

# Depth dependent dilation factor

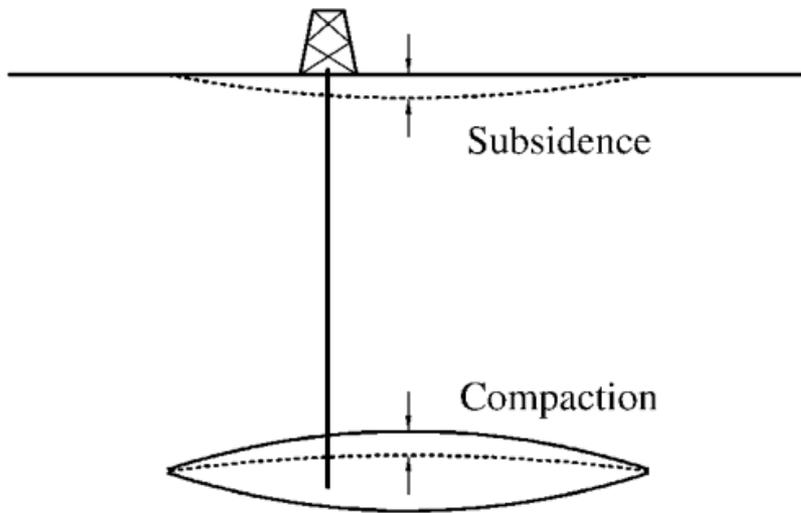
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Faculty of Engineering  
Department of Geoscience and Petroleum  
RoSe meeting  
Trondheim, 25<sup>th</sup> April, 2017

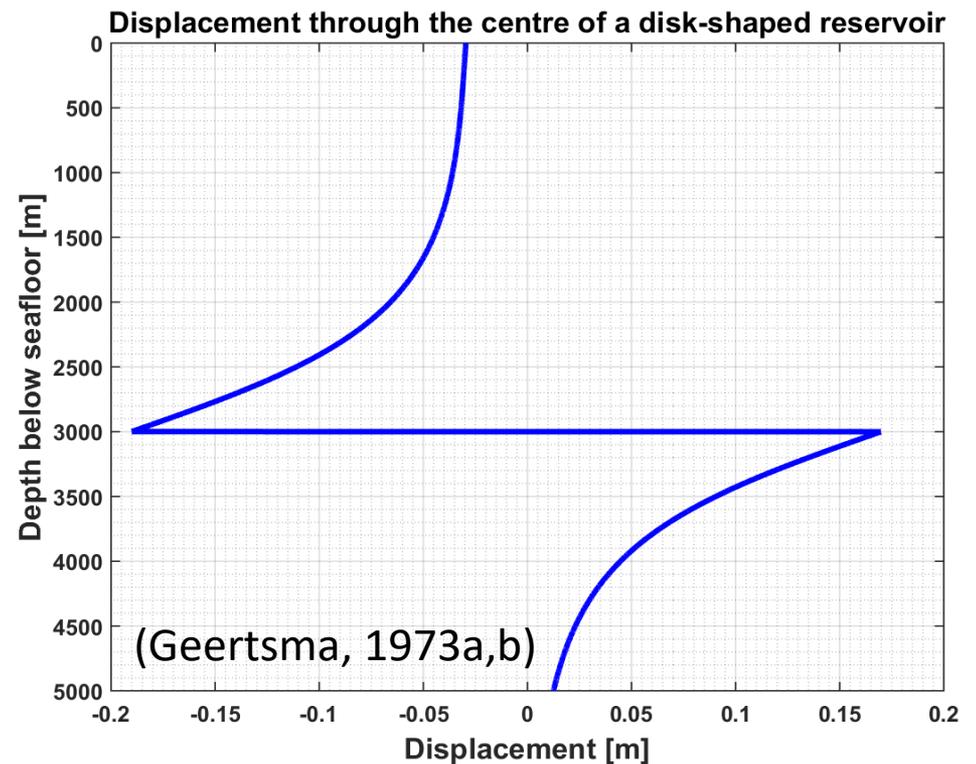
# Outline

- Introduction
- The dilation factor – Lab measurements and interpretation
- Modelling and discussion
- Conclusions

# Depleting reservoir Compaction and Subsidence



(Figure courtesy: Fjær et al., 2008)



# Introduction

## 4D travelttime analysis

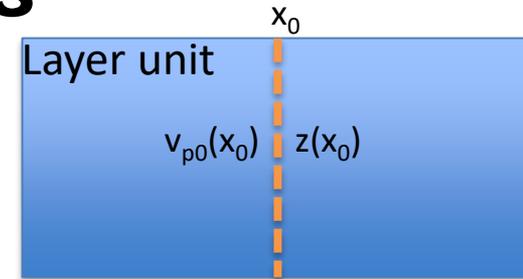
$t_0$  = two-way vertical time thickness of unit  
 $x_0$  = coordinate position along a line  
 $z$  = thickness of formation unit  
 $v_{p0}$  = vertical P-wave velocity of unit  
 $\Delta$  = changes in physical parameters  
 $\alpha$  and  $R$  = ratio between relative velocity  
 and thickness changes

$$t_0(x_0) = \frac{2z(x_0)}{v_{p0}(x_0)}$$

$$\frac{\Delta t_0(x_0)}{t_0(x_0)} \approx \frac{\Delta z(x_0)}{z(x_0)} - \frac{\Delta v_{p0}(x_0)}{v_{p0}(x_0)}$$

$$\frac{\Delta v_{p0}(x_0)}{v_{p0}(x_0)} = \alpha \frac{\Delta z(x_0)}{z(x_0)}$$

$$R = -\alpha$$



(Landrø and Stammeijer 2004)

(Røste et al., 2005)

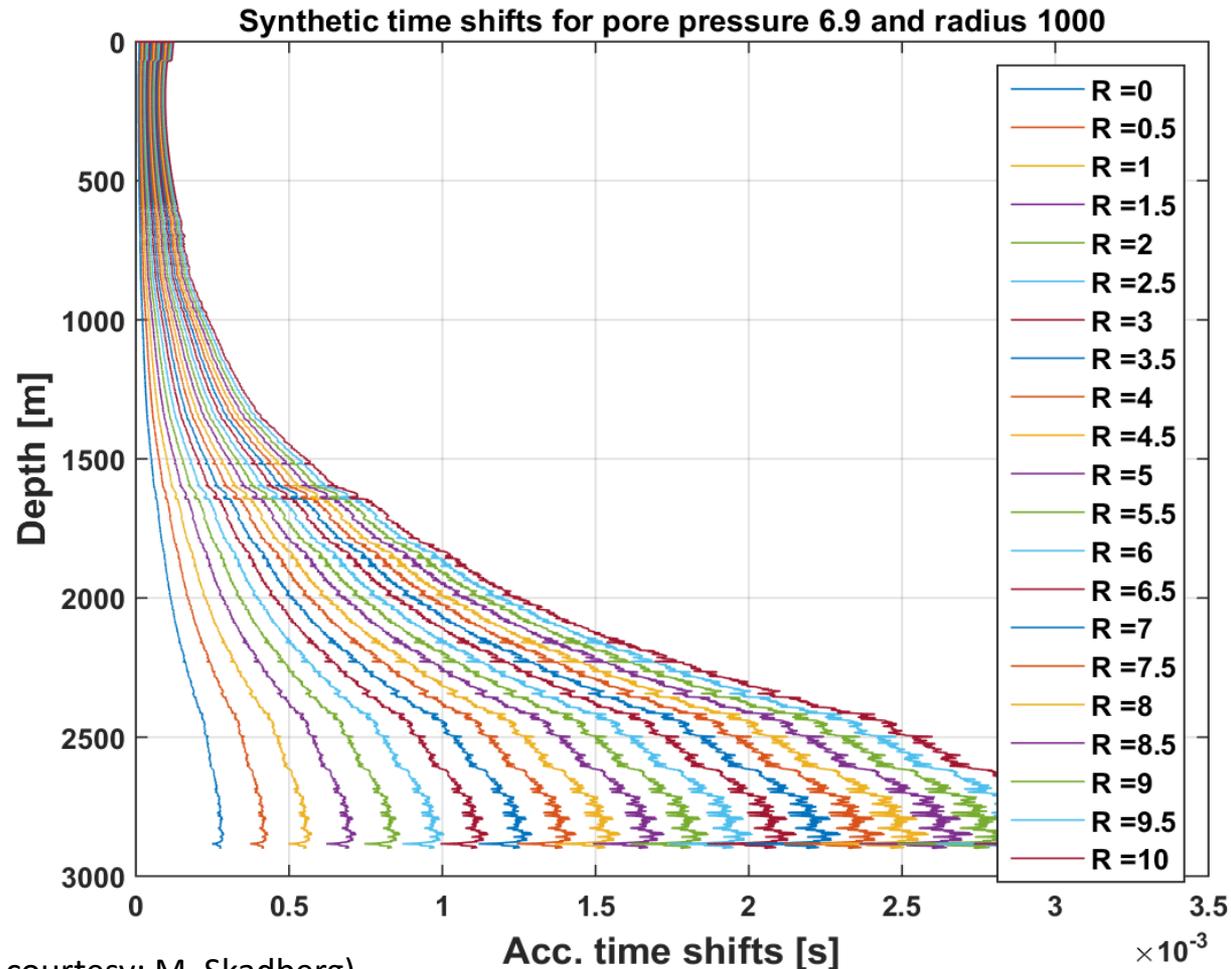
(Hatchell et al., 2005)

