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Viscoacoustic sensitivity analysis using partial Hessian matrix.

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April 24, 2018

Outline

1. Introduction
2. Methodology
3. Results
4. Conclusions

1 Introduction

2 Methodology

3 Results

4 Conclusions

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- Sensitivity analysis to determine the trade-off between parameters.
  - Two or more variables have similar effects on the data during the inversion.
  - The result depends on the acquisition configuration for each model. [2]

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- Accounting with an homogeneous background model allows to calculate analytical Hessian.
- For this work a viscoacoustic model has been used.

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Misfit function

$$e(\mathbf{m}) = \frac{1}{2}\int d\mathbf{x}_r \int d\mathbf{x}_s \int dt \left[d^{obs}(\mathbf{x}_r, \mathbf{x}_s, t) - d(\mathbf{x}_r, \mathbf{x}_s, t)\right]^2 \quad (1)$$

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$\Delta\hat{\mathbf{m}} \rightarrow$ Estimated model perturbation.

- The estimate $\Delta\hat{\mathbf{m}}$ is related to the true parameter estimate perturbation $\Delta\mathbf{m}$ via [1]

$$\Delta\hat{m}(x) = \alpha \int d\mathbf{x}'\mathbf{H}(\mathbf{x}, \mathbf{x}')\Delta\mathbf{m}(\mathbf{x}'), \quad (2)$$

- The approximate Hessian can be expressed in terms of the Modeling operator $\mathbf{J}$ [4]

$$\mathbf{H}(\mathbf{x},\mathbf{x}') = \int dt \int dS(\mathbf{x}_s) \int dS(\mathbf{x}_r) \mathbf{J}(\mathbf{x}_r,\mathbf{x},t)\mathbf{J}(\mathbf{x}_r,\mathbf{x}',t), \qquad (3)$$

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- The modeling operator provides a linear relation between Δm and Δd [3, 5]

$$\Delta d(\mathbf{x}_r\mathbf{x}_s,t) = \int d\mathbf{x}'\mathbf{J}(\mathbf{x}_r,\mathbf{x}',t)\Delta\mathbf{m}(\mathbf{x}'), \qquad (4)$$

$\mathbf{J}$ is defined as $\mathbf{J} = [J_\alpha, J_Q]$.

- Re-expressing the Hessian

$$H^{ij}(x,x') = \int d\omega \int ds(x_s) \int ds(x_r) J^i(x_r,x,\omega) J^j(x_r,x',\omega) \quad (5)$$

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- For this case of study

$$\mathbf{H} = \begin{bmatrix} H^{\alpha\alpha} & H^{\alpha Q} \\ H^{Q\alpha} & H^{QQ} \end{bmatrix} \quad (6)$$

The parameter estimates are:

$$\widehat{\Delta\alpha} = H^{\alpha\alpha}\Delta\alpha + H^{\alpha Q}\Delta Q \quad (7)$$

$$\widehat{\Delta Q} = H^{Q\alpha}\Delta\alpha + H^{QQ}\Delta Q. \quad (8)$$

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$$\alpha = \sqrt{\frac{M}{\rho}} \qquad M = M_0 \left| \frac{\omega}{\omega_0} \right|^{2\gamma} e^{i\pi\gamma \mathrm{sgn}(\omega)}$$

$$\gamma = \frac{1}{\pi} \arctan\left(\frac{1}{Q}\right) \qquad c = \sqrt{\frac{M_0}{\rho}}$$

$$M_0 = \rho c_0^2 \cos^2\left(\frac{\pi\gamma}{2}\right)$$

- The specific model parameters can be obtained by using

$$\mathbf{J}^{c} = \mathbf{J}^{\alpha}(\mathbf{x}_r, \mathbf{x}, \omega)\left(\frac{\partial \alpha}{\partial c}\right),$$

$$\mathbf{J}^{Q} = \mathbf{J}^{\alpha}(\mathbf{x}_r, \mathbf{x}, \omega)\left(\frac{\partial \alpha}{\partial Q}\right).$$

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- Where

$$J^\alpha = \frac{\eta \exp\left[\frac{\omega r1}{c_0\epsilon}\left(-\tan(\xi/2) + i\left(1 + \frac{\xi c_o \epsilon}{2\omega r_1}\right)\right)\right]}{c_0 \cos\left(\frac{\xi\epsilon}{2}\right)}. \tag{9}$$

