Why and How do we want to enhance low frequencies?

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Let's start

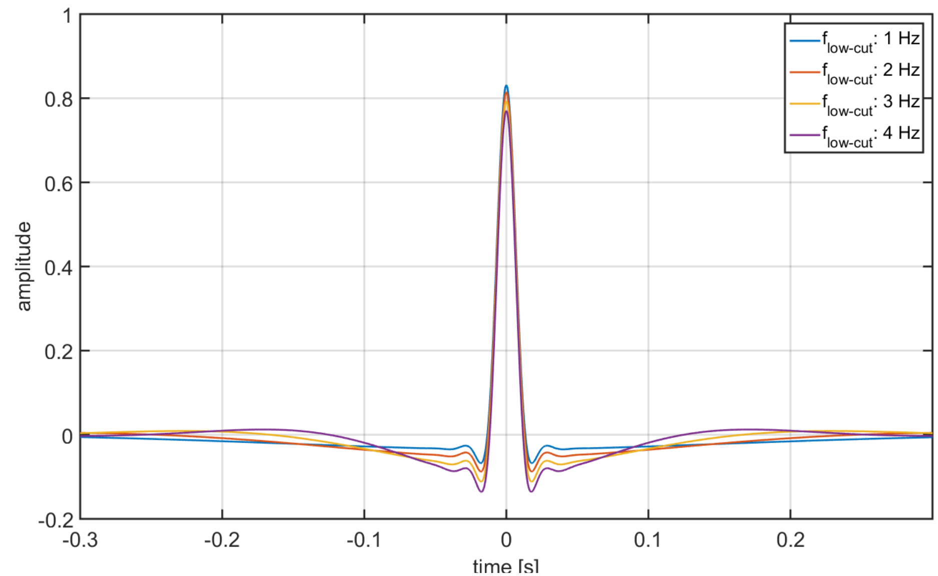
- What are low frequencies?
  → ~ 0.1 Hz - 5 Hz
- Why do we need low frequencies?
- How can we handle the problem?
  ⇨ Two options

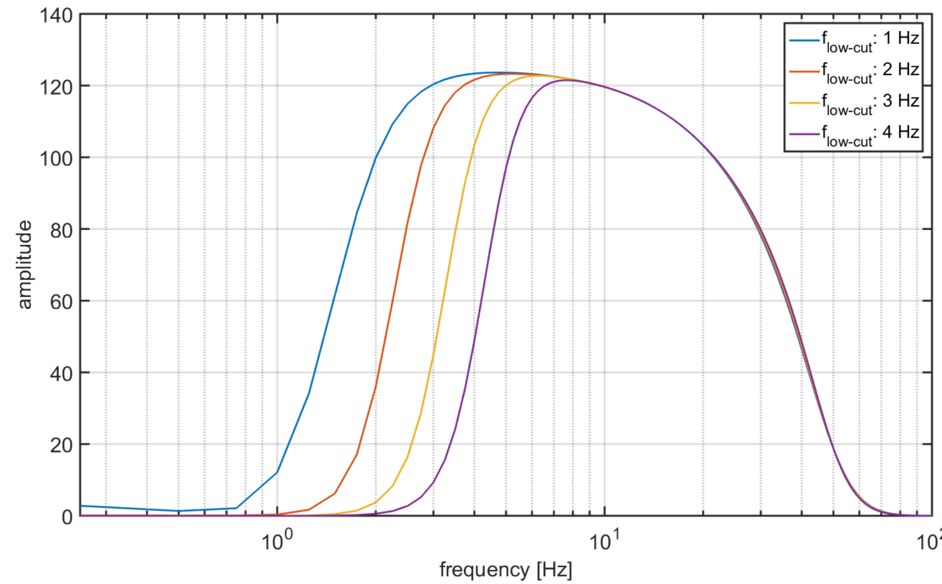
  1. Combined Elastic Waveform and Gravity Inversion
  2. Mechanism for low frequencies in seismic acquisition
- Outlook

Why do we need low frequencies?