$$\left[ \frac{\Delta t_0(x_0)}{t_0(x_0)} \approx (1 + R(x_0)) \frac{\Delta z(x_0)}{z(x_0)} = (1 + R(x_0)) \varepsilon_a(x_0) \right]_{layer}$$

# Introduction

## Accumulated travelttime shifts

$$\Delta T = 2 \int_0^Z (1 + R(z)) \frac{\varepsilon_a}{v(z)} dz$$



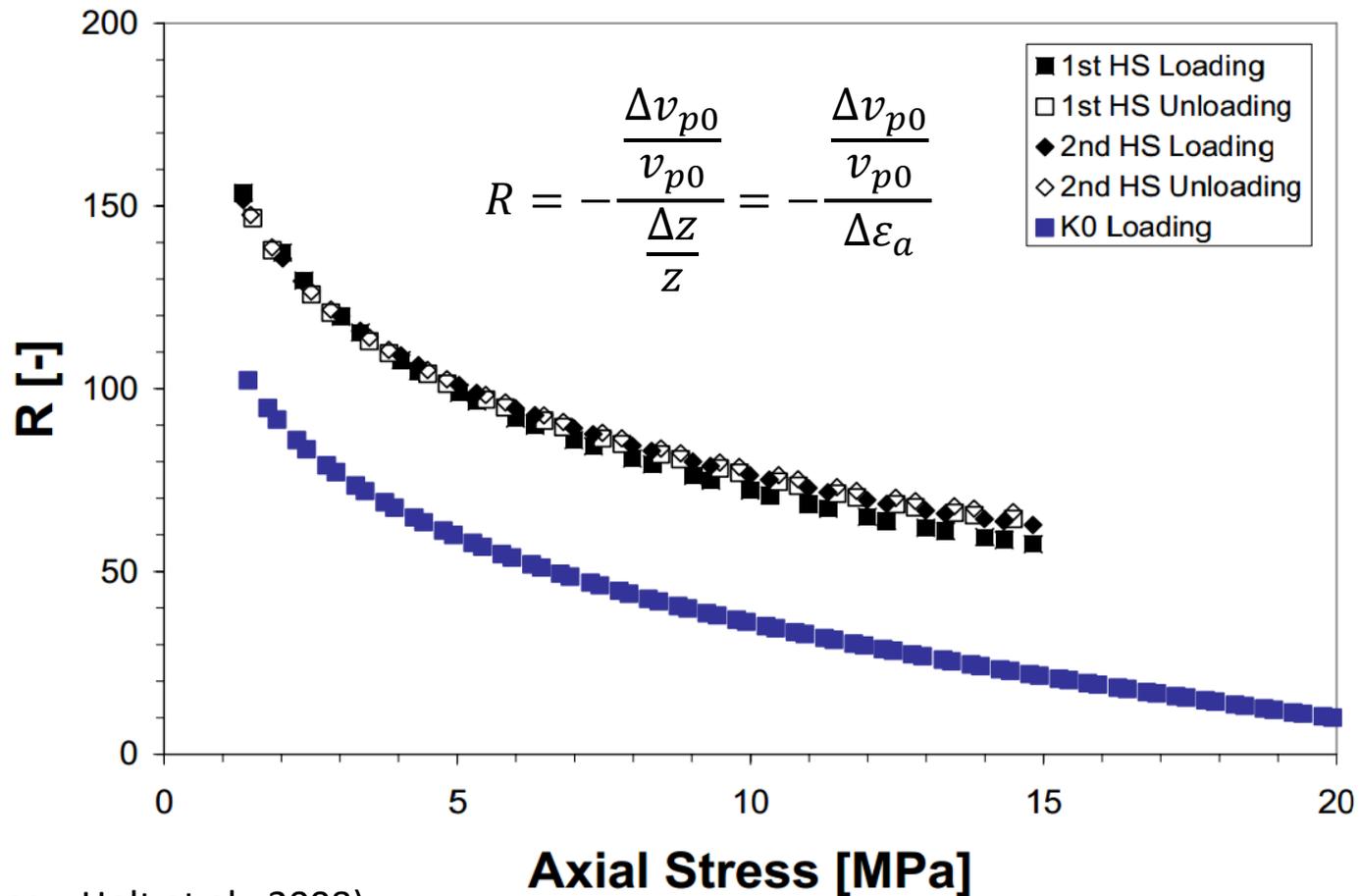
(Figure courtesy: M. Skadberg)

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# Dilation factor vs. axial stress

