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2.5D Frequency Domain EM Modeling in Conductive TIV Media

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SUMMARY

We present a framework for 2.5D EM modeling in conductive TIV media based on integral equations. The method is developed for mCSEM purposes, thus assuming inline polarization of the source and crossline invariant media. The framework is tested numerically on mCSEM scale and conductivity range, but without the free surface. We show that the results are reasonable when tested against an isotropic 1D modeling software.

Introduction

CSEM data are now widely accepted as an useful tool for hydrocarbon exploration, and robust and accurate modelling tools are thus wanted. It is well known that electromagnetic data in the frequency domain may be modelled by Lippman-Schwinger type integral equations (Green, 1828), and we present here an approach for 2.5D frequency domain modelling of EM data, inspired by the approach of Abubakar et al. (2005), but extended to media that are transversely isotropic in the vertical direction (TIV). We further test the theory numerically on an example on marine CSEM scale. The results show that the 2.5D TIV EM modelling approach is feasible for mCSEM modelling.

Governing equations

Consider a background model $\mathcal{B}$, with given electromagnetic parameters. We denote the electric conductivity by σ, electric permittivity by ϵ and magnetic permeability by μ, and let the permeability take the value it has in vacuum throughout this abstract. We assume there in this background is a scattering object, which differs from the background in electric conducivity and is enclosed in a bounded domain $\mathcal{D}$. It is well known that when neglecting diplacement currents, which is feasible for mCSEM purposes where we consider low frequencies and conductive media (Wiik, 2008), the electric field in the frequency domain at a point $\mathbf{x} = (x, y, z)$ in space is given by (Green, 1828; Zhdanov et al., 2006)

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}^{\mathrm{inc}}(\mathbf{x}) + \int_{\mathcal{D}} G_{E,\mathcal{D}}(\mathbf{x}, \mathbf{x}_0)\, \sigma_0(\mathbf{x}_0)\, \chi(\mathbf{x}_0)\, \mathbf{E}(\mathbf{x}_0)\, \mathrm{d}\mathbf{x}_0, \tag{1}$$

which is a type 2 Fredholm integral equation. Here $\mathbf{E}$ denotes the electric field vector, $\mathbf{E}^{\mathrm{inc}}$ denotes the incident field in the background model from a known electric source, while $G_{E,\mathcal{D}}$ is the electric Green's tensor. Moreover, σ_0 is the electric conductivity of the background model and $\chi = \frac{\sigma}{\sigma_0} - 1$ is the dimensionless contrast between the background and the real profile of $\mathcal{D}$. This imply that the support $\mathrm{supp}(\chi) \subset \mathcal{D}$.

We will for now for the sake of illustration restrict ourselves to an isotropic, homogeneous background, following Abubakar et al. (2005), and firstly consider an isotropic scattering object. We further define the Fourier transform

$$\hat{u}(k_x) = \int_{-\infty}^{\infty} e^{\mathrm{i}k_x x} u(x)\, \mathrm{d}x, \tag{2}$$

and the corresponding inverse. We now place the source and the receivers along the x-axis and let the source be a horizontal electric dipole polarized in the same direction (in-line). Further, we make the 2.5D assumption of medium invariance along one axis, here the y-axis. We thus apply the Fourier transform in the y-direction to equation (1) and find

$$\hat{\mathbf{E}}(\mathbf{x}_T, k_y) = \hat{\mathbf{E}}^{\mathrm{inc}}(\mathbf{x}_T, k_y) + \int_{\mathbf{x}_T \in \mathcal{D}} \hat{G}_{E,\mathcal{D}}(\mathbf{x}_T, \mathbf{x}_{0,T}, k_y)\, \chi(\mathbf{x}_{0,T})\, \hat{\mathbf{E}}(\mathbf{x}_{0,T})\, \mathrm{d}\mathbf{x}_{0,T}, \tag{3}$$

which is now a 2D integral, as opposed to equation (1) which is 3D. Here χ is the same as earlier due to the assumption of medium invariance, which ensures that it does not vary with y. Also, $\mathbf{x}_T = (x, z)$, and k_y is the Fourier parameter due to the Fourier transform. Moreover, as we have chosen a homogeneous background, we find an explicit expression for the 2.5D Green's tensor (Abubakar et al., 2005):

$$\hat{G}_{E,\mathcal{D}}(\mathbf{x}_T, \mathbf{x}_{0,T}, k_y) = \left(k_0^2 I + \hat{\nabla}\hat{\nabla}\right) \hat{g}(\mathbf{x}_T - \mathbf{x}_{0,T}, k_y), \tag{4}$$

where

$$\hat{g}(\mathbf{x}_T, k_y) = \frac{\mathrm{i}}{4} H_0^{(1)}(\gamma_0 |\mathbf{x}_T|), \tag{5}$$

