	74.6	76		
W, MMSTB	46.0	137		
Associated water, MMSTB	27.3	118		
Μ	0.51	2.25		

method involves a trial and error optimization of two variables, whereas our method requires optimization of only one variable. Convergence is less ambiguous in our case. Notice, for instance, how well the straight line matches the data in Fig. 2, and then notice the scatter of the open circles with the curve in Fig. 15 of the authors' work. Because the goal is to estimate the OGIP, we do not consider this difference a material shortcoming in the authors' work. The linearity of the data in Fig. 2 gives us confidence that the model is applicable and the OGIP estimate is good.

Summary

In summary, the advantages of our method are as follows:

1. No range of OGIP estimates needs to be presumed.

2. No family of backcalculated \bar{c}_{e} -*p* curves as a function of the OGIP needs to be computed.

3. No family of \bar{c}_e -*p* curves as a function of *M* needs to be computed.

4. No subjective matching of \bar{c}_e -p curves is necessary.

5. It is completely analogous to the popular work of Havlena and Odeh.

6. Only one plot (*F* vs. E_t) instead of two plots (p/z vs. G_p and \overline{c}_e vs. p) is required.

Another advantage of our method is that its extension to gascondensate systems is trivial if one adopts more general definitions of *F* and E_g (see Walsh *et al.*^{3,4} and Walsh⁵⁻⁷).

Although we believe our method is simpler, we openly admit that there is nothing wrong with the method of Fetkovich *et al.* As our example illustrates, both methods yield comparable OGIP estimates; thus, the choice is purely a matter of preference.

Nomenclature

 B_g = gas FVF, res bbl/1,000 scf

- B_{tw} = two-phase water/gas FVF, res bbl/STB
- B_w = water FVF, res bbl/STB

- \bar{c}_e = cumulative effective isothermal compressibility, Lt²/m, psi⁻¹
- c_f = instantaneous rock (formation) isothermal compressibility, Lt²/m, psi⁻¹
- \bar{c}_f = cumulative rock (formation) isothermal compressibility, Lt²/m, psi⁻¹
- E_f = rock (formation) net expansivity, res bbl/res bbl
- E_g = gas phase net expansivity, res bbl/1,000 scf
- $\vec{E_t}$ = total net expansivity, res bbl/1,000 scf
- E_w = water phase net expansivity, res bbl/STB
- F = total fluid withdrawal, L³, res bbl
- $G = \text{OGIP}, L^3, 1,000 \text{ scf}$
- G_{fgi} = original free GIP, L³, 1,000 scf
- G_p = cumulative net gas produced, L³, 1,000 scf
- G_s = original dissolved GIP, L³, 1,000 scf
- M = ratio of associated and reservoir PV's, dimensionless
- $p = \text{pressure, m/Lt}^2$, psia
- R_{sw} = dissolved gas/water ratio, 1,000 scf/STB
- S_{wi} = reservoir initial water saturation, dimensionless
- V_p = system (reservoir + associated) PV, L³, res bbl
- V_{pA} = associated PV, L³, res bbl
- V_{pR} = reservoir PV, L³, res bbl
- W = original water in place, L³, STB
- W_e = encroached water, L³, res bbl
- W_p = cumulative net water produced, L³, STB

Subscripts

- A = associated
- f = formation (rock)
- g = gas
- i = initial condition
- R = reservoir
- w = water

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SI Metric Conversion Factors

bbl $ imes$ 1.589 873	$E - 01 = m^3$
$ft^3 \times 2.831685$	$E - 02 = m^3$
psi × 6.894 757	E + 00 = kPa

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