## Dry glass bead measurements



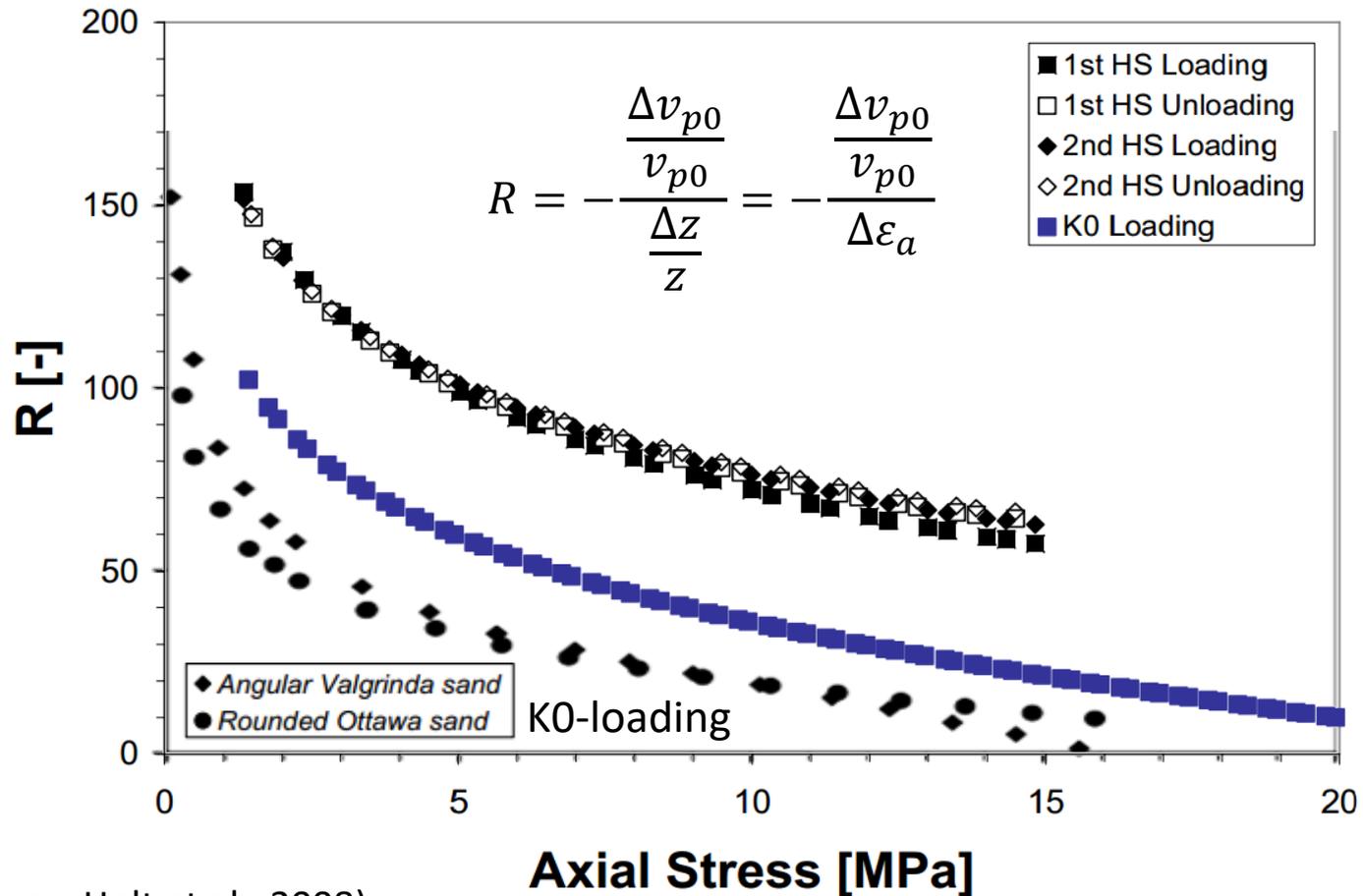
(Figure courtesy: Holt et al., 2008)

# Interpretation from lab experiments

- The dilation factor is stress path dependent
  - R-value for hydrostatic stress path  $>$  R-value for K0 loading

# Dilation factor vs. axial stress

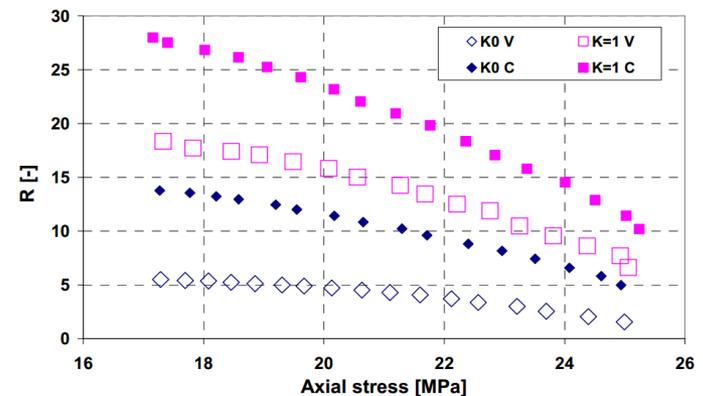
## Dry glass beads vs. unconsolidated dry sand



(Figure courtesy: Holt et al., 2008)

# Interpretation from lab experiments

- The dilation factor is dependent on the stress path
  - R-value for hydrostatic stress path > R-value for K0 loading path
- The R-value also dependent on grain contact conditions
  - .....but also “lithology”
- Lab. experiments show increased R-values due the coring process (Holt et al., 2008).



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# The “Hertz-Mindlin” grain contact model

## Hydrostatic loading Infinite or zero grain contact friction

Effective bulk modulus

$$K_{dry} = \left( \frac{C_p^2 (1 - \phi)^2 G_{ma}^2 \sigma}{18\pi^2 (1 - \nu_{ma})^2} \right)^{\frac{1}{3}}$$

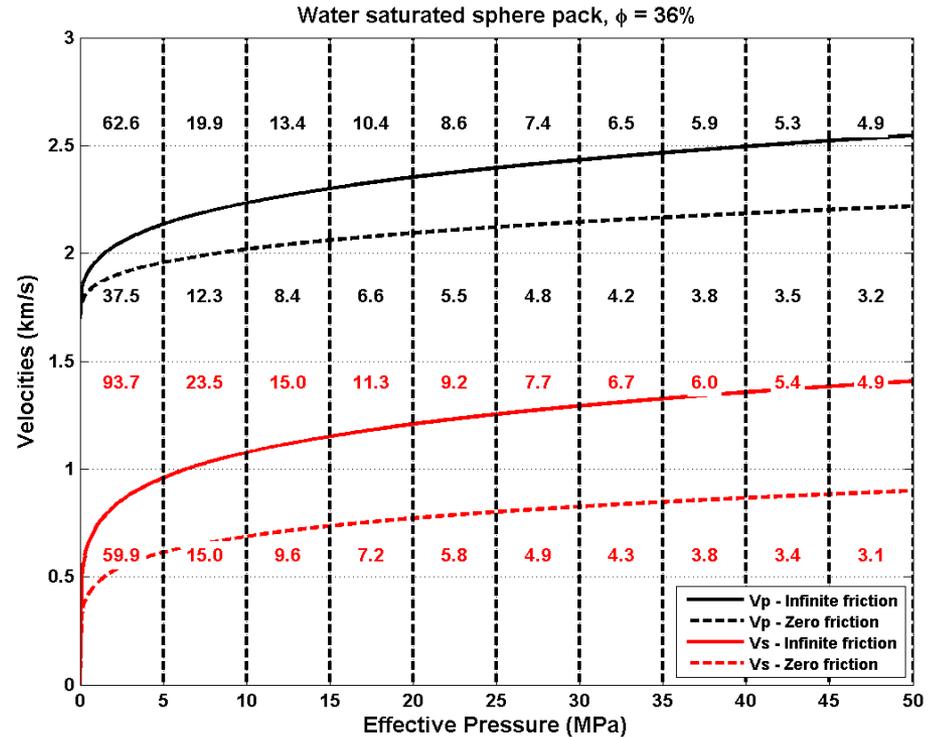
Effective shear modulus ( $\infty$  friction)

$$G_{dry} = \frac{3}{5} \left( \frac{5 - 4\nu_{ma}}{2 - \nu_{ma}} \right) K_{dry}$$

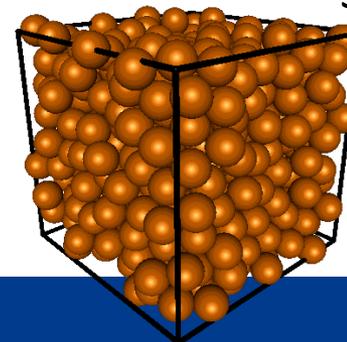
Effective shear modulus (zero grain friction)

$$G_{dry} = \frac{3}{5} K_{dry}$$

$\sigma$  = Differential stress  
 $\phi$  = Pore volume or Porosity  
 $\nu_{ma}$  = Grain Poisson's ratio  
 $G_{ma}$  = Grain shear modulus  
 $C_p$  = # grain contact points-



