Optimum seismic signal estimation with complicated models of coherent and random noise

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Summary

A method is described for optimally estimating seismic signal with a complicated, more realistic as compared with the conventional one, mathematical model of the multichannel record. The method is based on the Wiener criterion and related multichannel filter. The signal is supposed to be contaminated by random noise and an arbitrary number of coherent noise wavetrains. The signal and coherent noise wavetrains bear individual trace-independent waveforms, whereas their amplitudes and arrival times vary from trace to trace in arbitrary manner. Random noise is assumed to be a stationary zero-mean Gaussian stochastic process uncorrelated in space and with the same to within a scale factor, the variance, autocorrelation function on different channels. The signal and coherent noise waveforms are statistically independent zero-mean stationary stochastic processes and hence uncorrelated with each other and with random noise. For the method to be feasible, two essentially different approaches, which are based on the same record model, have been suggested to determine the necessary signal and noise parameters. The effectiveness of a simplified version of the method in subtracting severe coherent noise is demonstrated with field data.

Introduction

In the 1960s-1970s, the problem of optimally estimating the seismic signal contaminated by coherent and random noise attracted much attention in the literature. For that purpose, the majority of publications exploited multichannel Wiener filters (Schneider *et al.* 1965; Meyerhoff 1966; Sengbush and Foster 1968; Galbraith and Wiggins 1968; Cassano and Rocca 1973, 1974). Then the situation changed and the interest of geophysicists in these methods gradually was lost because of their insufficient effectiveness. Their place was taken by non-optimum methods, such as *f-k* filtering and the Radon transform, implying simplified mathematical models of the data. These methods are usually faster and more cost-effective. In our opinion, the optimum methods are often less effective because they exploit imperfect mathematical models of the record and are not supplied with reliable estimates of the required parameters. For this reason, in this study, we have made an attempt to rehabilitate the multichannel Wiener filter and reanimate the interest of geophysicists in it. With this purpose in mind, we utilize a more complicated and adequate model of the record. This mathematical model has been used to design more effective methods for evaluating the seismic signal and to supply them with more reliable estimates of the parameters needed.

Theory and method

Suppose that the *i*th trace of the record that consists of N traces may be written as:

$$u_i(t) = a_i s\left(t - \tau_{(s)i}\right) + \sum_{l=1}^{L} b_{il} r_l\left(t - \tau_{(r)il}\right) + n_i(t), \quad i = 1, \ldots, N \,. \quad (1)$$

Here the signal component is described by the first term. It is assumed to have an identical waveform $s(t)$ on each trace, with arbitrary trace-dependent amplitudes a_i and time delays $\tau_{(s)i}$. The second term represents an arbitrary superposition of coherent noise wavetrains with individual waveforms $r_l(t),\ l = 1, \ldots, L$. Each wavetrain, as well as the signal, bears arbitrary amplitudes b_{il} and time delays $\tau_{(r)il}$. Random noise is expressed by the third term. It is supposed to be a stationary zero-mean Gaussian stochastic process uncorrelated from trace to trace with identical to within a scale factor, the variance σ_i^2, autocorrelations. The signal and coherent noise waveforms are also assumed to be statistically independent zero-mean stationary stochastic processes and hence uncorrelated with each other and with random noise. Due to the above assumptions, the cross-spectrum between channels *i* and *j* may be expressed as:

$$\begin{aligned} R_{ij}(\omega) &= a_i a_j R_{(s)}(\omega) \exp\left[i\omega\left(\tau_{(s)i} - \tau_{(s)j}\right)\right] + \\ &\sum_{l=1}^{L} b_{il} b_{jl} R_{(r)l}(\omega) \exp\left[i\omega\left(\tau_{(r)il} - \tau_{(r)jl}\right)\right] + \sigma_i^2 R_{(n)}(\omega)\delta_{ij}, \end{aligned} \quad (2)$$

where $R_{(s)}(\omega)$, $R_{(r)l}(\omega)$ and $R_{(n)}(\omega)$ are the power spectra of $s(t)$, $r_l(t)$ and any $n_i(t)$, respectively, at an angular frequency ω, while δ_{ij} signifies the Kronecker delta.

