Traveltime approximations for a layered transversely isotropic medium

B.Ursin and A. Stovas, NTNU Dept.Petroleum Eng. and Applied Geophysics,Trondheim, Norway*

Summary

To derive a new fractional approximation for traveltime squared for a layered transversely isotropic medium with vertical symmetry axis we use the Taylor series for the phase velocity in terms of the horizontal slowness.

Introduction

The standard hyperbolic approximation (Dix, 1955) of the P-wave reflection traveltime commonly used in seismic data processing is correct for a homogeneous isotropic medium and a plane reflector. Taner and Koehler (1969) derived a Taylor series approximation of traveltime squared as function of offset (horizontal source-receiver separation) for a stack of horizontal layers from traveltime and offset expressed by the horizontal slowness or ray parameter. The standard hyperbolic approximation, expressed by the zero-offset traveltime and the RMS- or NMO-velocity is not valid for large offset. The shifted hyperbola (Malovichko, 1978; Castle, 1988, 1994; de Bazelaire, 1988) is an approximation which uses a third parameter, a heterogeneity coefficient, to improve the accuracy at larger offsets.

For a homogeneous transversely isotropic medium with vertical symmetry axis (a VTI medium), the hyperbolic approximation is valid only for small offsets and the velocity is NMO-velocity which is different from the vertical velocity (Thomsen, 1986). Tsvankin and Thomsen (1994) showed numerically that the fourth-order Taylor series of reflection traveltime squared rapidly looses accuracy with increasing offset, and they proposed a non-hyperbolic fractional approximation with better numerical performance.

Here we consider multiple converted and reflected qP-qSV waves or multiple reflected SH waves in a horizontally layered medium considering of isotropic and VTI layers where the source and receiver are not necessarily in the same layer. The traveltime and offset (horizontal source-receiver separation) are first expressed in terms of the group velocities and group angles, the angles the rays/group velocity vectors) make with the vertical axis. Using results from Ursin and Hokstad (2003) these are next expressed in terms of the phase velocities and phase angles. The phase slownesses are expanded in Taylor series in terms of the horizontal slowness or ray parameter (Stovas and Ursin, 2003). For a stack of isotropic layers traveltime and offset are functions of horizontal slowness with coefficients that depend on the velocity moments of the layered medium. We show that exactly the same structure can be obtained for a stack of VTI layers where the coefficients are computed recursively from the vertical velocities and anisotropy parameters without assuming weak anisotropy. From this we can directly use the results from Taner and Koehler (1969) or Hubral and Krey (1980) to write the Taylor series for traveltime squared in terms of even powers of offset. The NMO-velocities are as given by Thomsen (1986) and Tsvankin and Thomsen (1994) but the higher-order coefficients are different. Similarly we get the shifted hyperbolic approximation as in Castle (1994) but with coefficients generalized for a layered VTI medium. The higher-order terms in the Taylor series are used to derive a new fractional approximation for traveltime squared which is correct up to the sixth power in offset. By dropping some terms this approximation reduces to the one proposed by Stovas and Ursin (2004). This simplified fractional approximation and the shifted hyperbola are both expressed by the same three parameters: the zero-offset traveltime, the NMO-velocity and a heterogeneity coefficient.

The different traveltime approximations are compared in a numerical example using a model from Ursin and Hokstad (2003).

Taylor series approximation

We consider wave propagation in a stack of homogeneous VTI. For a multiple transmitted and reflected SH-wave or multiple transmitted, reflected and converted qP-qSV-wave, the traveltime is

$$T = \sum_k \frac{\Delta z_k}{V_k \cos\alpha_k} = \sum_k \frac{\Delta z_k}{v_k \cos\theta_c}\left(1 + \frac{p}{v_k} v_k'\right), \quad (1)$$

where the sum is taken over each layer as the wave passes though. For layer number k: Δz_k = layer thickness, V_k = group velocity, v_k = phase velocity, α_k = group angle (ray angle), θ_k = phase angle, $p = \sin\theta_k / v_k$ = horizontal slowness (ray parameter) and $v_k' = dv_k/dp$ = derivative of the phase velocity.

The offset, or horizontal distance between source and receiver, is given by

$$x = \sum_k \Delta z_k \tan\alpha_k = \sum_k \frac{v_k \Delta z_k}{\cos\theta_k}\left(p + \frac{v_k'}{v_k^3}\right), \quad (2)$$

We express the phase velocity by

$$\frac{1}{v_k^2} = \frac{1}{v_{0,k}^2} - p^2 S_k, \quad (3)$$

where $v_{0,k}$ is the vertical velocity in layer number k.

