Shear wave singularities in tilted orthorhombic media

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April 24th, 2017
RoSe Meeting
Trondheim, Norway

NTNU – Trondheim
Norwegian University of
Science and Technology

Outline

Background
Objectives

→ Occurrence of singularities in ORT media,

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- → Traveltime parameters in the vicinity of singularity points.

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[1] Crampin (1981, 1991); Vavryčuk (2003a); Grechka (2015)

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Background

S-wave singularities

Crampin (1991):
"In all anisotropic solids, there are directions of propagation, known as shear-wave singularities, where the split shear-waves have the same phase-velocities."

S-waves peculiarities

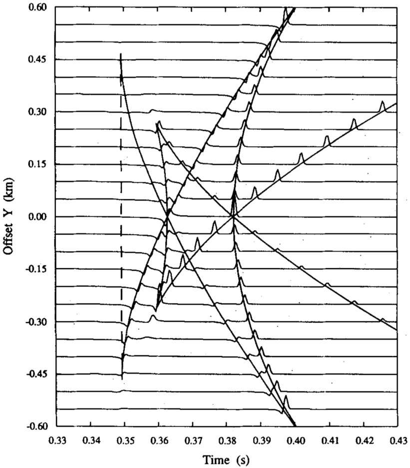

Figure 1: Synthetic seismogram in the vicinity of a singularity in an olivine sample (from Rümpker and Thomson, 1994).

Background and motivation

Why study S-wave singularities?

→ Modelling[1],

[1] Vavryčuk (2001)

Background and motivation

Why study S-wave singularities?

→ Modelling[1],

→ Microseismic monitoring[2].

[1] Vavryčuk (2001)
[2] Vavryčuk (2013); Grechka (2015)

S-wave singularities classification

Three types[1]:

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3. Point singularity (conical point, acoustic axis): orthorhombic and lower symmetry media.

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S-wave singularities

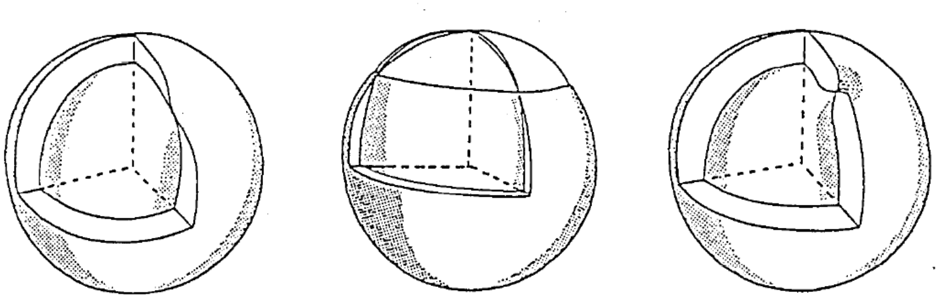


Figure 2: From left: kiss, line, and point singularities (scale is exaggerated). From Crampin (1991).

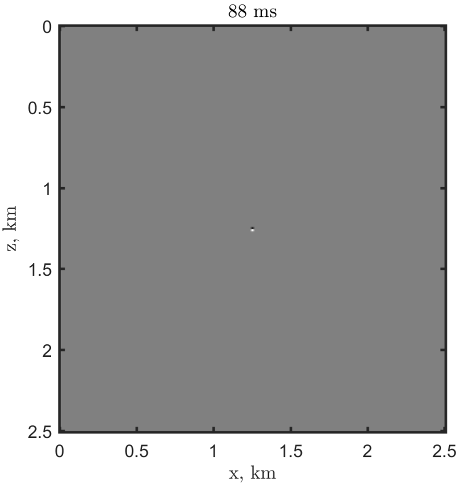
S-wave singularities

Point singularity

Courtesy of Mike Naylor

What is a medium's complexity?

