Accuracy of finite-difference modeling

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Overview

1. Introduction
2. Physical modeling
3. Finite-difference modeling
4. Results
5. Conclusions

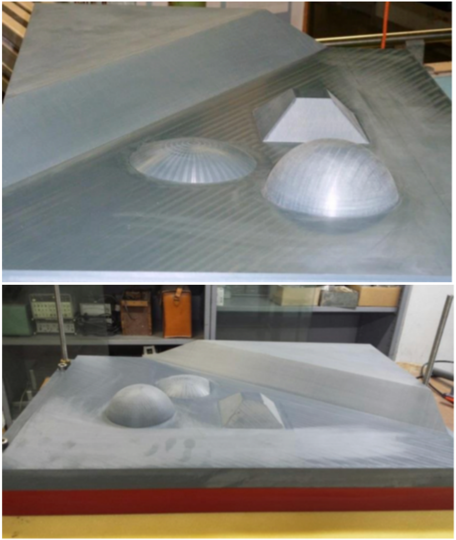
Introduction

- Finite-difference modeling is extensively used in imaging and inversion of seismic data
- The use of high-order spatial derivatives apparently allows the use of coarse grids
- Acceptable run-times even for very large models using modern CPUs and GPUs
- Simple and efficient coding
- Models represented by simple regular grids

Introduction

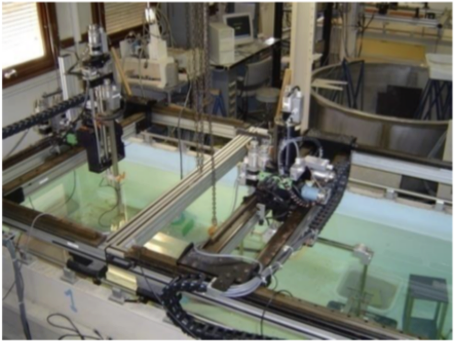
- Complex, curved interfaces are difficult to represent with regular grids
- Plane, sloping layers are known to be problematic for finite-difference methods
- There is a need for calibrating finite-difference modeling against data from known models

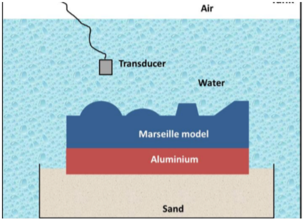
The BENCHIE model



(courtesy of Favretto-Cristini, 2015,[1])

The BENCHIE model

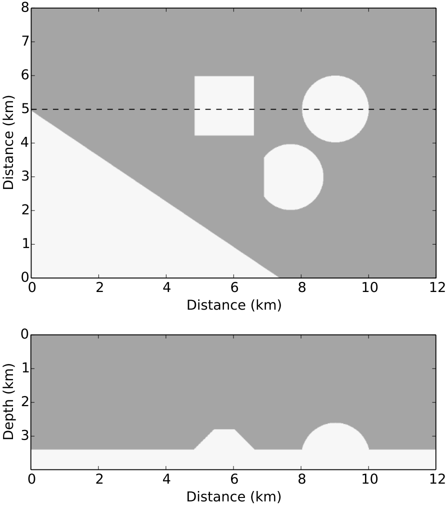




(courtesy of Favretto-Cristini, 2015,[1])

The BENCHIE model

Scale factor: 20000

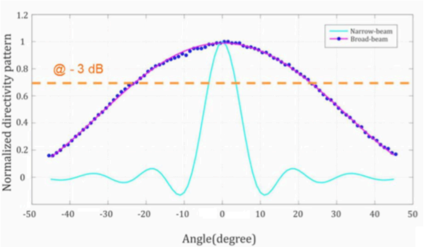


The BENCHIE model

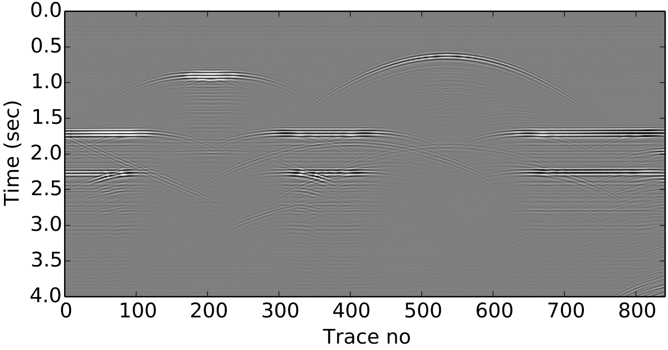
Physical properties

Material	V_p	V_s	Density	Q_p	Q_s
Water	1480	0.0	1000	∞	∞
PVC	2220	1050	1412	42	32