1. Higher resolution

- Higher frequencies reduce width of main lobe
- Lower frequencies reduce side lobes
- Improved peak-to-sidelobe ratio from 5.6 (4 Hz) to 12.3 (1 Hz)

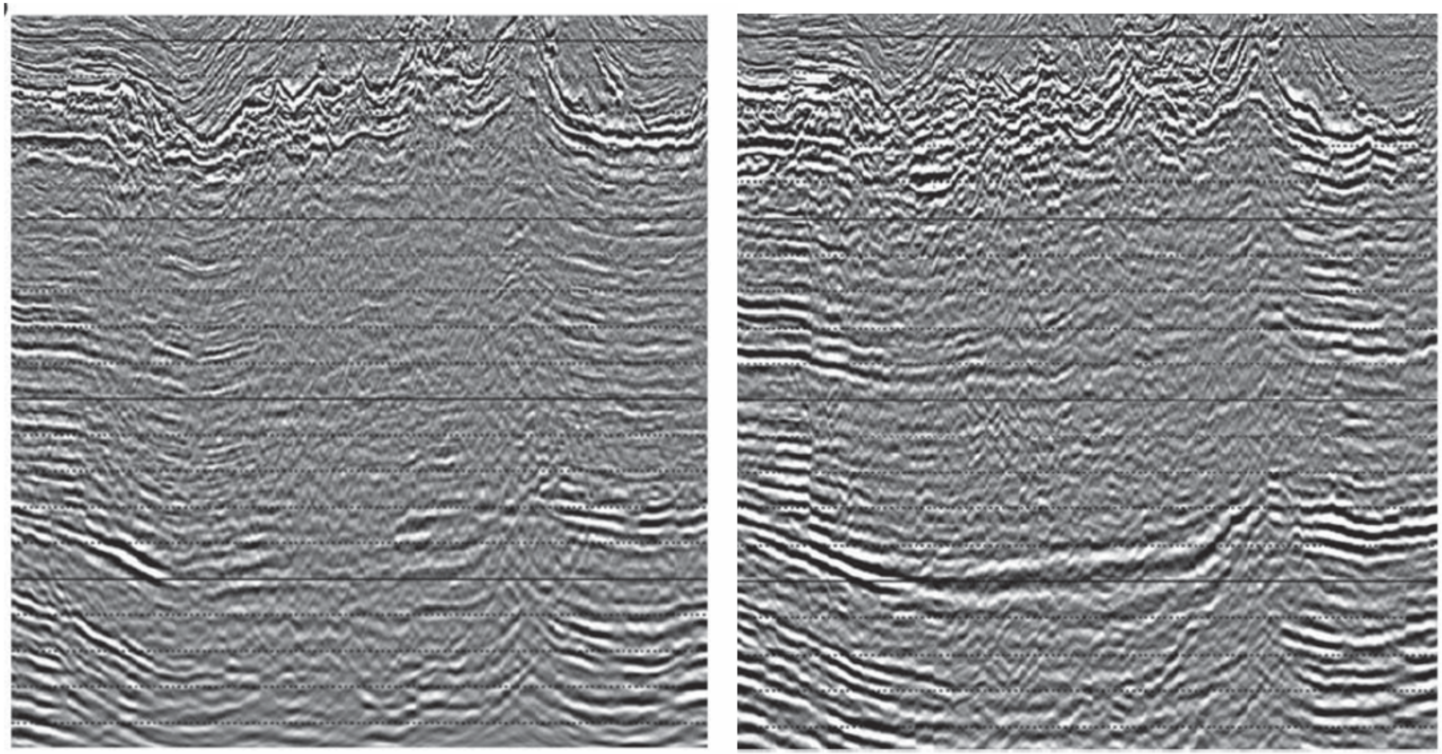




2. Better penetration

- Less attenuation for lower frequencies

Conventional acquisition | Broadband acquisition



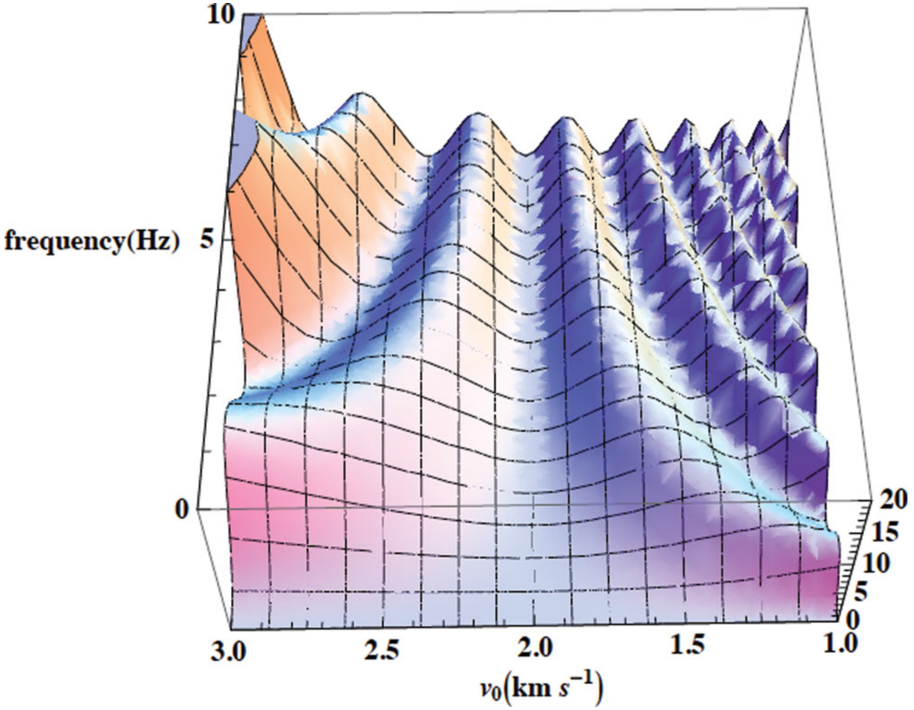
Source: Kroode et al., 2013

3. Full Waveform Inversion

- Highly non-linear problem

- Non-linearity reduced by:
  - **low frequencies** (data space)
  - good initial model (model space)
  - sequential inversion
  - additional information