Next we expand the function $1 + S_k$ in Taylor series

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$$1+S_k=\sum_{j=0}^{\infty}a_{j,k}\left(pv_{0,k}\right)^{2j} \tag{4}$$

For example, for qP-wave: $v_{0,k}=\alpha_{0,k}$, $a_{0,k}=1+2\delta_k$, $a_{1,k}=2(\varepsilon_k-\delta_k)\left[1+2\delta_k\gamma_{0,k}^2/\left(\gamma_{0,k}^2-1\right)\right]$, $\gamma_{0,k}=\alpha_{0,k}/\beta_{0,k}$. That gives

$$T=\sum_k\frac{\Delta z_k}{v_{0,k}}\frac{1+\sum_{j=0}^{\infty}ja_{j,k}\left(pv_{0,k}\right)^{2j+2}}{\sqrt{1-\sum_{j=0}^{\infty}a_{j,k}\left(pv_{0,k}\right)^{2j+2}}}, \tag{5}$$

and

$$x=p\sum_k\Delta z_k v_{0,k}\frac{\sum_{j=0}^{\infty}(1+j)a_{j,k}\left(pv_{0,k}\right)^{2j}}{\sqrt{1-\sum_{j=0}^{\infty}a_{j,k}\left(pv_{0,k}\right)^{2j+2}}}, \tag{6}$$

By eliminating the square roots in equation (5) and (6) and expanding in Taylor series, we can write

$$T(p)=T(0)\sum_{n=0}^{\infty}b_n\mu_{2n}p^{2n} \tag{7}$$

and

$$x(p)=pT(0)\sum_{n=0}^{\infty}b_n\mu_{2n+2}p^{2n} \tag{8}$$

Here

$$b_n=\begin{cases}1, & n=0\\ \dfrac{1*3*\ldots*(2n-1)}{2^n n!}, & n=1,2,\ldots\end{cases}, \tag{9}$$

$$T(0)=\sum_k\frac{\Delta z_k}{v_{0k}}=\sum_k t_{0,k} \tag{10}$$

with $t_{0,k}$ being vertical one-way traveltime in each layer as the ray passes through, and

$$\begin{aligned}
\mu_0&=1\\
\mu_2&=\frac{1}{T(0)}\sum_k v_{0,k}^2t_{0,k}a_{0,k}=v_{NMO}^2\\
\mu_4&=\frac{1}{T(0)}\sum_k v_{0,k}^4t_{0,k}\left[a_{0,k}^2+4a_{1,k}\right]\\
\mu_6&=\frac{1}{T(0)}\sum_k v_{0,k}^6t_{0,k}\left[a_{0,k}^3+4a_{1,k}a_{0,k}+8a_{2,k}\right]
\end{aligned} \tag{11}$$

These parameters we can compute from

$$\mu_{2n}=\frac{1}{T(0)}\sum_k t_{0,k}\mu_{2n,k} \tag{12}$$

where a recursive scheme for computing $\mu_{2n+2,k}$ from the previous $\mu_{2n,k}$ values and the Taylor coefficients $a_{j,k}$ depending on the layer parameters given by

$$\begin{aligned}
&\mu_{2,k}=a_{0,k}v_{0,k}^2,\quad \text{for } n=0,\\
&b_n\mu_{2n+2,k}=a_{n,k}(n+1)v_{0,k}^{2n+2}\\
&+\sum_{j=0}^{n-1}a_{j,k}v_{0,k}^{2j+2}\mu_{2n-2j,k}\left[(j+1)b_{n-j}-jb_{n-j-1}\right],\quad \text{for } n=1,2,\ldots
\end{aligned} \tag{13}$$

with $\mu_{0,k}=1$. The Taylor series for $T(p)$ and $x(p)$ in equation (7) and (8) are of exactly the same form as for a stack of homogeneous isotropic layers. We can therefore directly write (Taner and Koehler, 1969; Hubral and Krey, 1980)

$$T(x)^2=T(0)^2+\frac{x^2}{v_{NMO}^2}+c_2x^4+c_3x^6+\ldots \tag{14}$$

where

$$\begin{aligned}
c_2&=\frac{1}{4T(0)^2}\frac{\mu_2^2-\mu_4}{\mu_2^4}\\
c_3&=\frac{1}{8T(0)^4}\frac{2\mu_4^2-\mu_2\mu_6-\mu_2^2\mu_4}{\mu_2^7}
\end{aligned} \tag{15}$$

The shifted hyperbola

The standard hyperbolic traveltime approximation (Dix, 1955) is obtained by only using two terms in the Taylor series in equation (14). An improved approximation is the shifted hyperbola (de Bazelaire, 1988; Castle, 1988)

$$T(x)=T(0)+\frac{T(0)}{S}\left[\sqrt{1+\frac{x^2S}{T(0)^2v_{NMO}^2}}-1\right] \tag{16}$$

where

$$S=\frac{\mu_4}{\mu_2^2} \tag{17}$$

For a stack of VTI layers we use the same formula, with the new definition of μ_2 and μ_4 given in equation (11).