The BENCHIE model



The BENCHIE model



Finite-difference modeling

Visco-elastic equations

$$\begin{aligned} u_i(\boldsymbol{x},t) &= \partial_j \sigma_{ij}(\boldsymbol{x},t) \\ \sigma_{ij}(\boldsymbol{x},t) &= \lambda(\boldsymbol{x},t) * e_{kk}(\boldsymbol{x},t)\delta_{ij} + 2\mu(\boldsymbol{x},t) * e_{ij}(\boldsymbol{x},t) + q_{ij}(\boldsymbol{x},t) \\ e_{ij}(\boldsymbol{x},t) &= \frac{1}{2}[\partial_j u_i(\boldsymbol{x},t) + \partial_i u_j(\boldsymbol{x},t)] \end{aligned} \tag{1}$$

Where

- $\boldsymbol{x}, t$: Space, time
- $u_i(\boldsymbol{x},t)$: Displacement vector i
- $\sigma(\boldsymbol{x},t)$: Stress tensor
- $\lambda(\boldsymbol{x},t), \mu(\boldsymbol{x},t)$: Lameé parameters
- $q_{ij}(\boldsymbol{x},t)$: Source tensor

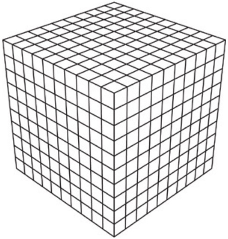
Finite-difference modeling

The Finite-difference method solves the Visco-elastic equations by approximating derivatives with finite-differences

$$\partial_x u \approx \frac{1}{\Delta x} \sum_{l=1}^{8} \alpha_l [u(\boldsymbol{x} + l\Delta x) - u(\boldsymbol{x} - (l-1)\Delta x)] \qquad (2)$$