Objective function

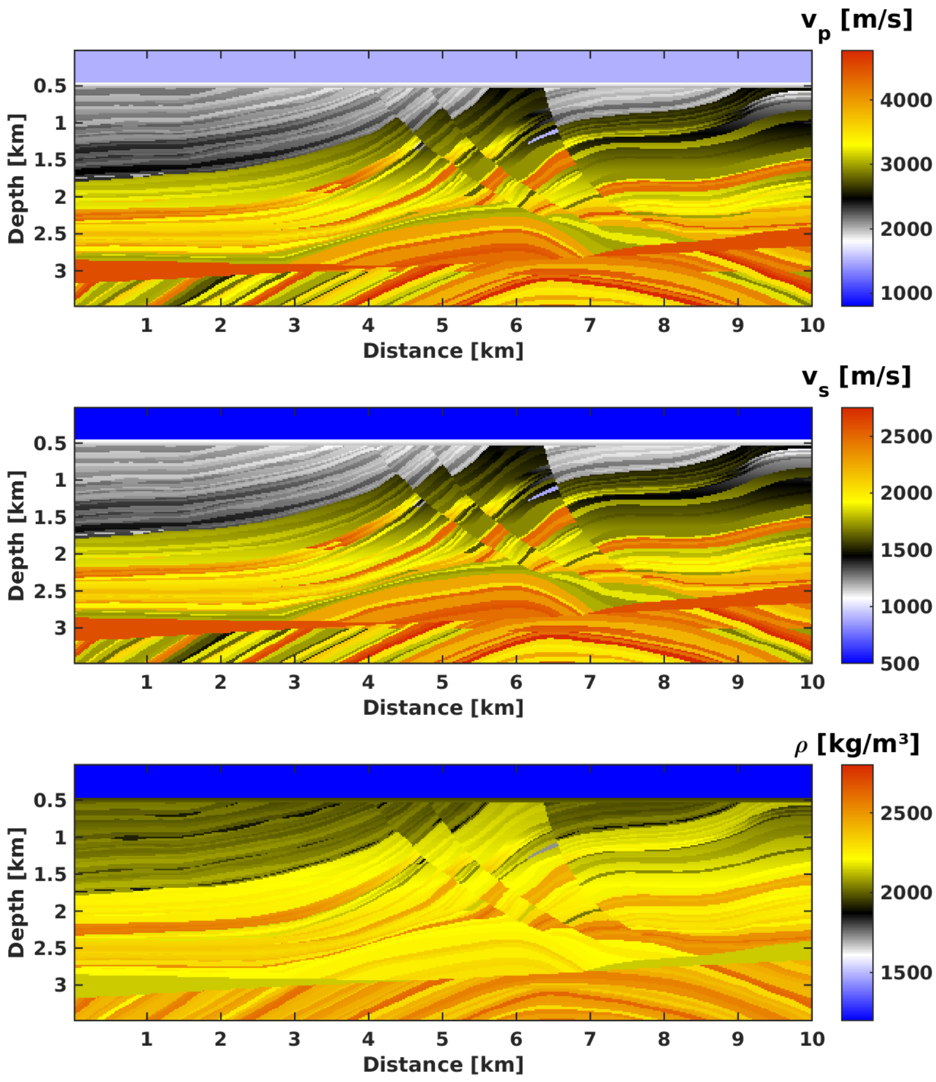


Source: Alkhalifah, 2012

How can we handle the problem?