Fractional approximations

Tsvankin and Thomsen (1994) proposed a fractional traveltime approximation for VTI media. A new approximation, which is correct up to the order x^6, is the following

$$T(x)^2 = T(0)^2 + \frac{x^2}{v_{NMO}^2} + \frac{c_2 x^4}{1+Bx^2} \qquad (18)$$

with

$$B = -\frac{c_3}{c_2} = -\frac{1}{2T(0)^2}\frac{2\mu_4^2 - \mu_2\mu_6 - \mu_2^2\mu_4}{\mu_2^3\left(\mu_2^2 - \mu_4\right)} \qquad (19)$$

Assuming μ_4 to be large, and dropping other terms, we obtain

$$B \approx \frac{1}{T(0)^2}\frac{\mu_4}{\mu_2^3} \qquad (20)$$

Stovas and Ursin (2004) have proposed an approximation for a single VTI layer which can be extended to the multi-layer case as

$$T(x)^2 = T(0)^2 + \frac{x^2}{v_{NMO}^2} - \frac{Gx^4}{v_{NMO}^4\left[T(0)^2 + \frac{x^2}{v_{NMO}^2}(1+4G)\right]} \qquad (21)$$

where G now is given by

$$G = \frac{1}{4}\left[\frac{\mu_4}{\mu_2^2} - 1\right] = \frac{1}{4}[S-1] \qquad (22)$$

It can be verified that the approximation in equation (21) is the same as the one in equation (18) with the approximate value of B given by equation (20).

Numerical examples

We compare all approximations for the layered VTI model from Ursin and Hokstad (2003) including 13 layers. Figure 1 and 2 show the absolute value of the traveltime errors for the different approximations for the depth 0.75km and 2.24km, respectively. It is seen that in all cases (with the exception of SVSV reflection in Figure 2), the fractional approximation (18), (19) has the best performance. The fractional approximation (18), (20) and the shifted hyperbola approximation (16) give similar results.

Conclusions

We have derived new approximations for the PP, SVSV, PS (SP) and SHSH reflection traveltimes for a layered VTI media not using the weak-anisotropy assumption. The comparison made for the layered model shows that the fractional approximation is more accurate.

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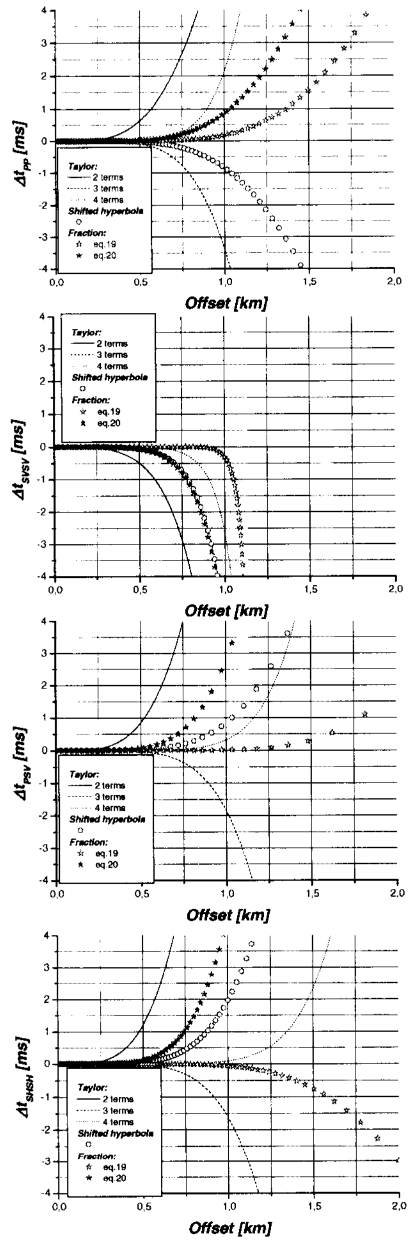

Figure 1. Traveltime errors for reflections from the bottom of layer 5 (at 0.75 km).

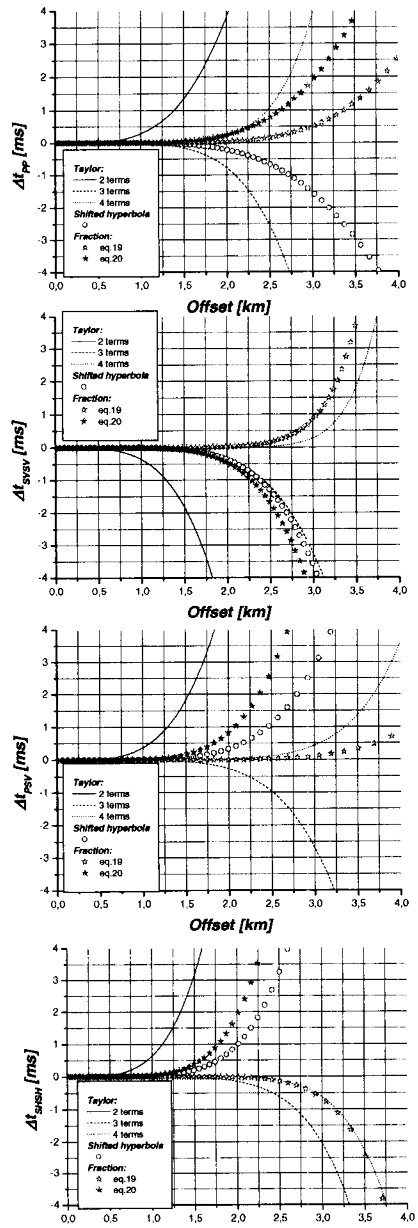

Figure 2. Traveltime errors for reflections from the bottom of layer 13 (at 2.24 km).