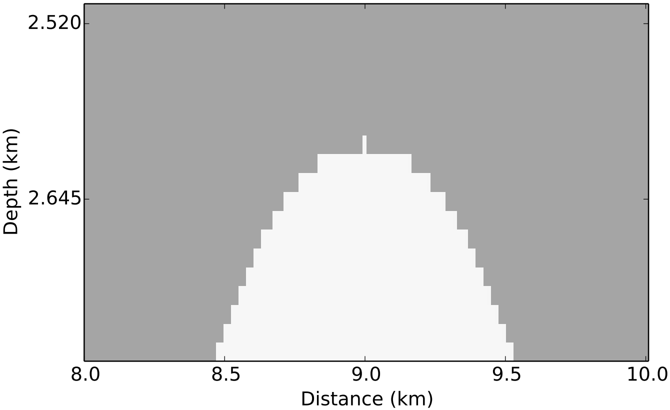
Where

- u: Wavefield
- α: Precomputed coefficients
- Δx: Distance between grid-points

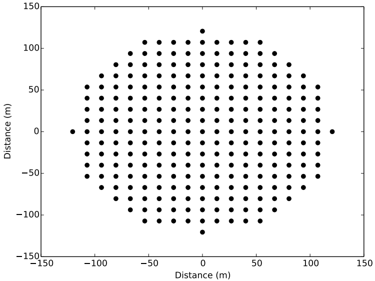


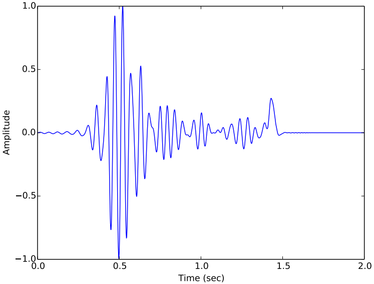
Finite-difference modeling

Use $\Delta x = 13.4$m giving grid-points/wavelength $= 4.0$

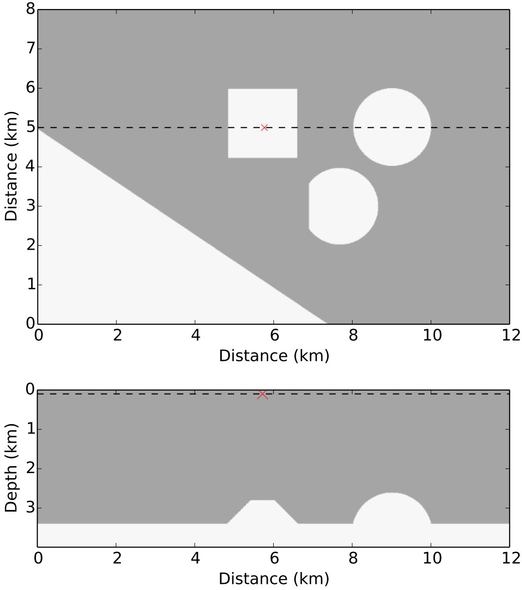


Finite-difference modeling

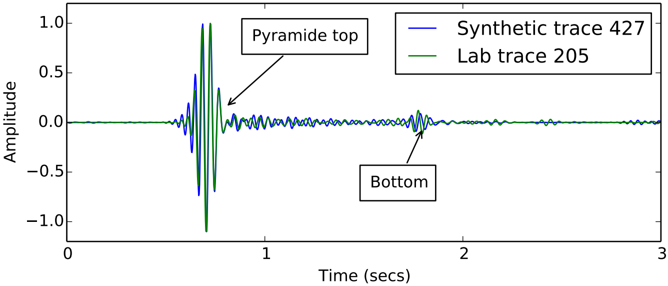




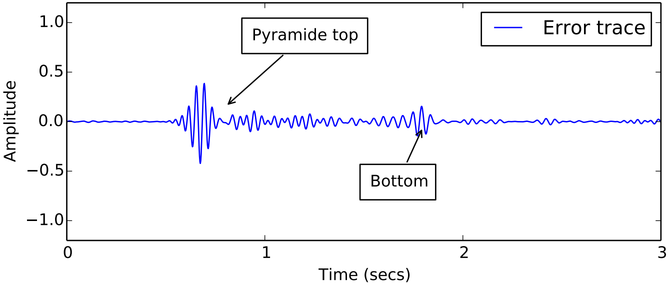
Results



Results

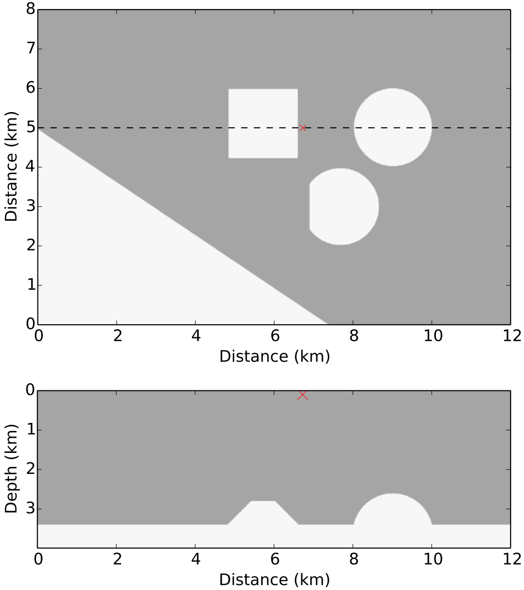


Pyramide top
Synthetic trace 427
Lab trace 205
Bottom
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
Time (secs)

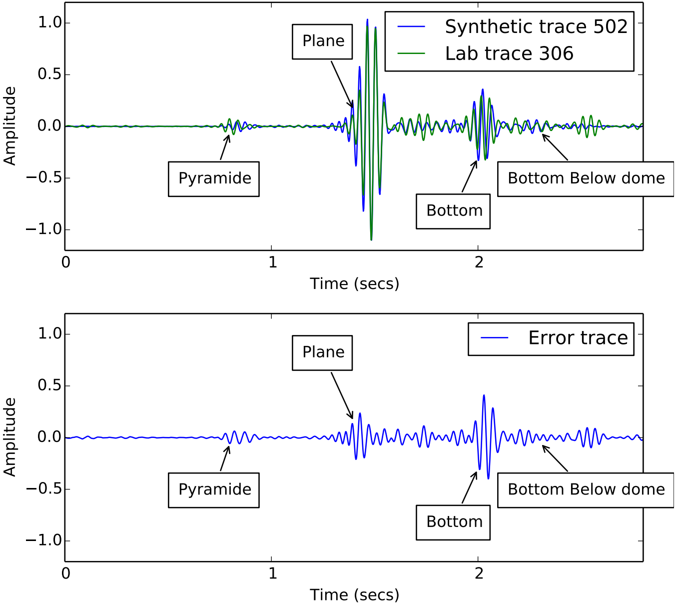


Pyramide top
Error trace
Bottom
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
Time (secs)