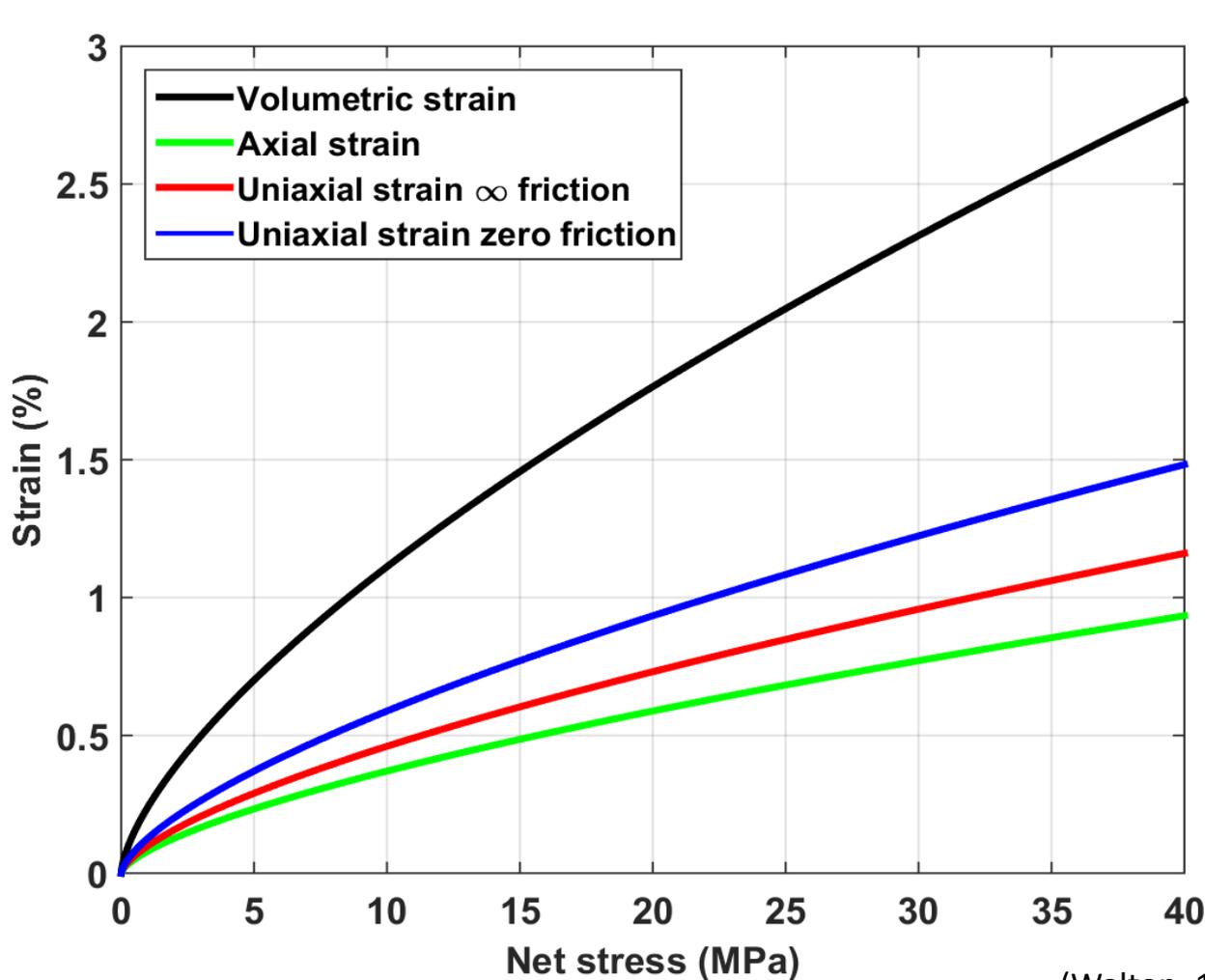
Sphere assembly



S. Torquato, 2000

# Stress-strain relation – Contact theory

## Hydrostatic and K0 loading conditions



$$\varepsilon_{vol\_HS}^e = 3 \left( \frac{3\pi(1 - \nu_{ma})\sigma}{2C_p(1 - \varphi)G_{ma}} \right)^{\frac{2}{3}}$$

$$\varepsilon_{ax\_uni}^e = \left( \frac{3\pi(1 - \nu_{ma})\sigma_{ax}}{C_p(1 - \varphi)G_{ma}} \right)^{\frac{2}{3}}$$

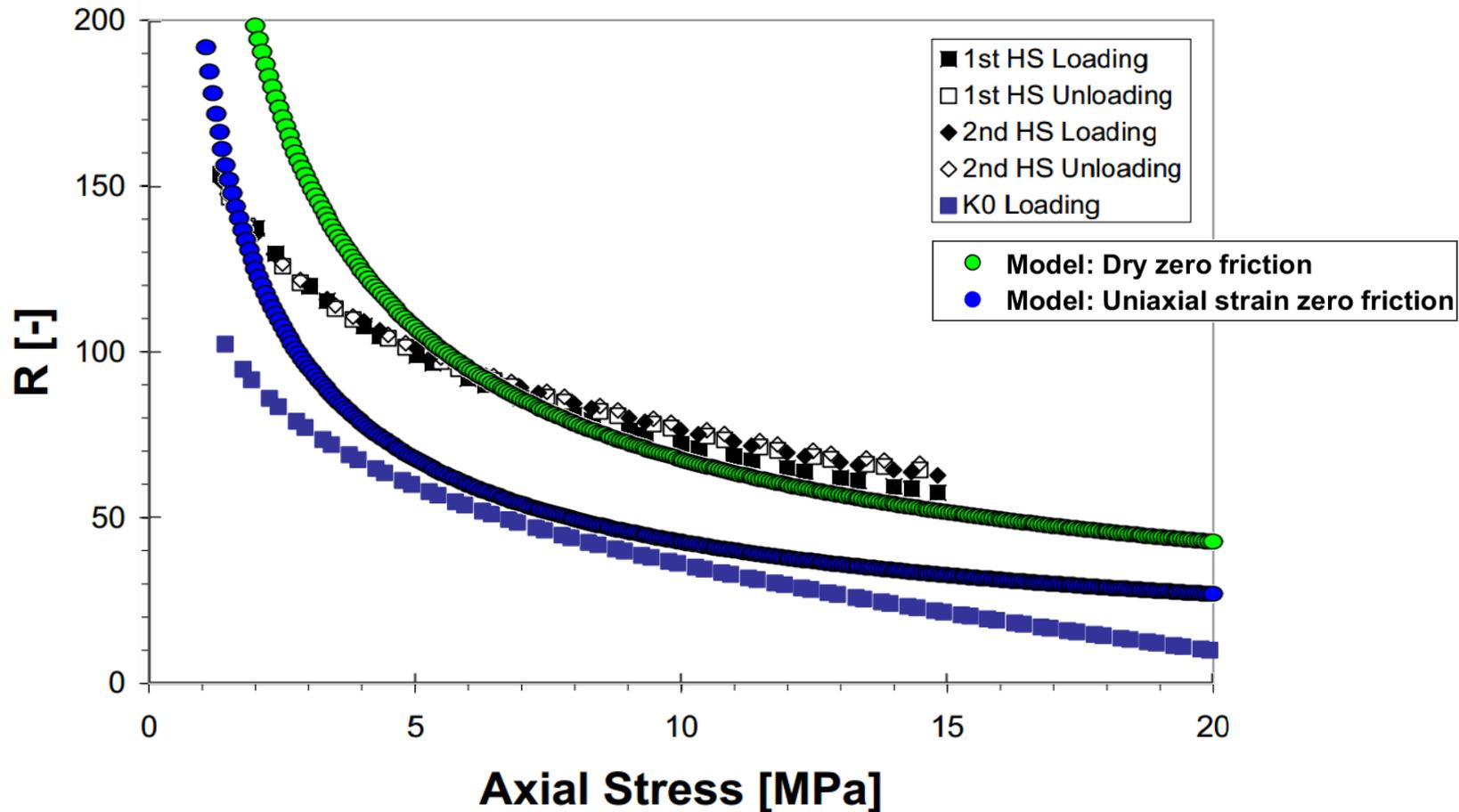
$$\varepsilon_{ax\_uni}^e = \left( \frac{3\pi(1 - \nu_{ma})(2 - \nu_{ma})\sigma_{ax}}{C_p(1 - \varphi)(3 - 2\nu_{ma})G_{ma}} \right)^{\frac{2}{3}}$$

$$\varepsilon_{ax\_HS}^e = \left( \frac{3\pi(1 - \nu_{ma})\sigma}{2C_p(1 - \varphi)G_{ma}} \right)^{\frac{2}{3}}$$