1. Combined Elastic Waveform and Gravity Inversion

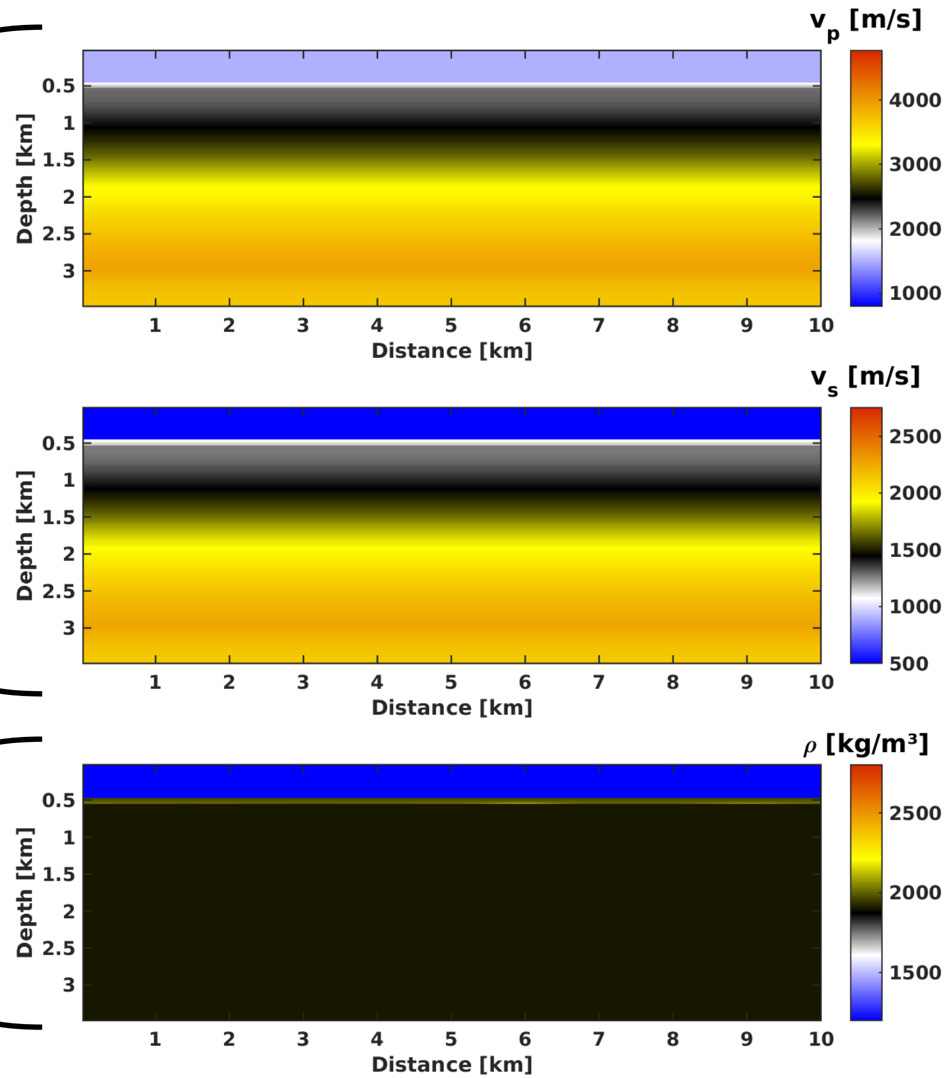
True 2D Marmousi-II model



Initial model