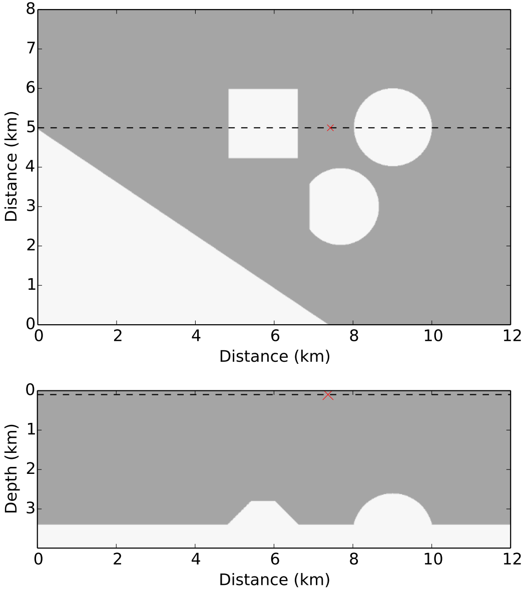
Results



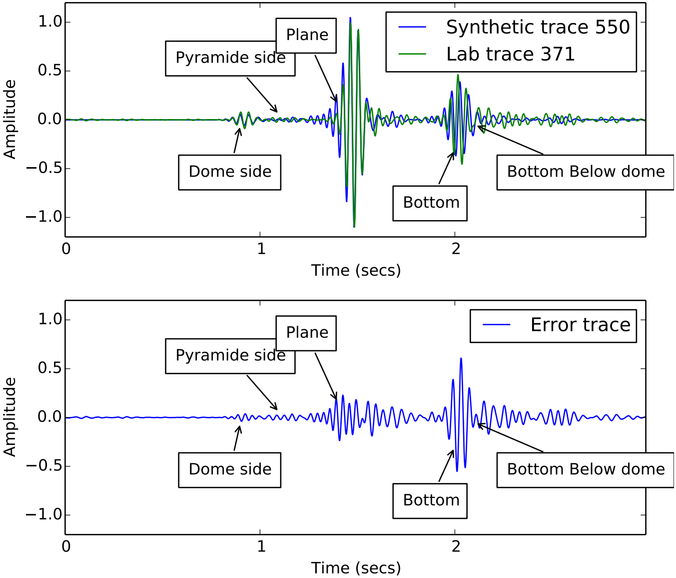
Results


Synthetic trace 502
Lab trace 306
Plane
Pyramide
Bottom
Bottom Below dome
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0
1
2
Time (secs)
Error trace
Plane
Pyramide
Bottom
Bottom Below dome
Amplitude
Time (secs)

Results

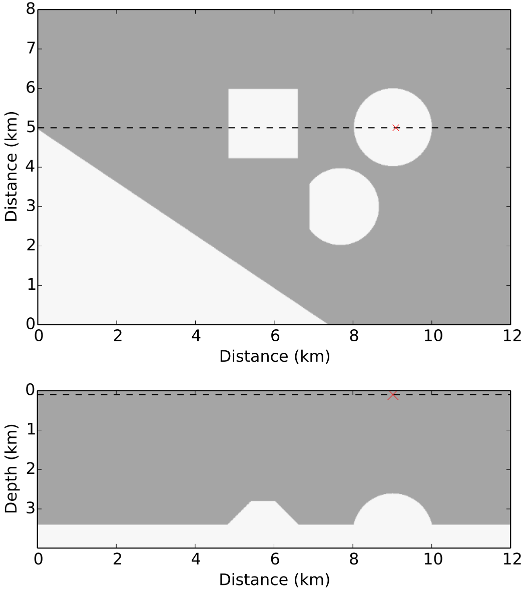


Results



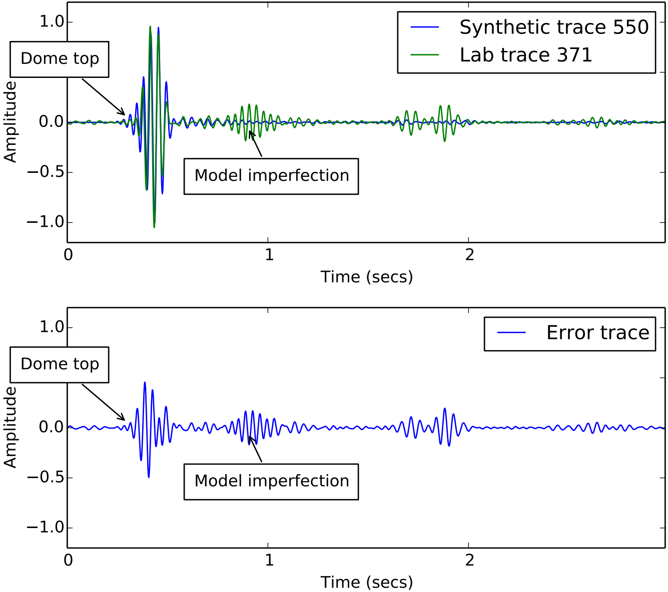
Synthetic trace 550
Lab trace 371
Plane
Pyramide side
Dome side
Bottom
Bottom Below dome
Amplitude
Time (secs)
1.0
0.5
0.0
−0.5
−1.0
0
1
2
Error trace
Plane
Pyramide side
Dome side
Bottom
Bottom Below dome
Amplitude
Time (secs)