I is the identity tensor, $\hat{\nabla} = [\partial_x, -\mathrm{i}k_y, \partial_z]$, $k_0^2 = \mathrm{i}\omega\mu_0\sigma_0$ is the squared background wavenumber, $H_0^{(1)}$ is the Hankel function of the first kind and order zero and $\gamma_0^2 = k_0^2 - k_y^2$ is a modified square wavenumber. As usual, ω denotes the angular frequency. For simplification we will from now omit denoting the arguments in equation (3), and write the equations in operator form:

$$\hat{\mathbf{E}}^{\rm inc} = \hat{\mathbf{E}} - \hat{\mathbf{G}}_{E,\mathcal{D}}\chi\hat{\mathbf{E}}. \tag{6}$$

Here $\hat{\mathbf{G}}_{E,\mathcal{D}}$ is a matrix containing the corresponding integral operators.

TIV anisotropy

We will here extend the work by Abubakar et al. (2005), and that presented in this abstract on isotropic 2.5D EM modelling, to a medium that is transversely isotropic in the vertical direction (TIV medium). By this we mean that the conductivity is a diagonal dyad of the form

$$\sigma = \begin{pmatrix} \sigma_h & 0 & 0 \\ 0 & \sigma_h & 0 \\ 0 & 0 & \sigma_v \end{pmatrix}, \tag{7}$$

where σ_h and σ_v denotes the horizontal and vertical conductivities, respectively. Thus, by TIV we mean that the currents do not flow equally in the vertical and horizontal directions. We still restrict ourselves to a homogeneous, isotropic background, and thus the Green's functions stay the same as for the isotropic 2.5D case. This is not a severe restriction, as the background will typically be seawater, which is isotropic, perhaps with a free surface if desired. Further, all the equations presented for the 2.5D isotropic case will symbolically look the same, but one must keep in mind that the contrast χ is now a dyad on the form

$$\chi = \begin{pmatrix} \chi_h & 0 & 0 \\ 0 & \chi_h & 0 \\ 0 & 0 & \chi_v \end{pmatrix}, \tag{8}$$

where $\chi_h = \frac{\sigma_h}{\sigma_0} - 1$ and $\chi_v = \frac{\sigma_v}{\sigma_0} - 1$ are the horizontal and vertical contrasts, respectively. Thus, the different field components of the electric field are acted upon by different contrasts. TIV anisotropy is often referred to by stating σ_h and the ratio $\Upsilon = \frac{\sigma_h}{\sigma_v}$. We may then write

$$\sigma = \sigma_v \begin{pmatrix} \Upsilon & 0 & 0 \\ 0 & \Upsilon & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Solving the equations

To determine the electric field at an observation point outside $\mathcal{D}$, we observe that we need first to solve equation (3) for $\mathbf{x}_T \in \mathcal{D}$. Afterwards, we may then compute the electric and magnetic field at an arbitrary point outside $\mathcal{D}$ using again equation (3), and the corresponding equation for the magnetic field. The difficulties occur when solving for $\mathbf{x}_T \in \mathcal{D}$, when the unknown $\hat{\mathbf{E}}$ is both on the left hand side and inside the integral. For this we use a conjugate gradient method as described by van den Berg (1984), where we minimize the cost functional

$$F\left(\hat{\mathbf{E}}\right) = \frac{\left\|\hat{\mathbf{E}}^{\rm inc} - \hat{\mathbf{E}} + \hat{\mathbf{G}}_{E,\mathcal{D}}\chi\hat{\mathbf{E}}\right\|^2_{L^2(\mathcal{D})}}{\left\|\hat{\mathbf{E}}^{\rm inc}\right\|^2_{L^2(\mathcal{D})}}, \tag{10}$$

for a set of $k_y \in \mathbf{k_y}$.

For a successful modelling, the set $\mathbf{k_y}$ needs to be chosen appopriately. I.e., it has to be sufficient to represent the correct field when we after minimizing the cost functional for each $k_y \in \mathbf{k_y}$ transform back to the (x, y, z, ω)-domain from (x, k_y, z, ω)-domain, where we have solved the equations. To ensure this, we use the results presented in Abubakar et al. (2005); Mitsuhata (2000); Kong et al. (2008). Especially the upper bound suggested by Mitsuhata (2000) is useful.