Wavefronts



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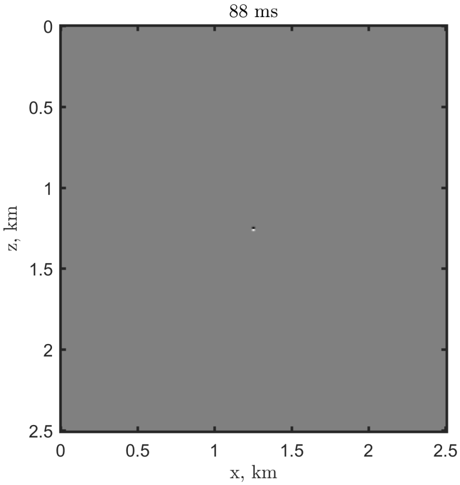

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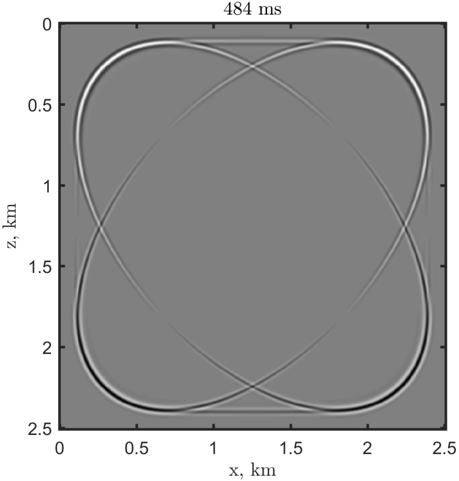
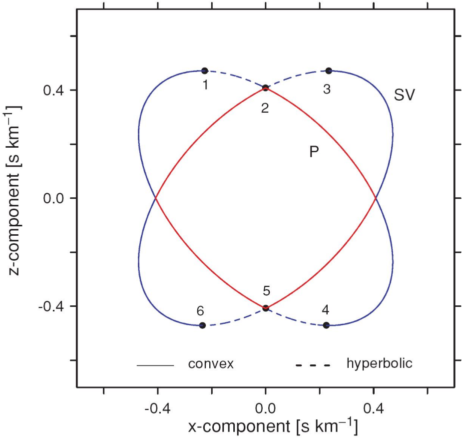


Restricted VTI[1]

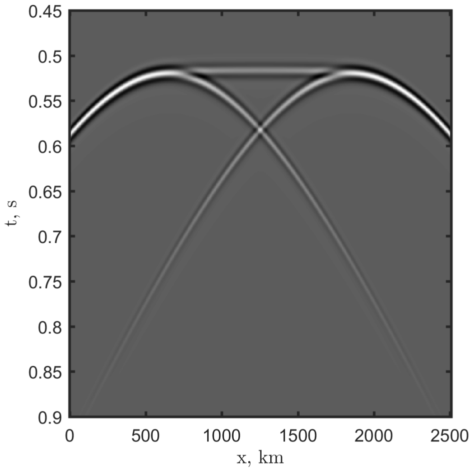
Slowness surface[2]



[1] Payton (1992)
[2] Vavryčuk (2003b)