Results



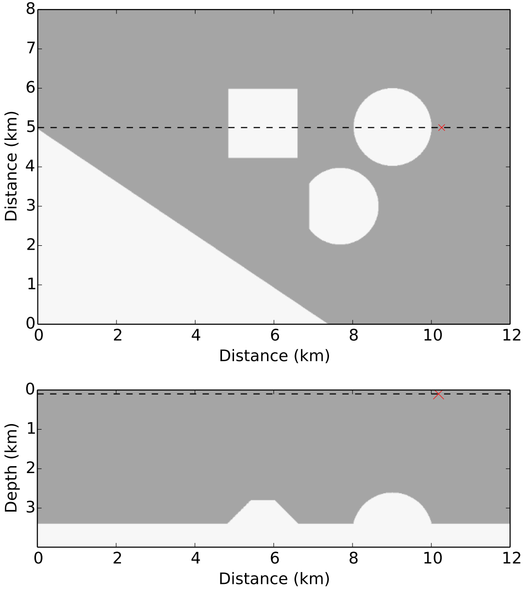
8
7
6
5
4
3
2
1
0
Distance (km)
0
2
4
6
8
10
12
Distance (km)
0
1
2
3
Depth (km)
0
2
4
6
8
10
12
Distance (km)

Results

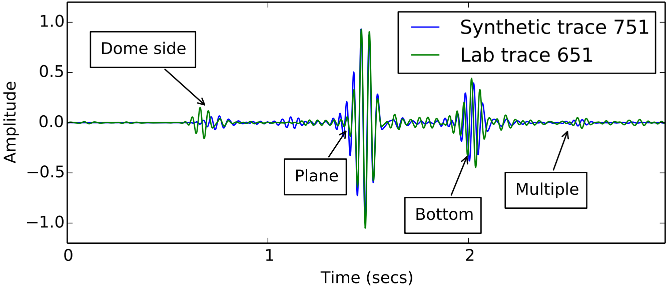


Synthetic trace 550
Lab trace 371
Dome top
Model imperfection
Amplitude
1.0
0.0
−0.5
−1.0
0
1
2
Time (secs)
Error trace
Dome top
Model imperfection
Amplitude
1.0
0.0
−0.5
−1.0
0
1
2
Time (secs)

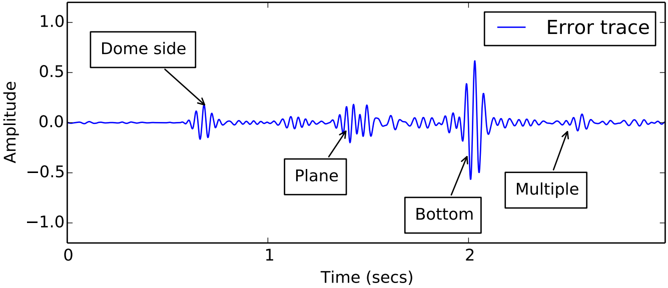
Results



Results



Synthetic trace 751
Lab trace 651
Dome side
Plane
Bottom
Multiple
Amplitude
Time (secs)
1.0
0.5
0.0
−0.5
−1.0
0
1
2



Error trace
Dome side
Plane
Bottom
Multiple
Amplitude
Time (secs)
1.0
0.5
0.0
−0.5
−1.0
0
1
2

Conclusions

- Coarse grid Finite-difference not accurate for plane surfaces
- Coarse grid Finite-difference not accurate for tilting plane
- Coarse grid Finite-difference not accurate for spherical surface

References

[1] Nathalie Favretto-Cristini and Paul Cristini. Benchmarking of numerical methods dedicated to wave propagation and wave-based imaging against laboratory data in complex environments description of the experimental set-up & data acquisition zero-offset & multi-offset data at 500 khz nathalie favretto-cristini & paul cristini version 1. Technical report, Laboratory of Mechanics and Acoustics-Marseilles (LMA), 2015.