Where, $r_1 = r + r_s$, $\epsilon = \left|\frac{\omega}{\omega_0}\right|^{\xi/\pi}$, $\xi = \arctan\left(\frac{1}{Q}\right)$, $\eta = -\frac{2\omega^2}{r r_s}$

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- The expressions of the Hessian are

$$H^{cc} = \int d\omega \int ds(x_s) \int ds(x_r) \frac{\eta\eta' s(\omega) s^*(\omega)}{c_0^2} \Lambda \tag{10}$$

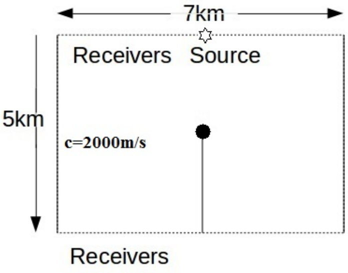
$$H^{cQ} = \int\int\int \frac{\kappa}{c_0} \left[\frac{1}{2} \tan\left(\frac{\xi}{2}\right) - \frac{1}{\pi} \ln\left|\frac{\omega}{\omega_0}\right| + \frac{i}{2} \right] \Lambda \tag{11}$$

$$H^{Qc} = \int\int\int \frac{\kappa}{c_0} \left[\frac{1}{2} \tan\left(\frac{\xi}{2}\right) - \frac{1}{\pi} \ln\left|\frac{\omega}{\omega_0}\right| - \frac{i}{2} \right] \Lambda \tag{12}$$

$$H^{QQ} = \int\int\int \frac{\kappa}{Q^2+1} \left[\frac{1}{4} \tan^2\left(\frac{\xi}{2}\right) - \frac{1}{\pi} \tan\left(\frac{\xi}{2}\right) \ln\left|\frac{\omega}{\omega_0}\right| + \frac{1}{4} \right] \Lambda, \tag{13}$$

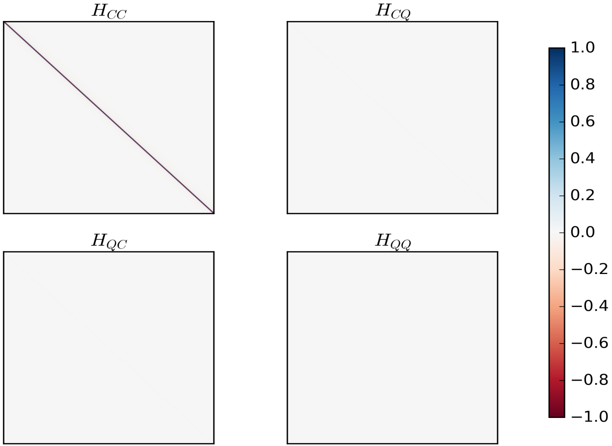
where, $R = r' + r'_s - r - r_s$, $\xi = \arctan\left(\frac{1}{Q}\right)$, $\kappa = \frac{\eta\eta' s(\omega) s^*(\omega)}{Q^2+1}$,
$\eta = -\frac{2\omega^2}{r r_s}$, $\eta' = -\frac{2\omega^2}{r' r'_s}$, $\Lambda = \exp\left[\frac{i\omega R}{c_0 \left|\frac{\omega}{\omega_0}\right|^{\xi/\pi}}\right] \exp\left[\frac{-\omega R \tan\left(\frac{1}{2}\xi\right)}{c_0 \left|\frac{\omega}{\omega_0}\right|^{\xi/\pi}}\right]$.

Acquisition model



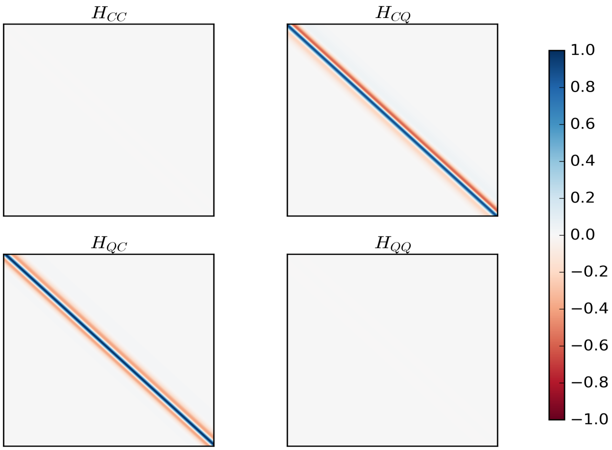
Homogeneous model with a perturbation at the center.