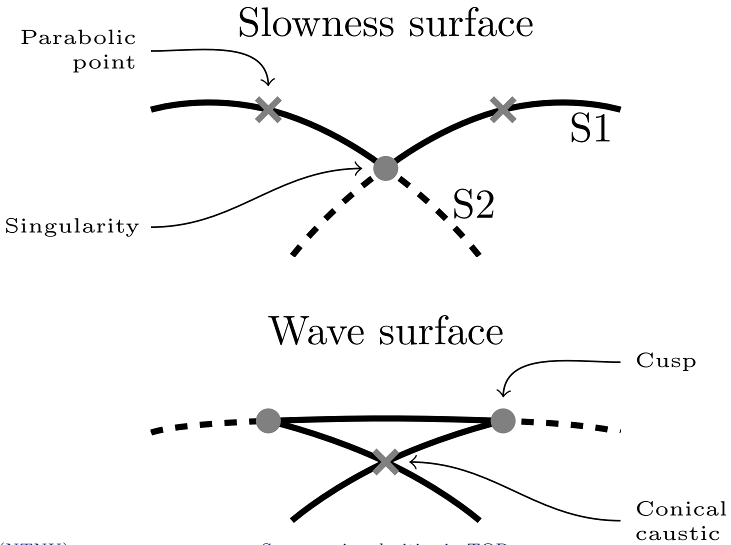
Restricted VTI

Recording



Point singularity

Phase–group correspondence



Conical points in orthorhombic media

- Maximum allowed number[1]: 16: 4×each plane + 4×outside,

[1] Musgrave (1985)

Conical points in orthorhombic media

- Maximum allowed number[1]: 16: $4\times$each plane + $4\times$outside,
- Minimum[2]: 0.

[1] Musgrave (1985)
[2] Alshits and Lothe (1979)

Conical points in orthorhombic media

Perturbation of ISO media to ORT

$$c_{ijkl}^{(\mathrm{ORT})} = c_{ijkl}^{(\mathrm{ISO})} + \varepsilon \tilde{c}_{ijkl}^{(\mathrm{ORT})}, \tag{1}$$

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All the singularity directions are found[1].

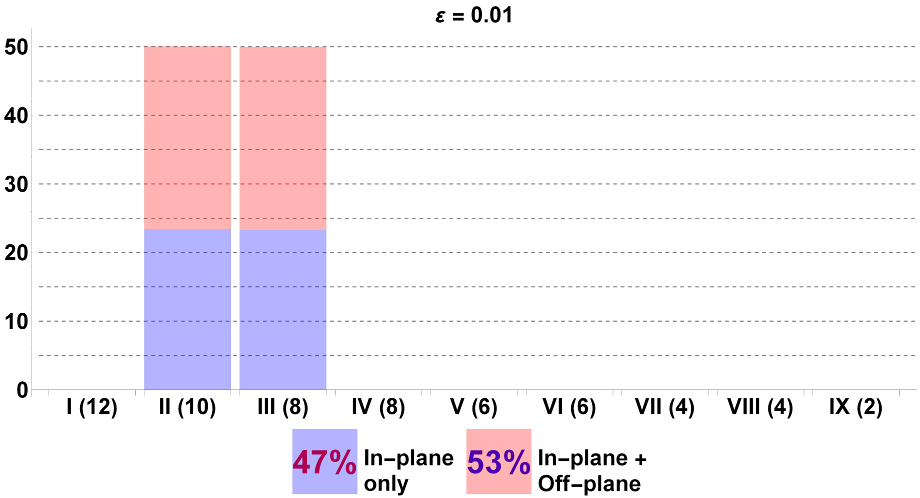
[1] Boulanger and Hayes (1998)

Conical points in orthorhombic media

In-plane singularities: distribution classes

Class	X	Y	Z	Multiplicity	Total # of in-plane S
I	2	2	2	1	12
II	1	2	2	3	10
III	1	1	2	3	8
IV	0	2	2	3	8
V	0	1	2	6	6
VI	1	1	1	1	6
VII	0	0	2	3	4
VIII	0	1	1	3	4
IX	0	0	1	3	2