(Walton, 1987; Holt et al. 2007; Duffaut et al, 2011)

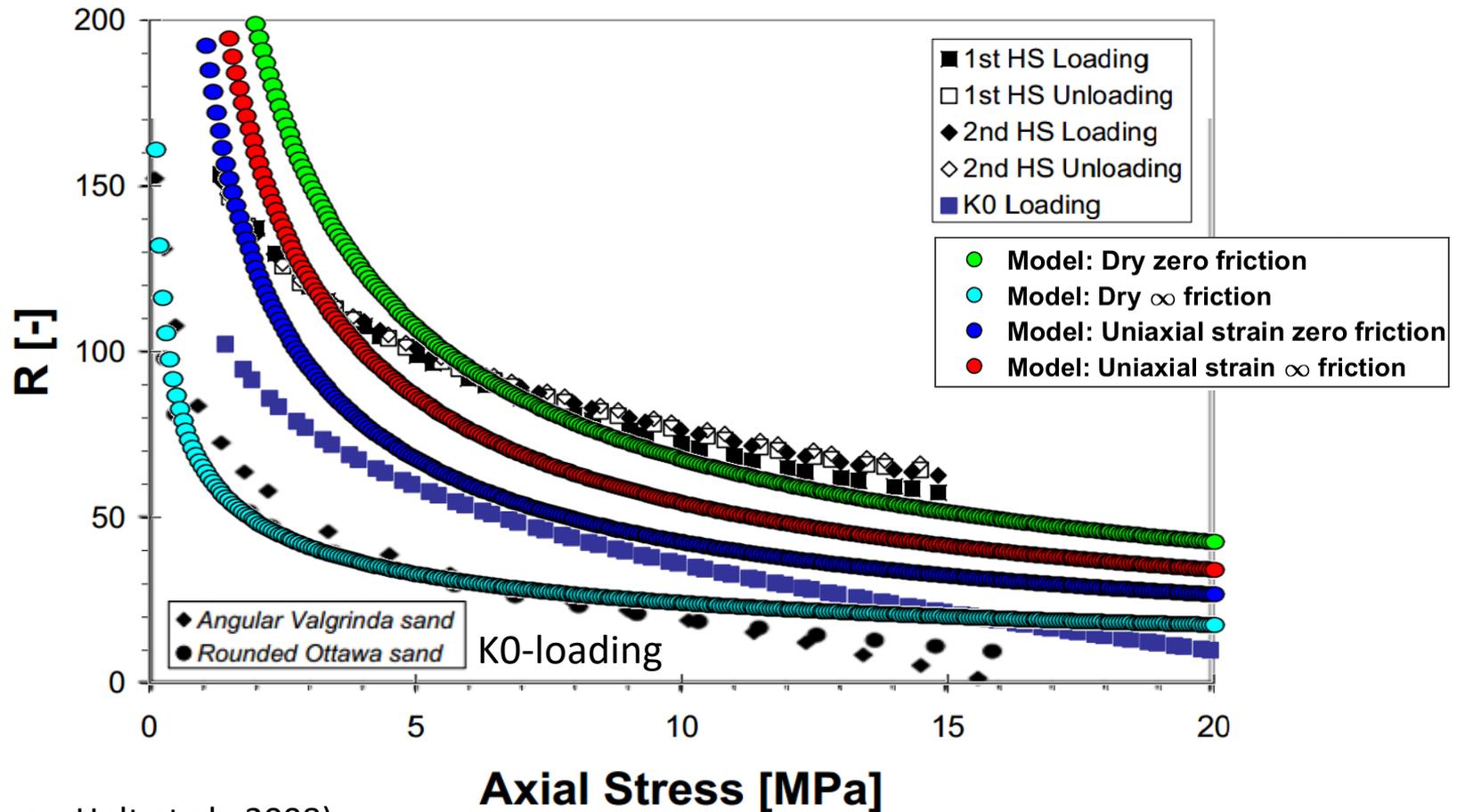
# Dilation factor vs. axial stress

## Dry glass bead measurements vs. grain contact theory



# Dilation factor vs. axial stress

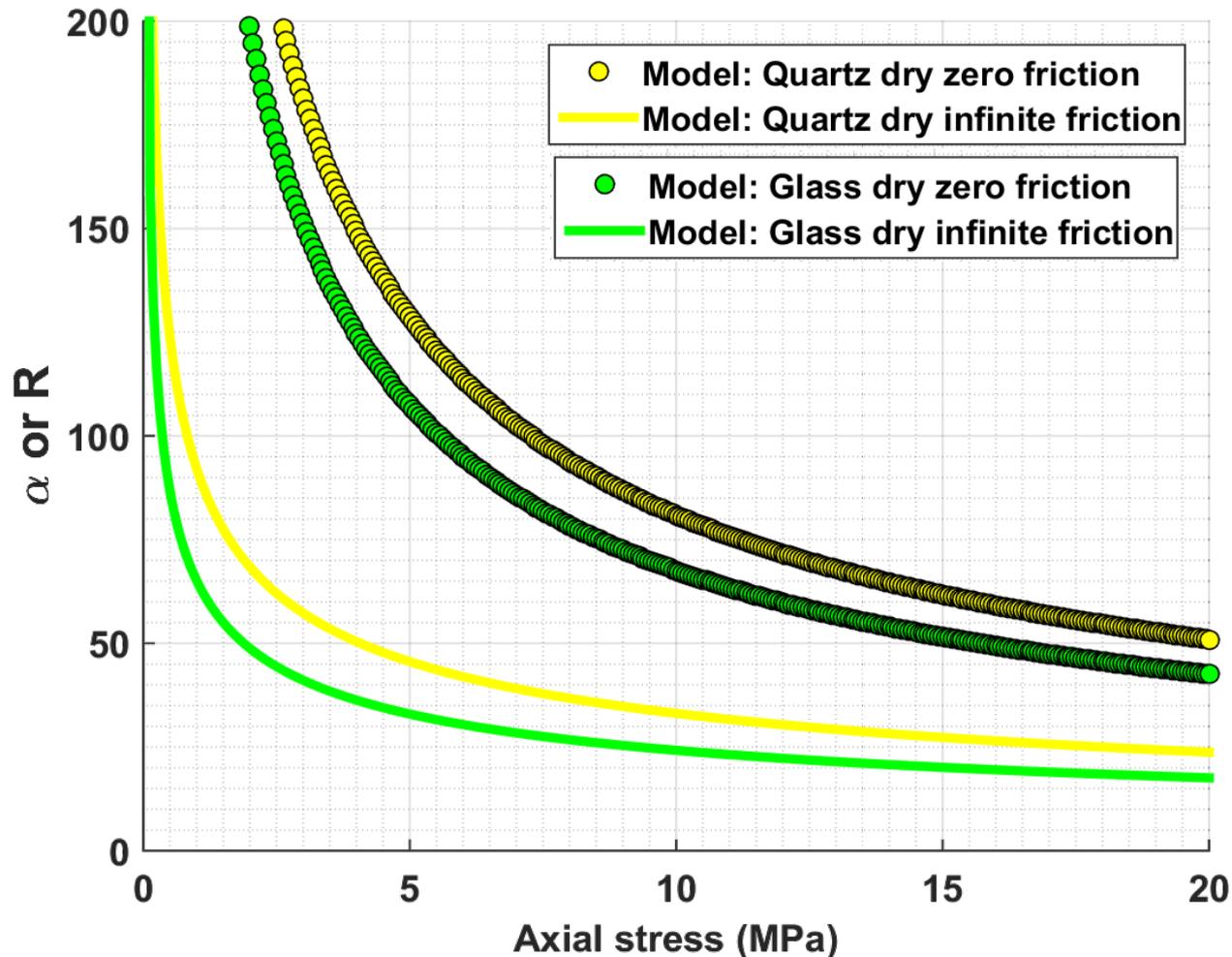
## Dry glass beads and sand vs. grain contact theory