Reflection experiment



No trade-off between parameters. No attenuation $Q = 10,000$.

Reflection experiment



Hessian with high attenuation, $Q = 10$.

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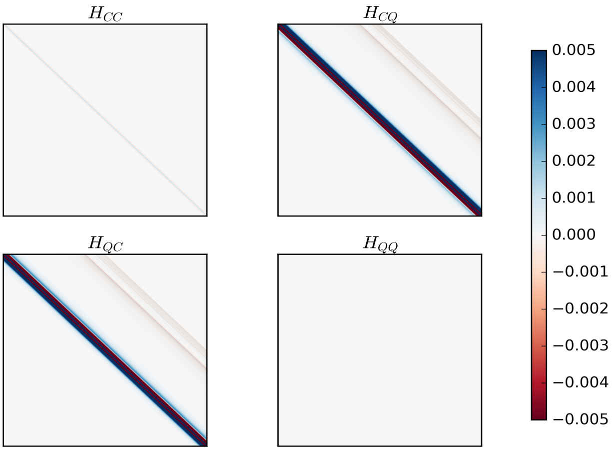
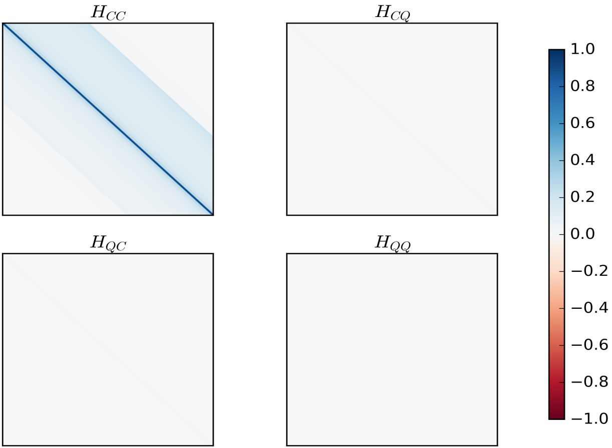


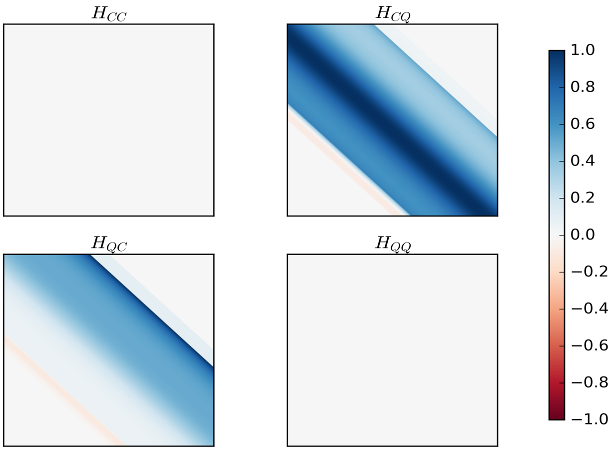
Hessian with low attenuation, $Q = 50$.

Transmission experiment



No trade-off between parameters. No attenuation, $Q = 10,000$.

Transmission experiment



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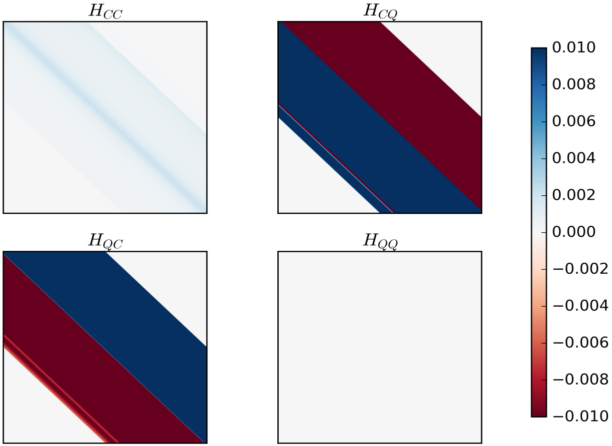

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- It is not possible under this model and parametrization to distinguish between the effects of the perturbations of the parameters of the model in the inversion.
- This results can be extrapolated to viscoelasticity.

Acknowledgments

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