P137

Our Experiences of 3D Synthetic Seismic Modeling with Tip-wave Superposition Method and Effective Coefficients

M. Ayzenberg (StatoilHydro), A. Aizenberg (Institute of Petroleum Geology and Geophysics), H.B. Helle* (Odin Petroleum AS), K.D. Klem-Musatov (Institute of Petroleum Geology and Geophysics), J. Pajchel (StatoilHydro) & B. Ursin (NTNU)

SUMMARY

We show that seismic waves travelling in the subsurface are subject to two physically different processes, namely propagation inside smoothly inhomogeneous layers and reflection and transmission at internal reflectors. We propose a new approach to modeling of the reflection data, which is based on the analytical description and numerical implementation of the two processes. The propagation is realized through the tip-wave superposition method, which is a Kirchhoff-based technique. The reflection and transmission is implemented in the form of effective reflection and transmission coefficients. We illustrate the advantages of using the tip-wave superposition method compared to a conventional Kirchhoff modeling. Also, we address the applicability of the plane-wave reflection and transmission coefficients in different models, and evaluate their performance compared to the effective reflection and transmission coefficients.

Introduction. Seismic waves travelling in the subsurface are subject to two physically different processes, namely propagation inside smoothly inhomogeneous layers and reflection and transmission at internal reflectors, which are formed by seismic parameter discontinuities. It is natural that synthetic modeling is performed according to this separation. Several modeling methods follow this principle; among them are ray tracing and various approaches of implementing the Kirchhoff integral. All of them are based on geometrical seismics, and therefore experience difficulties in the shadow zones and in the presence of steeply dipping reflectors and caustic triplications. We have been developing a new approach to modeling of the reflection data, which is based on the analytical description and numerical implementation of the two processes mentioned above. The propagation is realized through the tip-wave superposition method (TWSM), which is a Kirchhoff-based technique. The reflection and transmission are implemented in the form of effective reflection and transmission coefficients (ERC and ETC). We illustrate the advantages of using the TWSM compared to a conventional Kirchhoff modeling. Also, we address the applicability of the plane-wave reflection and transmission coefficients (PWRC and PWTC) in different models, and evaluate their performance compared to ERC and ETC.

Tip-wave superposition method. Kirchhoff integral is a way of describing the process of wave propagation in the subsurface mathematically. The integral can be considered as an operator acting over the values of the reflected (transmitted) wavefield at the reflector. Then the Kirchhoff propagator is based on Huygens' principle. It states that every point of the reflector is the virtual source of a secondary wave, which is spherical in homogeneous and weakly curved layers. Secondary waves (often referred to as the Green's function), with adjusted amplitudes, propagate towards the receivers and interfere to produce the reflection response. Application of the Kirchhoff approach requires knowledge of the Green's function as well as the values of the reflected (transmitted) wavefields at the reflector (Ursin, 2004).

Following the work of Klem-Musatov et al. (2009), we implement the Kirchhoff propagator according to Huygens' principle. We split the reflector into small rhombic elements. Typical linear size of an element is a fraction of the wavelength. Then the Kirchhoff-integral propagator becomes the sum of the integrals over the elements. When evaluated within a seismic-frequency approximation, the integral over an element produces a wave beam. Thus the propagator is the sum of the wave beams bringing the reflected (transmitted) wavefield from all the elements of the reflector towards the receiver. The wave beams have phases corresponding to the traveltimes from the elements to the receiver. Those wave beams belonging to the Fresnel zone of the specular ray contribute most to the reflected (transmitted) wavefield registered at the receiver. The beams not belonging to the Fresnel zone interfere destructively and cancel out.

The wave beam contains a narrow plane-wave ray tube propagating from the rhombic element. It also contains four edge-diffracted conical waves propagating from the edges as narrow conical ray tubes. Because each rhombic element has eight vertices (two vertices belonging to each edge), the wave beam contains eight tip-diffracted spherical waves propagating from vertices. Because the plane-wave and conical-wave ray tubes are narrow, the wavefield is mainly formed by the tip-diffracted spherical waves. Therefore, it is natural to use the term "tip-wave beam". This also explains why the corresponding method for synthetic seismic modeling got the name "tip-wave superposition method" (TWSM). Because each tip-wave beam describes the energy transport along the plane-wave ray tube and the energy diffusion along the edge-diffracted and tip-diffracted wavefronts, their superposition preserves all kinematical and dynamic properties of the reflected (transmitted) wavefield.