Let us obtain the best estimate of the signal component for the record described by equation (1). To this end, multichannel Wiener filtering may be applied. By filtering each trace with its corresponding filter along with summing the outputs, this procedure produces a single record that resembles closest in least squares sense a desired process $w(t)$. The spectral characteristics $H_i(\omega),\ j = 1, \ldots, N,$ of the filter satisfy the following set of linear equations:

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$$\sum_{j=1}^{N} H_j(\omega) R_{ij}(\omega) = R_i^{(w)}(\omega), \qquad (3)$$

where $R_{ij}(\omega)$ denotes the cross-spectrum of traces *i* and *j*, whereas $R_i^{(w)}(\omega)$ is the cross-spectrum of trace *i* and the desired output *w*(*t*).

Let the signal waveform, $s(t)$, be the desired output of the filter. Then

$$R_i^{(w)}(\omega) = a_i R_{(s)}(\omega) \exp(i\omega\tau_{(s)i}).$$

The system of equations (3) may be given in matrix notation as:

$$\mathbf{R}_L \mathbf{h} = \mathbf{p}, \qquad (4)$$

where

$$\mathbf{R}_L = \sum_{l=1}^{L} R_{(r)l} \mathbf{g}_l \mathbf{g}_l^* + \mathbf{R}_0 = \mathbf{G}\mathbf{B}\mathbf{G}^* + \mathbf{R}_0,$$

$$\mathbf{g}_l = \{b_{1l} \exp(-i\omega\tau_{(r)1l}), \ldots, b_{Nl} \exp(-i\omega\tau_{(r)Nl})\}^*,$$

$$\mathbf{R}_0 = R_{(s)} \mathbf{f}\mathbf{f}^* + R_{(n)} \mathbf{D},$$

$$\mathbf{f} = \{a_1 \exp(-i\omega\tau_{(s)1}), \ldots, a_N \exp(-i\omega\tau_{(s)N})\}^*,$$

$$\mathbf{D} = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_N^2\}, \quad \mathbf{G} = \{\mathbf{g}_j\} = \{b_{ij} \exp(i\omega\tau_{(r)ij})\},$$

$$\mathbf{B} = \mathrm{diag}\{R_{(r)1}, \ldots, R_{(r)L}\}, \quad \mathbf{p} = R_{(s)} \mathbf{f},$$

whereas the superscripted asterisk signifies Hermitian (complex conjugate) transpose. For shot, here and in the following the dependence on frequency is dropped. Inverting the composite matrix $\mathbf{R}_L$ yields (Horn and Jonhson 1986)

$$\mathbf{R}_L^{-1} = (\mathbf{I} - \mathbf{R}_0^{-1}\mathbf{G}\mathbf{V}^{-1}\mathbf{G}^*)\mathbf{R}_0^{-1},$$

with $\mathbf{I}$ being an identity matrix and $\mathbf{V} = \mathbf{B}^{-1} + \mathbf{G}^*\mathbf{R}_0^{-1}\mathbf{G}$. The sought-for solution is thus equal to

$$\mathbf{h} = \mathbf{R}_L^{-1}\mathbf{p} = R_{(s)}(\mathbf{I} - \mathbf{R}_0^{-1}\mathbf{G}\mathbf{V}^{-1}\mathbf{G}^*)\mathbf{R}_0^{-1}\mathbf{f}. \qquad (5)$$