Conical points distribution



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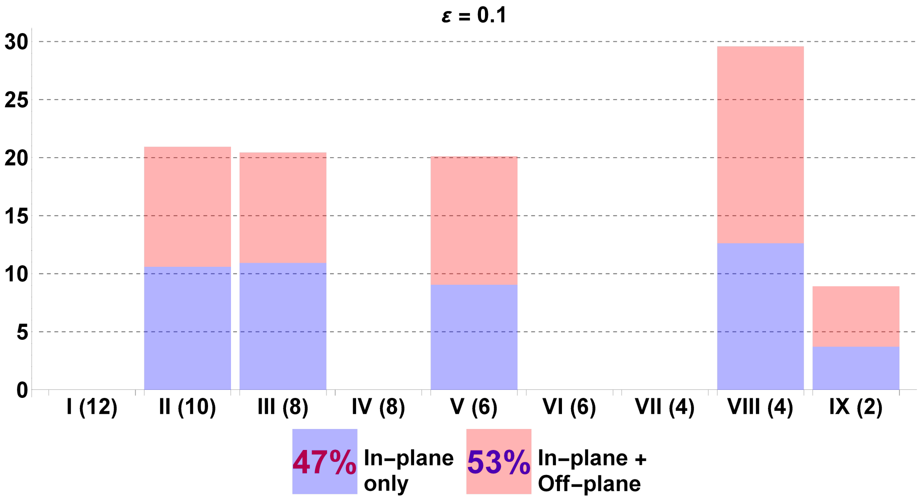

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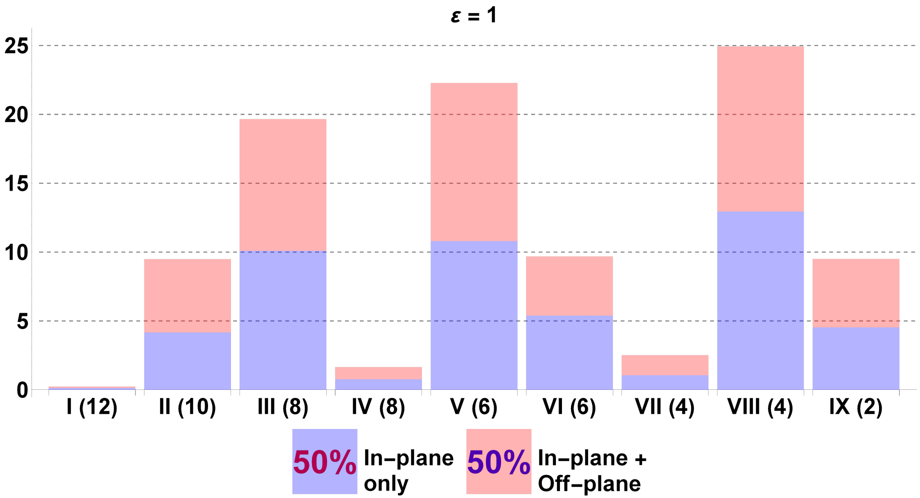

Conical points distribution

Dependence on the perturbation magnitude[1]

$$c_{ijkl}^{(\mathrm{ORT})} = c_{ijkl}^{(\mathrm{ISO})} + \varepsilon \tilde{c}_{ijkl}^{(\mathrm{ORT})}, \qquad (2)$$

[1] Vavryčuk (2005)

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$$c_{ijkl}^{(\mathrm{ORT})} = c_{ijkl}^{(\mathrm{ISO})} + \varepsilon \tilde{c}_{ijkl}^{(\mathrm{ORT})}, \tag{2}$$

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$\varepsilon \in [-1000, 1000]$,

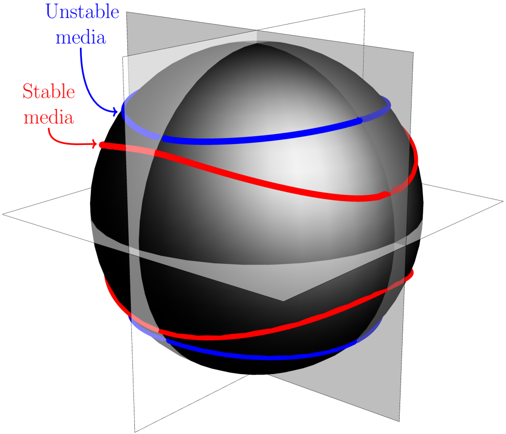
$$\tilde{c}_{ijkl}^{(\mathrm{ORT})} = \begin{bmatrix} -0.4117 & 0.4118 & 0.5525 & 0 & 0 & 0 \\ 0.4118 & 0.6576 & 0.6092 & 0 & 0 & 0 \\ 0.5525 & 0.6092 & -0.7989 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8755 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1565 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1606 \end{bmatrix}$$

Only off-planes singularity directions are considered.