In the presence of more than one reflector, TWSM implies evaluation of the tip-wave beams between each pair of two neighboring reflectors. Moreover, each layer is described by two tip-wave beam sets, namely those traveling upward from the lower reflector to the upper reflector, and those traveling downward. A set of the tip-wave beams form the so-called "layer matrix". The two layer matrices for a layer fully describe the wave propagation in this layer. For a specified wavecode, the corresponding layer matrices are sequentially multiplied to produce the wavefield recorded at the receiver. Thus TWSM is an event-oriented way of modeling the wavefield. It allows modeling of specified events of interest as well as the full wavefield, whenever needed (Ayzenberg et al., 2008). This property may be extremely useful in various modeling tasks, in particular in survey design and illumination studies.

Effective reflection and transmission coefficients. As opposed to conventional Kirchhoff modeling, we have developed and implemented another approach to evaluation of the reflected and transmitted wavefields at the reflector. We represent the incident wavefield by a set of plane waves incident at all possible angles. Then the reflected (transmitted) wavefield is described by the integral of these plane waves multiplied with the plane-wave reflection (transmission) coefficient evaluated for the corresponding incidence angle. For plane reflectors between homogeneous media, such plane-wave decompositions are represented by Fourier-Bessel integrals over horizontal slowness and are thus well-studied. However, generalization of the classical plane-wave decompositions for curved reflectors between inhomogeneous media does not appear to be straightforward. This is partly explained by rotation of the normal to the reflector. Therefore the set of plane waves does not easily split into up-going and down-going waves. This makes it difficult to introduce the vertical slowness component, which is an important constituent of a plane-wave decomposition.

Klem-Musatov et al. (2004) introduced rigorous theory of reflection and transmission for interfaces of arbitrary shape between laterally heterogeneous media. The authors showed that the reflected (transmitted) wavefield in the acoustic Kirchhoff integral can be represented by a generalized plane-wave decomposition. The new decomposition is written in curvilinear surface coordinates and uses generalized plane waves, which are defined locally for each particular reflection (transmission) point. They are computed for the local values of acoustic and elastic parameters at this point and have to be re-evaluated for other points. The split to up-going and down-going plane waves and the choice of the vertical slowness component are, of course, also local. Because the generalized plane-wave decomposition is written in curvilinear coordinates, it naturally incorporates the information about the local reflector geometry into the reflection (transmission) response.

Although the generalized plane-wave decomposition strictly describes the process of reflection and transmission at curved interfaces in inhomogeneous media, numerical computation of the four-fold Fourier integrals entering the decomposition is prohibitively expensive. For a plane reflector, the four-fold integrals can be reduced to a single Bessel integral, which can be implemented in an efficient way. For a curved reflector, the problem of reduction of the order of the four-fold integrals cannot be resolved. Instead, Ayzenberg et al. (2007) proved that the reflected (transmitted) wavefield may be approximately described by multiplication of the incident wavefield and the corresponding effective reflection (transmission) coefficient, ERC (ETC). The authors also suggested that instead of evaluating ERC and ETC for the actual curved reflector, the effective coefficients can be computed for a plane reflector tangential to the actual reflector at the incidence point and an apparent point source position with respect to the plane reflector. Within this approximation, the incidence angle is preserved. However, the apparent distance between the source and the reflector is calculated using geometrical principles known from optics.

Plane-wave coefficients depend on the incidence angle and the elastic parameters just above and just below the reflector. Effective coefficients depend on an additional dimensionless parameter, which is the product of the apparent distance and the wavenumber. Effective coefficients are thus frequency-dependent. They also incorporate the local interface curvature and correctly describe near-critical and post-critical reflection and include head waves.