To simplify this formula, we assume that the record at each frequency satisfies the following set of inequalities:

$$R_{(s)} R_{(n)}^{-1} c_s >> 1, \qquad (6)$$

$$R_{(r)i} R_{(n)}^{-1} c_{ii} >> 1, \qquad (7)$$

$$c_{ii} c_{jj} >> |c_{ij}|^2, \qquad i \neq j, \qquad (8)$$

$$c_s c_{ii} >> |c_{si}|^2, \qquad (9)$$

where

$$c_s = \mathbf{f}^*\mathbf{D}^{-1}\mathbf{f} = \tilde{\mathbf{f}}^*\tilde{\mathbf{f}} = \sum_{i=1}^{N} \frac{a_i^2}{\sigma_i^2},$$

$$c_{ij} = \mathbf{g}_i^*\mathbf{D}^{-1}\mathbf{g}_j = \tilde{\mathbf{g}}_i^*\tilde{\mathbf{g}}_j = \sum_{k=1}^{N} \frac{b_{ki} b_{kj}}{\sigma_k^2} \exp[i\omega(\tau_{(r)ki} - \tau_{(r)kj})],$$

$$c_{si} = \mathbf{g}_i^*\mathbf{D}^{-1}\mathbf{f} = \tilde{\mathbf{g}}_i^*\tilde{\mathbf{f}} = \sum_{k=1}^{N} \frac{a_k b_{ki}}{\sigma_k^2} \exp[i\omega(\tau_{(s)k} - \tau_{(r)ki})],$$

$$\tilde{\mathbf{g}}_i = \mathbf{D}^{-1/2}\mathbf{g}_i, \qquad \tilde{\mathbf{f}} = \mathbf{D}^{-1/2}\mathbf{f},$$

which imply that the signal and any coherent noise component prevail significantly over random noise at the output from related optimum weighted stacking (Tyapkin and Ursin 2005) and all the vectors $\tilde{\mathbf{g}}_i$ and the vector $\tilde{\mathbf{f}}$ are almost mutually orthogonal. Inequalities (6)-(9) enable (5) to be reduced to the form

$$\mathbf{h} = \left[\mathbf{I} - \mathbf{D}^{-1} \sum_{l=1}^{L} \frac{\mathbf{g}_l \mathbf{g}_l^*}{c_{ll}}\right] \frac{\mathbf{D}^{-1}\mathbf{f}}{c_s}, \qquad (10)$$

which is independent of the spectra of all the record components.

Equation (10) allows us to suitably interpret the sequence of operations needed to embody this filter.

1. Optimum weighted stacking (Tyapkin and Ursin 2005) of all the traces in order to estimate the shape of the coherent noise wavetrain $r_l(t)$. Prior to this operation, the related arrival times should be cancelled out in order to align coherent noise and remove its time delays.
2. Subtraction of the *l*th coherent noise component from all the traces with regard for its arrival time, amplitude, and waveform estimate. These two operations are repeated for all the coherent noise components in the descending order of their energy.
3. Optimum weighted stacking of the residual record for estimating the signal waveform $s(t)$. As well as in the previous case, the related time shifts are cancelled out prior to stacking.

The effectiveness of the suggested method is highly dependent on the accuracy of the necessary signal and noise parameter determination. Therefore, for the method to be feasible, two essentially different approaches, which are based on the same record model, have been suggested to determine the necessary signal and noise parameters. In the first of them, conventional velocity analysis is combined with some methods derived for optimum weighted stacking. When random noise is rather stable in space, the singular value decomposition (SVD) may be used instead of optimum weighted stacking (Tyapkin and Ursin 2005). It can be regarded as a simplified version of the method developed. Note: this technique was used to process field data. The second approach exploits analysis of the eigenvalues and eigenvectors of the cross-spectrum matrix.

Field data examples

The data demonstrated below may be considered the first results of testing the method with field data. The left-hand panel in Fig.1 presents a 96-trace shot gather acquired with a vibratory source and no geophone grouping in the field. In this record, the reflections are obscured by severe coherent source-generated noise including ground roll, air, refracted and guided waves up to the first breaks. Besides, on the left top part of the panel, one can see a technogenic event with an infinite apparent velocity. By examining Fig.1, we see that all the noise components are several times stronger that the signal. For this reason, before removing the noise, there is almost no reflection in the original data that can be identified.