(Figure courtesy: Holt et al., 2008)

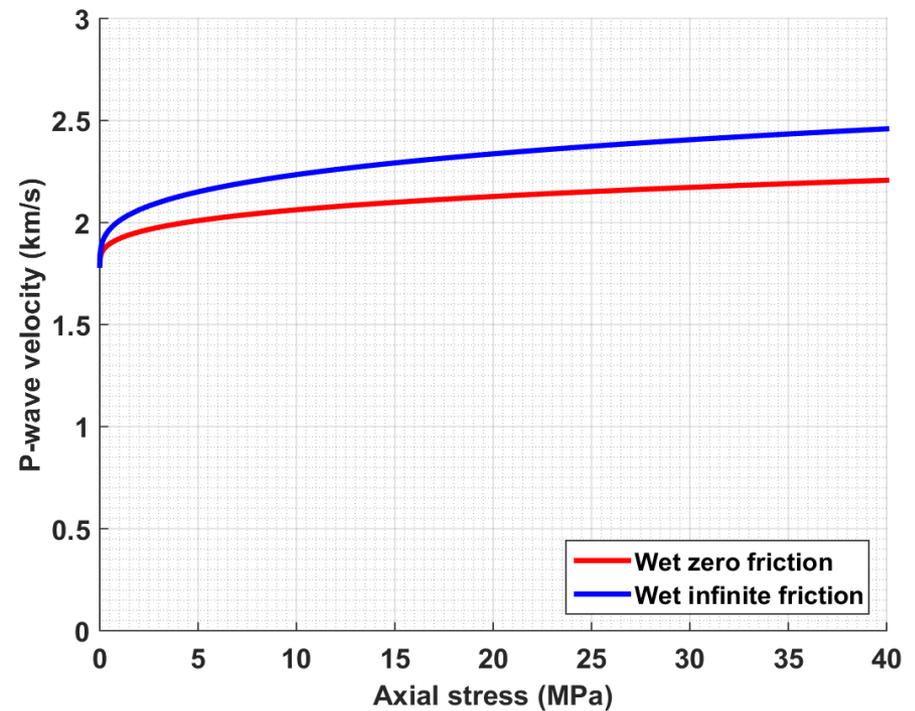
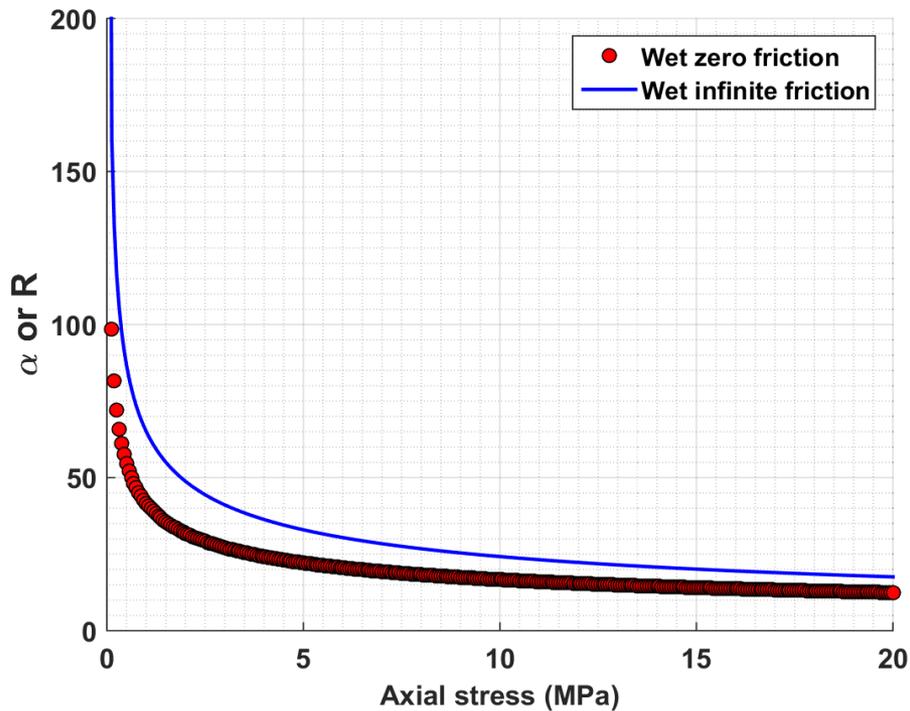
# Dilation factor vs. axial stress

Modelling the effect of rock stiffness (“lithology”)



# Dilation factor vs. axial stress

## Modelling wet glass beads – hydrostatic loading



# Summary

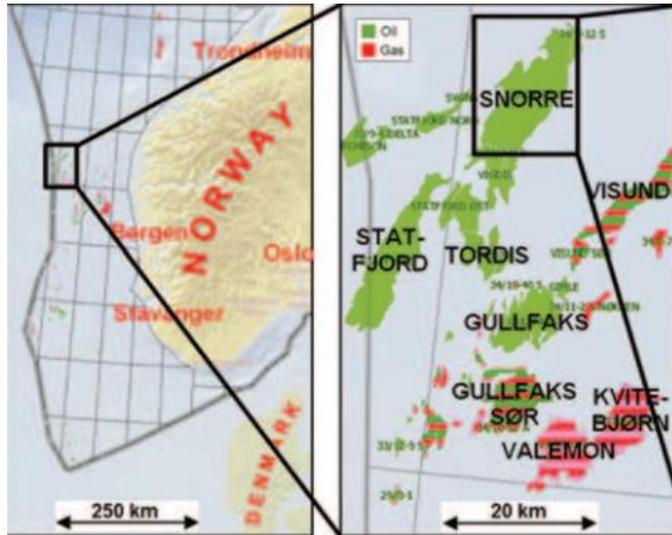
- Grain contact model (zero contact friction)
  - Hydrostatic and K0 loading path: R decrease with increasing axial stress similar to trends from lab measurements on glass beads
  - R for hydrostatic stress path > R for K0 loading path. R-level ok.
- Grain contact model (infinite contact friction)
  - Hydrostatic stress path: R value lower, but overall trend the same as for zero contact friction
  - K0 loading path: R values larger than that of zero grain friction
  - R for hydrostatic stress path < R for K0 loading path.
- Lowering of the matrix moduli reduces the R-value
  - Higher strains for lower matrix moduli