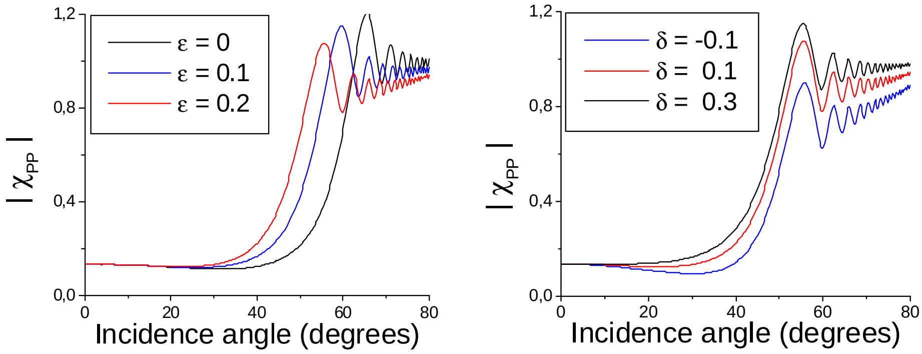

Figure 1 *Dependence of the PP-wave ERC on Thomsen parameters.*

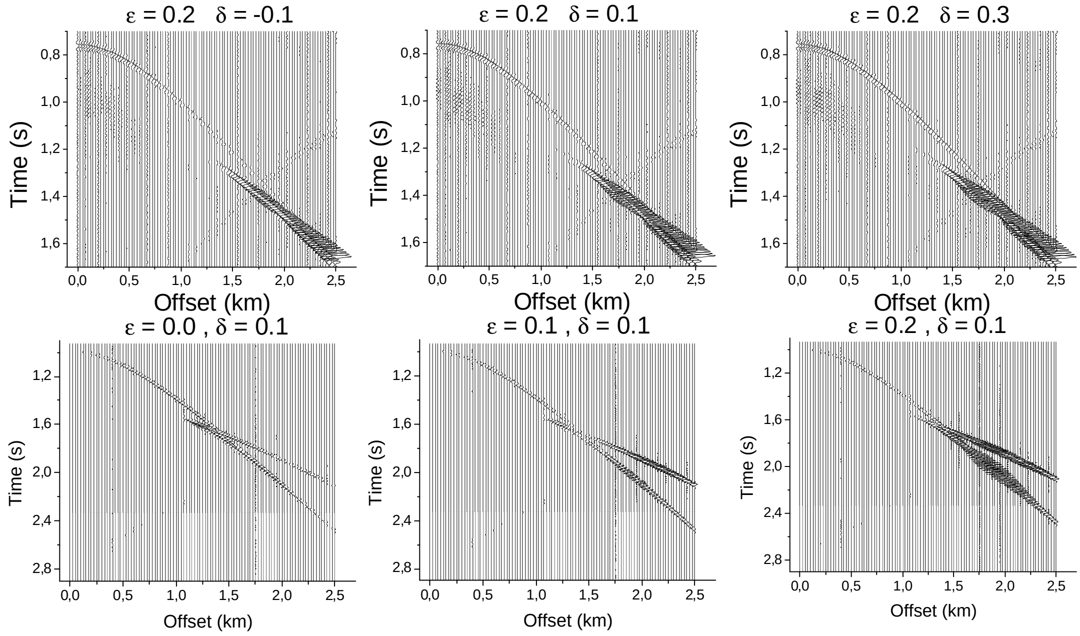

Figure 2 *Dependence of the reflected PP-wave amplitude on Thomsen parameters.*

Modeling. A good example of using the TWSM with ERC and ETC belongs to the group of anisotropic models. We consider a Gaussian-shaped reflector with the top of the anticline just below the source. The medium above the reflector is isotropic elastic with the velocities $v_{P1} = 2\,\text{km/s}$ and $v_{S1} = 1.2\,\text{km/s}$ and the density $\rho_1 = 2.15\,\text{g/cm}^3$. The lower medium is a bended TI layer with the symmetry axis orthogonal to the reflector at each point. The vertical velocities are $v_{P0} = 2.4\,\text{km/s}$ and $v_{S0} = 1.4\,\text{km/s}$ and the density is $\rho_2 = 2.35\,\text{g/cm}^3$. We vary the values of Thomsen parameters within the limits $0 < \varepsilon < 0.2$ and $-0.1 < \delta < 0.3$. Figures 1 and 2 show the ERC and corresponding seismograms for the PP-wave. Figures 3 and 4 provide the ERC and seismograms for the PS-wave. We observe that the amplitudes of the reflected waves follow the trends predicted by the plots of ERC. It is, however, difficult to estimate the exact effect of Thomsen parameters on the amplitudes for large offsets, partly because the reflection contains caustic triplications, and the ERC is “smeared” over it.

Conclusions. This paper comprises our experiences of seismic modeling with the TWMS and ERC and ETC. The method is numerically stable and capable of simulating complex wave phenomena, such as caustic triplications, diffractions, and head waves at curved reflectors. TWSM in combination with PWRC and PWTC resembles conventional 3D Kirchhoff modeling, which is computationally very efficient. However, amplitudes for near-critical and post-critical incidence angles are somewhat inaccurate. TWSM with ERC and ETC is computationally heavier, but produces much more accurate amplitudes.