To subtract the noise, which has a divergent, fan-like character, we apply the method suggested in (Tyapkin *et al.* 2004). It requires specifying lines of demarcation, much like a surgical mute, around the entire noise or each component (wavetrain), by picking segmented straight lines. Ones the lines of demarcation have been determined, each sector is mapped by shifting and stretching along the time axis into a new domain, where the segmented straight lines become horizontal. This procedure is intended to align the coherent noise events horizontally in order to favor the subsequent SVD. After that the coherent noise may successfully be approximated by a few dominant terms of the SVD and then subtracted from the record.

On the left-hand panel in Fig. 1, one can see four sectors within which the noise was subtracted successively. The top and bottom boundaries of each sector are marked by the same color. The result of subtracting the noise is depicted on the right-hand panel in Fig. 1. Comparing the result of processing and raw record, one can see that the severe noise is greatly diminished and the refinement of the data is significant, that favors a more confident identification and correlation of reflection events.

Figure 2 exhibits two time sections obtained after application of *f-k* filtering (top) and the above SVD-based method (bottom) for subtracting low-velocity coherent noise from a set of common midpoint gathers collected with a vibratory source. After subtracting the noise, in both cases, the left singular vector associated with the dominant singular value of the residual data was used for optimum stacking (Tyapkin and Ursin 2005). Note the remnants of the coherent noise after applying the *f-k* filter, specifically on the central and right-hand side of the top panel. This result can be attributed to spatial aliasing, which is a common problem with the performance of *f-k* filters. From Fig.2 it is evident that after applying the two-stage SVD-based technique, the section has a better S/N ratio and trace-to-trace continuity of reflected signals. Furthermore, a close examination shows that the SVD-based technique yields a better vertical resolution than the *f-k* filter. The improved vertical resolution is most obvious for the events marked with arrows. These improvements are due to the fact that our approach allows the coherent noise to be suppressed more effectively, with negligible effect on the signal.

Conclusions

The method developed can be a valuable tool in processing seismic data. Even a simplified variant of the method that implies random noise to be stable in space demonstrates its good performance in estimating signal of field data sets contaminated by severe coherent noise.

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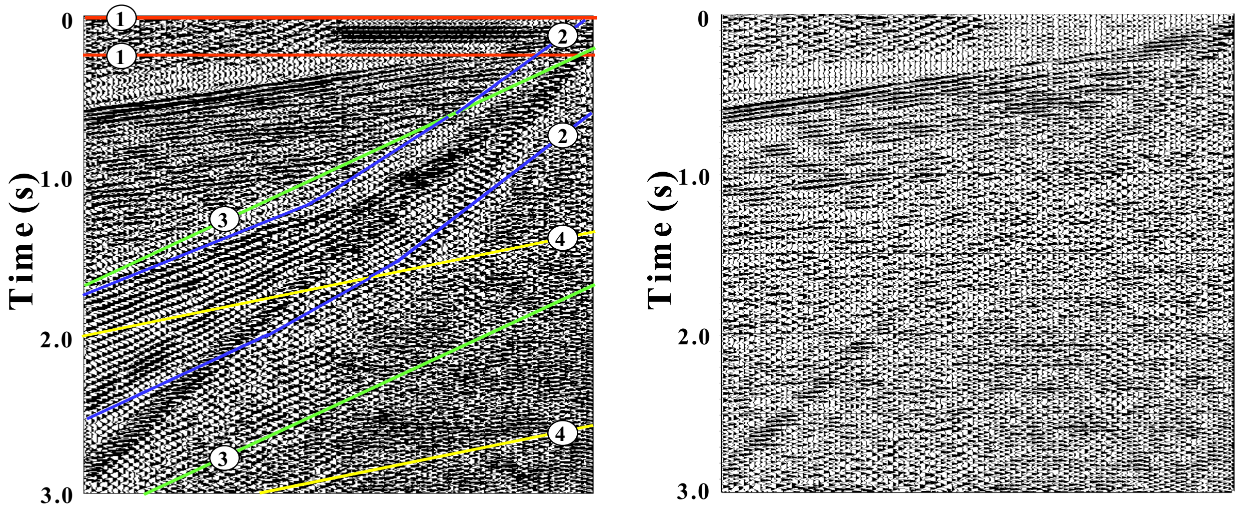