Numerical example

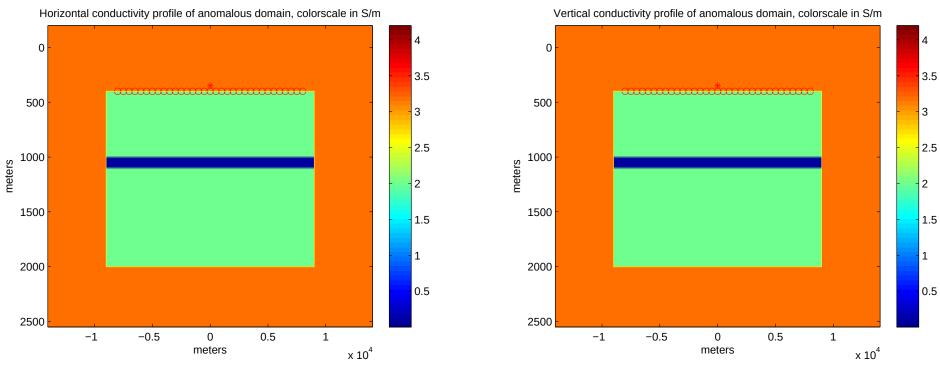
1D comparison, isotropic model

To compare the effect of anisotropy versus isotropy we test our code against an isotropic 1D code, based on the theory described by Ursin (1983), and elaborated on by Løseth and Ursin (2007). We choose a typical mCSEM conductivty of 2S/m as subsurface background, with a 100m thick resistive body placed between 1000m and 1100m depth. It is given a horizontal conductivity of $\sigma_h = 0.1$S/m and vertical conductivity of $\sigma_v = 0.025$S/m. That is, an anisotropy ratio $\Upsilon = 4$. This will hopefully yield insight into the effect of TIV anisotropy. The background conductivity in the homogeneous background was chosen to be a typical value for water, 3.2S/m. The free surface was neglected due to the homogeneous background assumption. The 1D code was validated by Wiik (2008). The model used is also shown in Figure 1.

The boundary of $\mathcal{D}$ is depicted as a yellow frame in Figure 1, and it was discretised into gridcells with dimensions $\Delta x = \Delta z = 20$m. The source is depicted as a star in Figure 1, placed at $x = 0$m, $z = 350$m, and is chosen as an electric point source with dipolemoment $3 \cdot 10^5$Am, operating frequency of 0.25Hz and is polarised in the x-direction. The receivers are placed at $z = 400$m, and $-8000\text{m} \leq x \leq 8000\text{m}$ with an uniform spacing of 500m, and are depicted as circles in Figure 1.

We choose in total 20 spectral values in the range 0m^{-1} to 0.04m^{-1} which were quadratically distributed over the interval, a configuration that yields at most 3% error in the incident field. This number of wavenumbers is somewhat less than, but in accordance with, the number of wavenumbers used by e.g. Mitsuhata (2000) and Kong et al. (2008), and the upper bound is chosen according to the criterion presented by Mitsuhata (2000).

For simplicity, we present only the inline component of the electric field. We observe from Figure 2 that the field is clearly most sensitive to the vertical conductivity in the resistor, but the response also clearly differs from the isotropic responses with the corresponding conductivities. This is in compliance with the comments made by Løseth and Ursin (2007). Thus, anisotropy yields distinct differences to the field propagation compared to an isotropic medium, and may be neccessary to give a sufficiently accurate description of the medium.

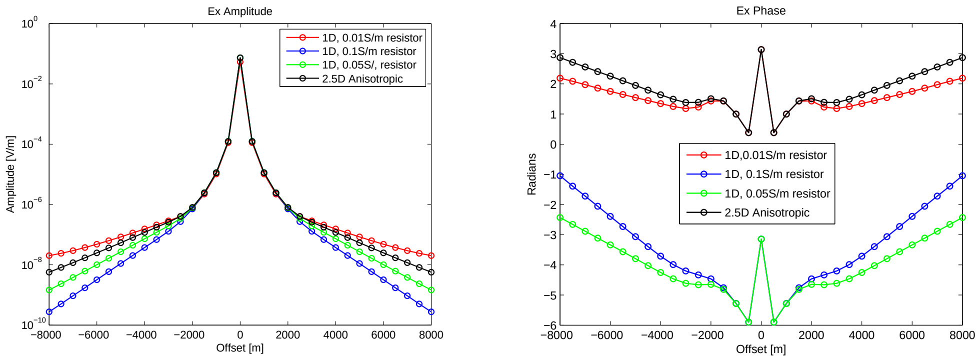


(a) Horizontal conductivity. (b) Vertical conductivity.

Figure 1: Model used in numerical example. The star denotes the source position, the circles denote receiver positions.

Conclusions

We have demonstrated an IE 2.5D TIV anisotropic modelling framework for low frequency electromagnetic field propagation in conductive media with conductivity contrasts. We find that the framework is capable of simulating the mCSEM experiment for hydrocarbon prospecting with respect to model size and conductivity ranges, and that one only has to solve between 20 and 30 2D problems to obtain reasonable 3D results under the 2.5D assumptions.



(a) Amplitude. (b) Phase.

Figure 2: Inline **E** field comparison between 1D code and 2.5D code on anisotropic model.

However, the approach is of course, due to the assumptions made, not sufficient in complex 3D geology, where the true 3D nature of the experiment is significant. Thus the method should be used with great care.

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