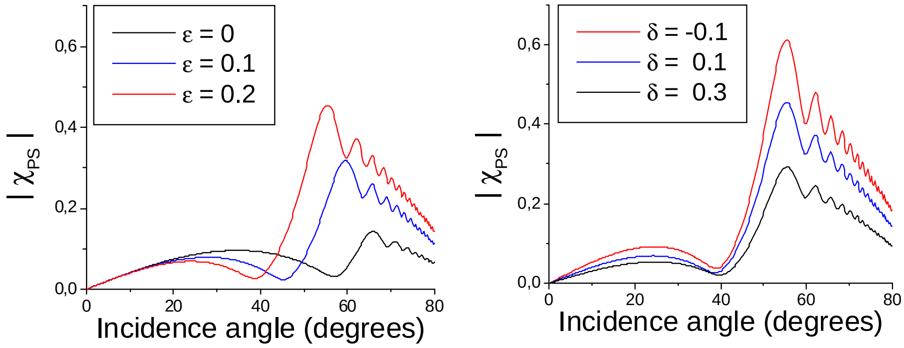

Figure 3 *Dependence of the PS-wave ERC on Thomsen parameters.*

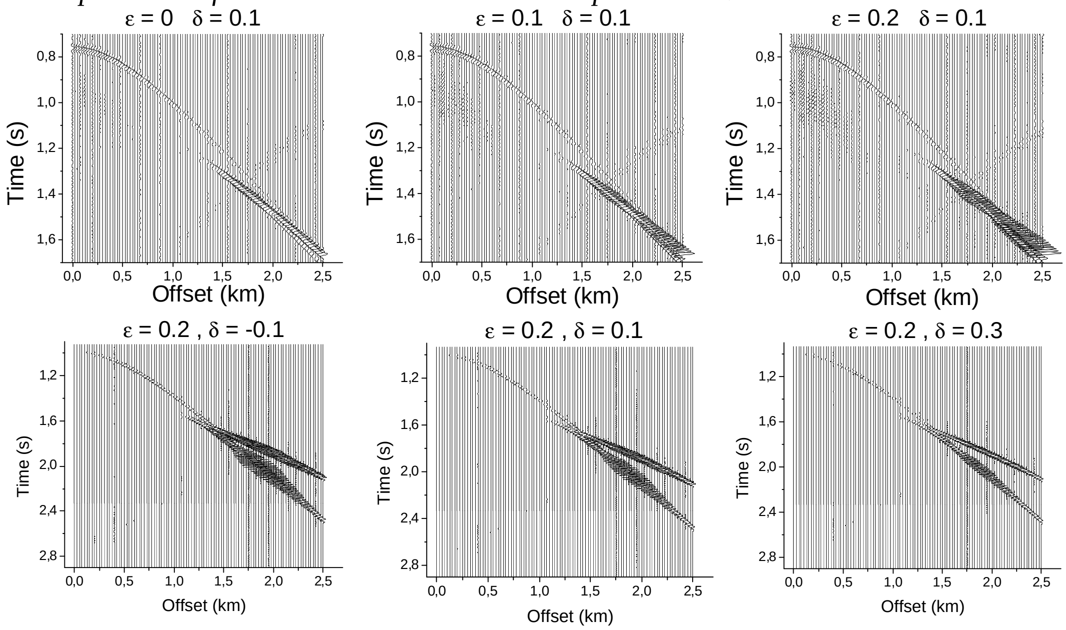

Figure 4 *Dependence of the reflected PS-wave amplitude on Thomsen parameters.*

Acknowledgements. We acknowledge StatoilHydro ASA (Bergen and Trondheim, Norway) and the Russian Foundation for Basic Research (grant 07-05-00671) for support of this study. Bjørn Ursin has received financial support from StatoilHydro ASA through the VISTA project and from the Norwegian Research Council through the ROSE project.

References

Ayzenberg, M.A., Aizenberg, A.M., Helle, H.B., Klem-Musatov, K.D., Pajchel, J. and Ursin, B. [2007] 3D diffraction modeling of singly scattered acoustic wavefields based on the combination of surface integral propagators and transmission operators. *Geophysics*, **72**, SM19-SM34.

Ayzenberg, M.A., Aizenberg, A.M., Helle, H.B., Klem-Musatov K.D., Pajchel, J. and Ursin, B. [2008] 3D modeling of acoustic Green's function in layered media with diffracting edges. *70th EAGE Conference & Exhibition*, Extended Abstracts, P-052.

Klem-Musatov, K., Aizenberg, A., Helle, H.B. and Pajchel, J. [2004] Reflection and transmission at curvilinear interface in terms of surface integrals. *Wave Motion*, **39**, 1, 77-92.

Klem-Musatov, K.D., Aizenberg, A.M., Pajchel, J. and Helle, H.B. [2009] Edge and Tip Diffractions - Theory and Applications in Seismic Prospecting. Tulsa, *SEG* (In Print).

Ursin, B. [2004] Tutorial: Parameter inversion and angle migration in anisotropic elastic media. *Geophysics*, **69**, 5, 1125-1142.