[1] Vavryčuk (2005)

Conical points distribution

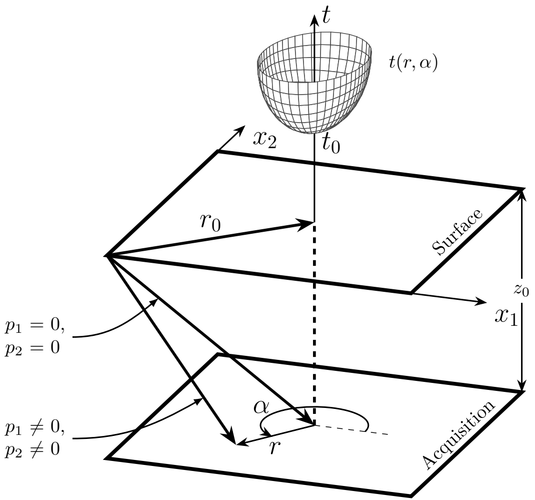
Dependence on the perturbation magnitude[1]



[1] Vavryčuk (2005)

Traveltime parameters.

The one-way propagation



The one-way propagation

Traveltime expansion about its minimum

$$t^2(r,\alpha) = {t_0}^2 + \frac{1}{V_n^2(\alpha)} r^2 + ..., \tag{3}$$

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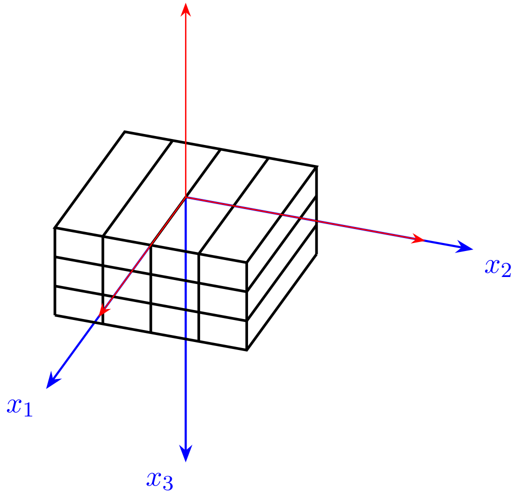
The traveltime minimum,

The normal moveout (NMO) velocity ellipse.

What if the orthorhombic symmetry planes are tilted?

Conical points in tilted orthorhombic media

Euler's angles θ and ψ



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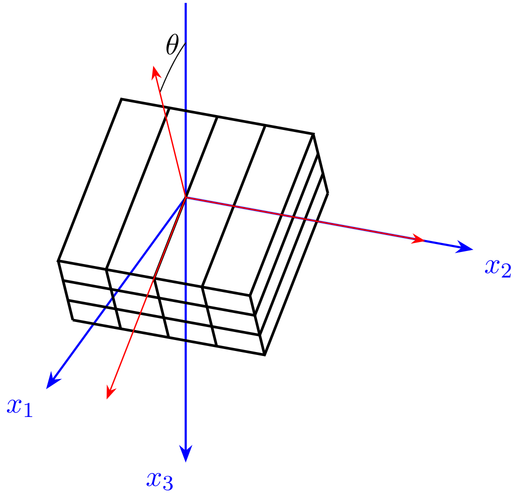