Figure 1. Common-shot gather before (left) and after (right) subtracting severe source-generated coherent noise. The boundaries of four sectors selected for processing are marked with different colors. Note that after using the SVD-based method, the noise was suppressed considerably and much of the underlying reflection was revealed.

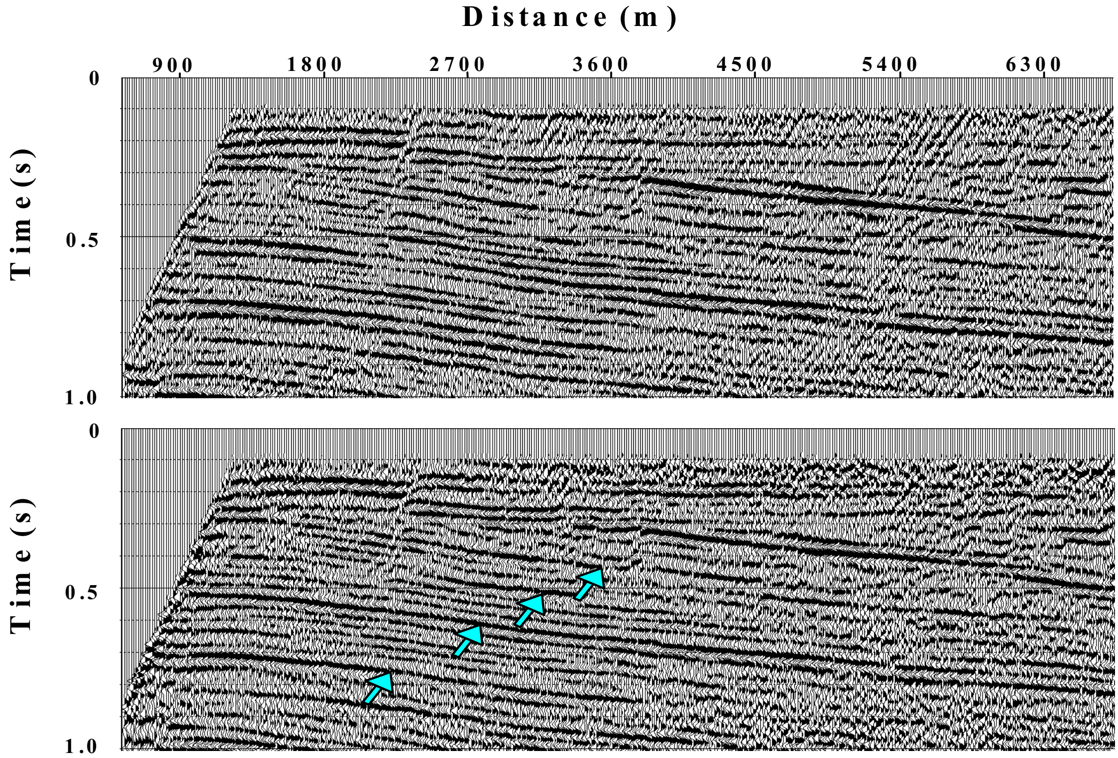

Figure 2: Stacks after application of *f-k* filtering (top) and after application of the SVD-based approach (bottom) for subtracting coherent noise from the same set of common midpoint gathers. The range of time dips or apparent velocities for applying both the *f-k* filter and the SVD-based technique was chosen to be the same. Except for these two processes, the same processing sequence with identical parameters was used for both sections. Note how the SVD-based approach successfully removed the remnants of coherent noise, specifically on the central and right-hand sides of the section, revealing the underlying reflections.

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