# Introduction

## Accumulated travelttime shifts

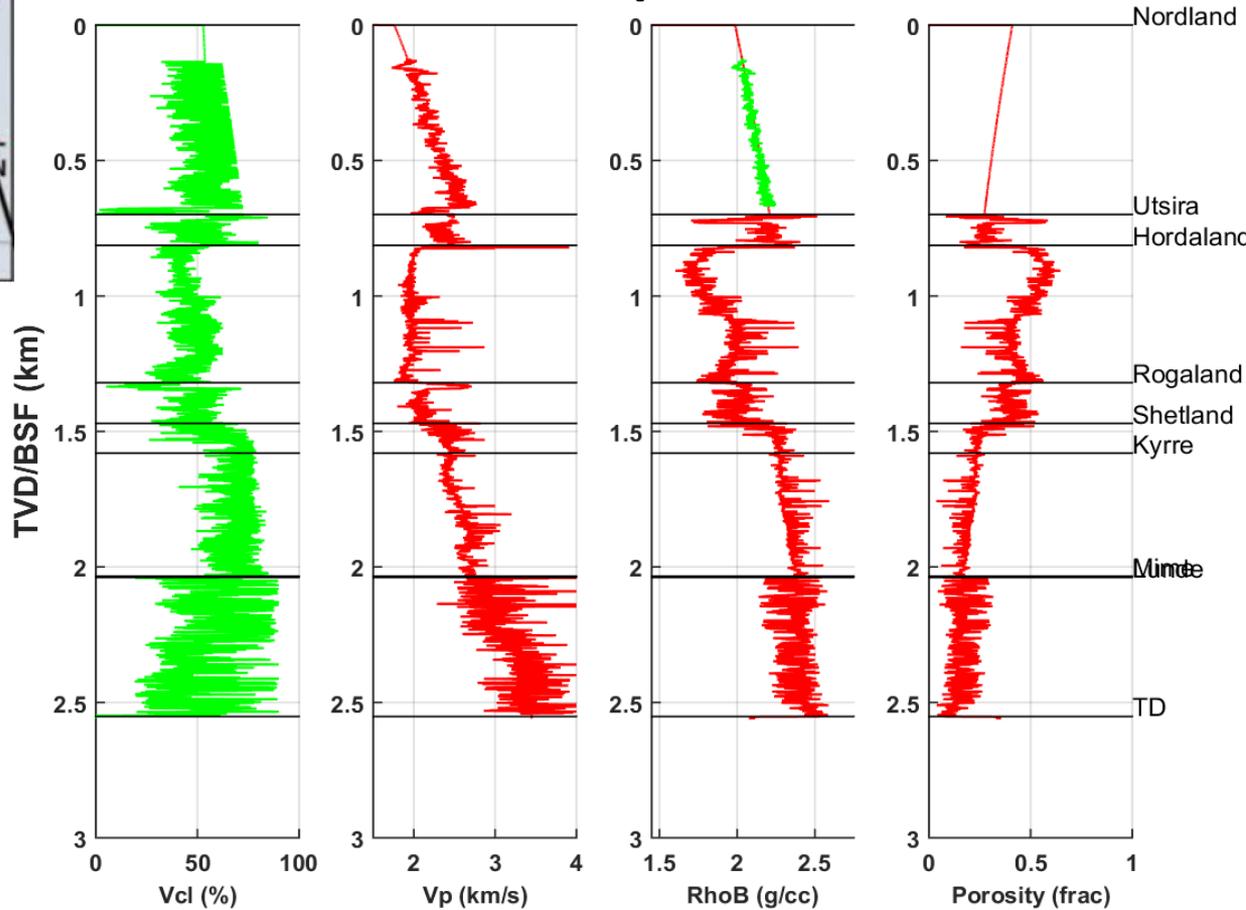
$$\Delta T = 2 \int_0^z (1 + R(z)) \frac{\varepsilon_a}{v(z)} dz$$

# The Snorre Field



(Figure courtesy: Røste et al., 2015)

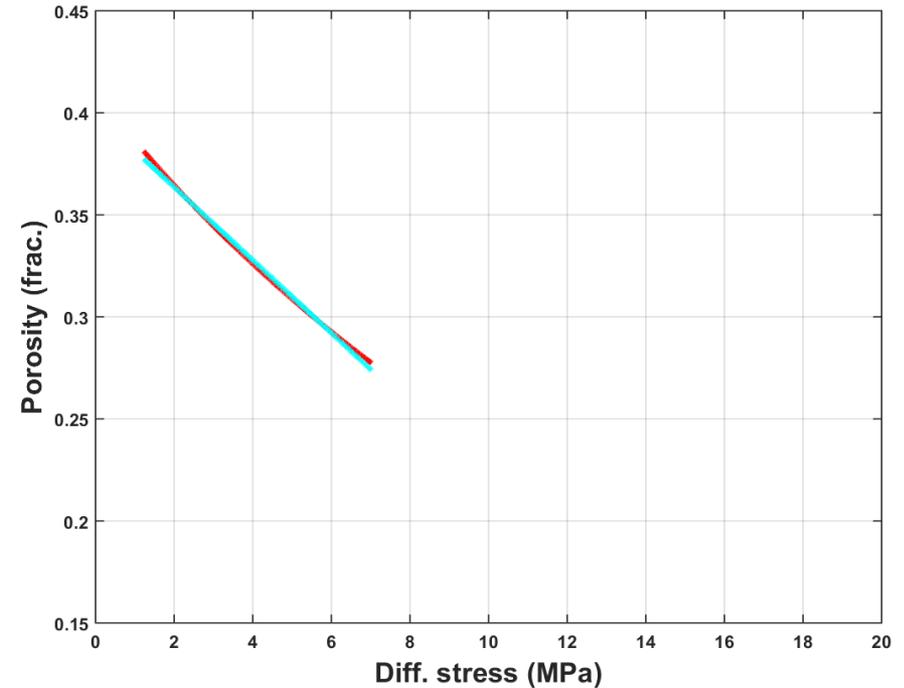
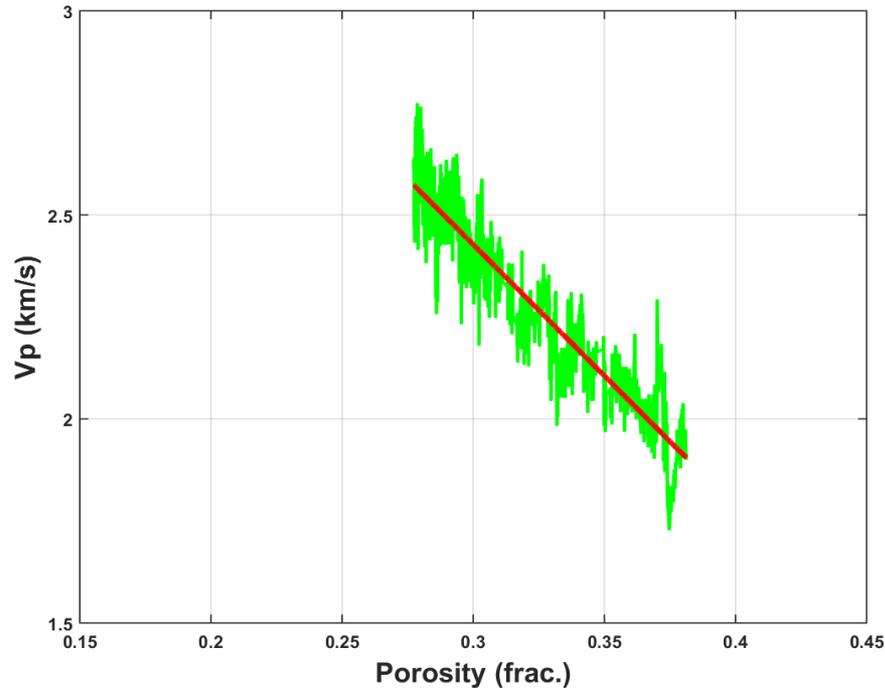
## Well panel