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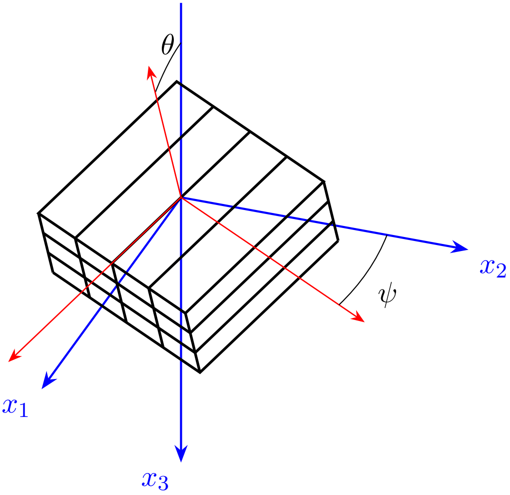
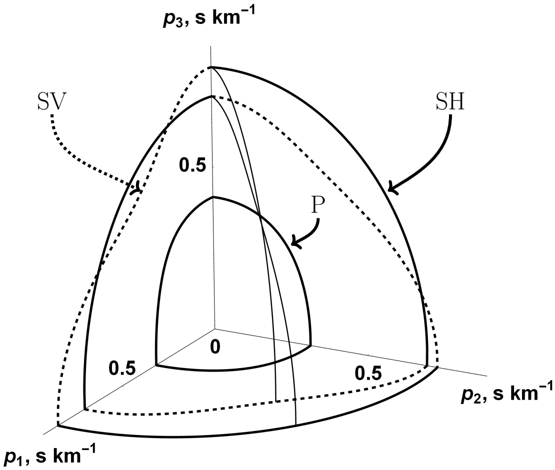


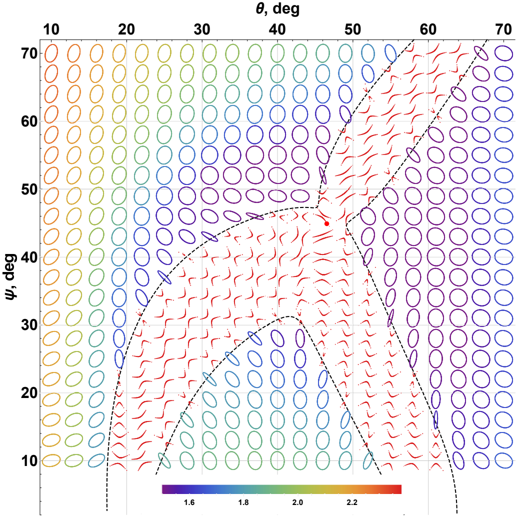
The traveltime parameters

The ORT model



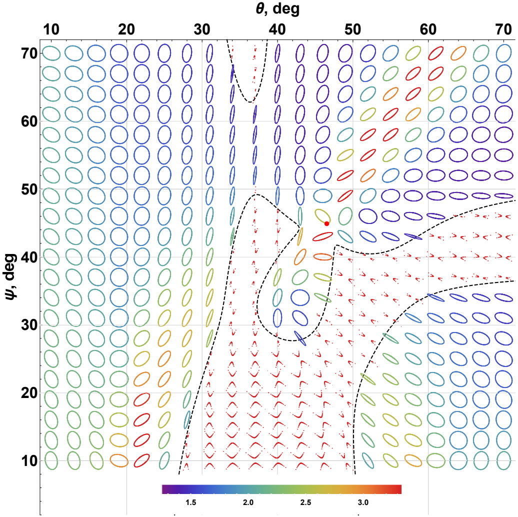
The traveltime parameters

The S1 NMO ellipse



The traveltime parameters

The S2 NMO ellipse



The traveltime parameters

The NMO ellipse

Higher-order traveltime parameters are also considered[1]

[1] Ivanov and Stovas (2017)

Conclusions

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Conclusions

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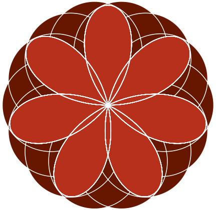
[1] Vavryčuk (2013, "Inversion for weak triclinic anisotropy from acoustic axes.")

Acknowledgments

Authors are thankful to the ROSE project for the financial support.



Thanks for your attention.

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