# The Nordland Group

## Estimated stress sensitivity of P-wave velocity

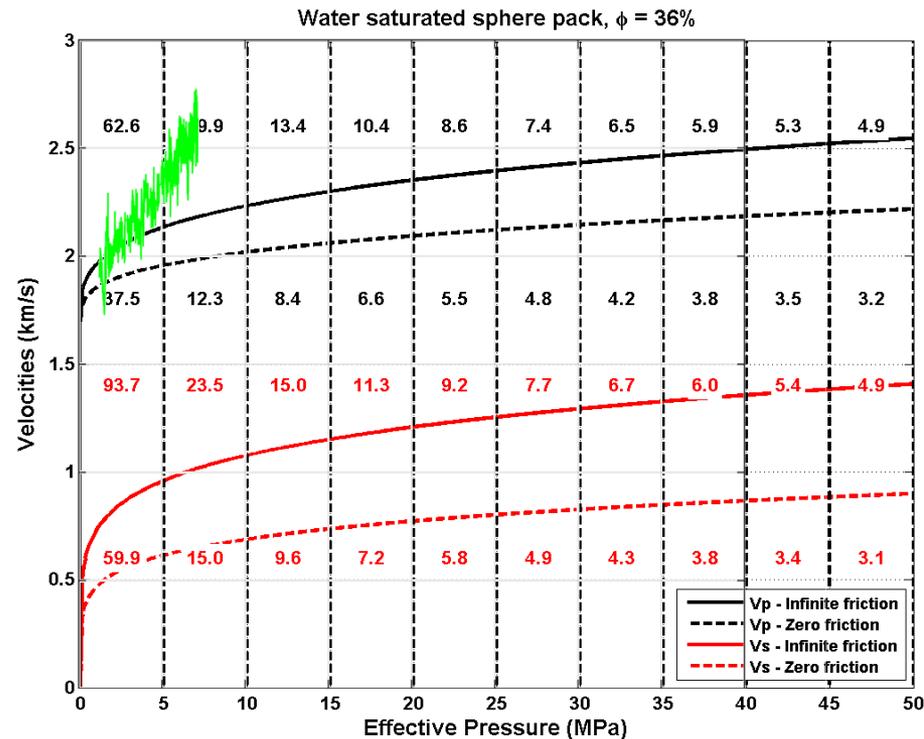
$$\frac{\Delta V_p}{\Delta \sigma'} \approx \frac{\Delta V_p}{\Delta \phi} \frac{\Delta \phi}{\Delta \sigma'} = -6432 \frac{m}{s} - 0.0179 \frac{1}{MPa} \approx 115 \frac{m/s}{MPa}$$



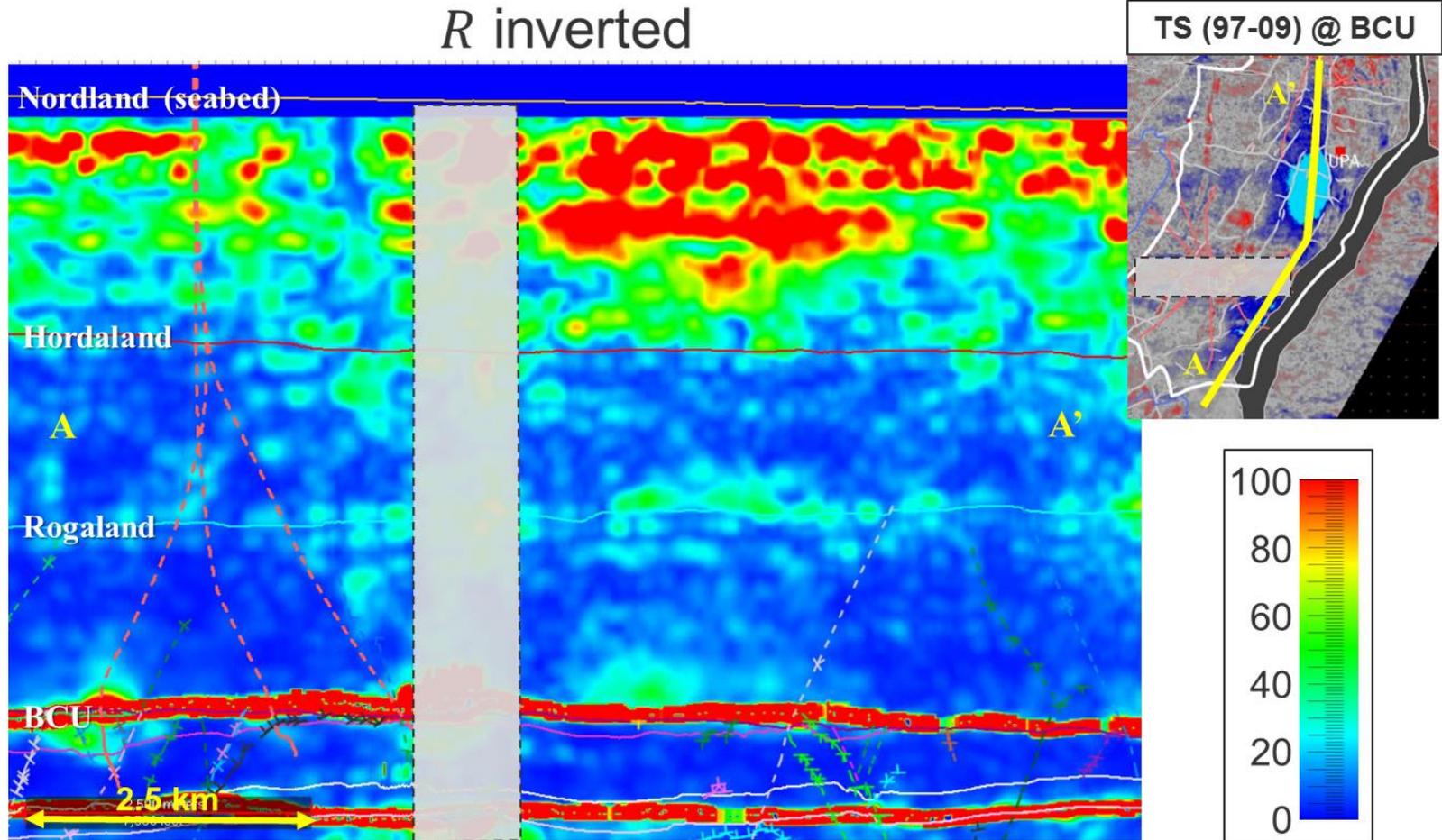
# The Nordland Group

## Estimated stress sensitivity of P-wave velocity

$$\frac{\Delta V_p}{\Delta \sigma'} \approx \frac{\Delta V_p}{\Delta \varphi} \frac{\Delta \varphi}{\Delta \sigma'} = -6432 \frac{m}{s} - 0.0179 \frac{1}{MPa} \approx 115 \frac{m/s}{MPa}$$



# Dilation factor estimated from 4D analysis



(Figure courtesy: Røste and Ke, 2017)

# Conclusions

- $\alpha$  or R is dependent upon stress path
- $\alpha$  or R is lithology dependent (stress-strain relation)
  - Lab. experiments show increased R-values due the coring process (Holt et al., 2008).
- $\alpha$  or R is like to vary with depth especially with sand layers
  - The thickness of these layers will have variable impact on the accumulated travelttime differences
- Grain contact theory simulating hydrostatic and K0 loading path explains fairly well the  $\alpha$  (or R) trends for unconsolidated sands.

Thank you