1D gradient of true model

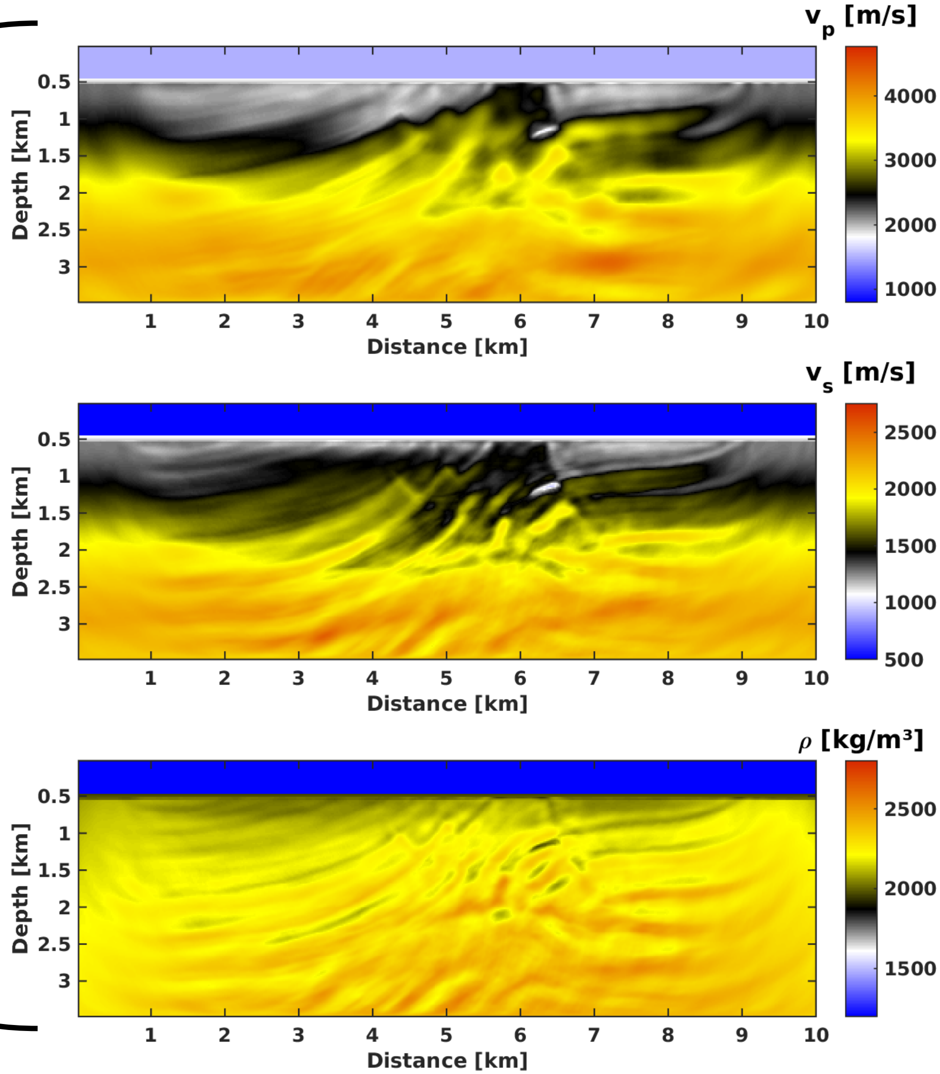
Water-column above constant halfspace



No empirical relations

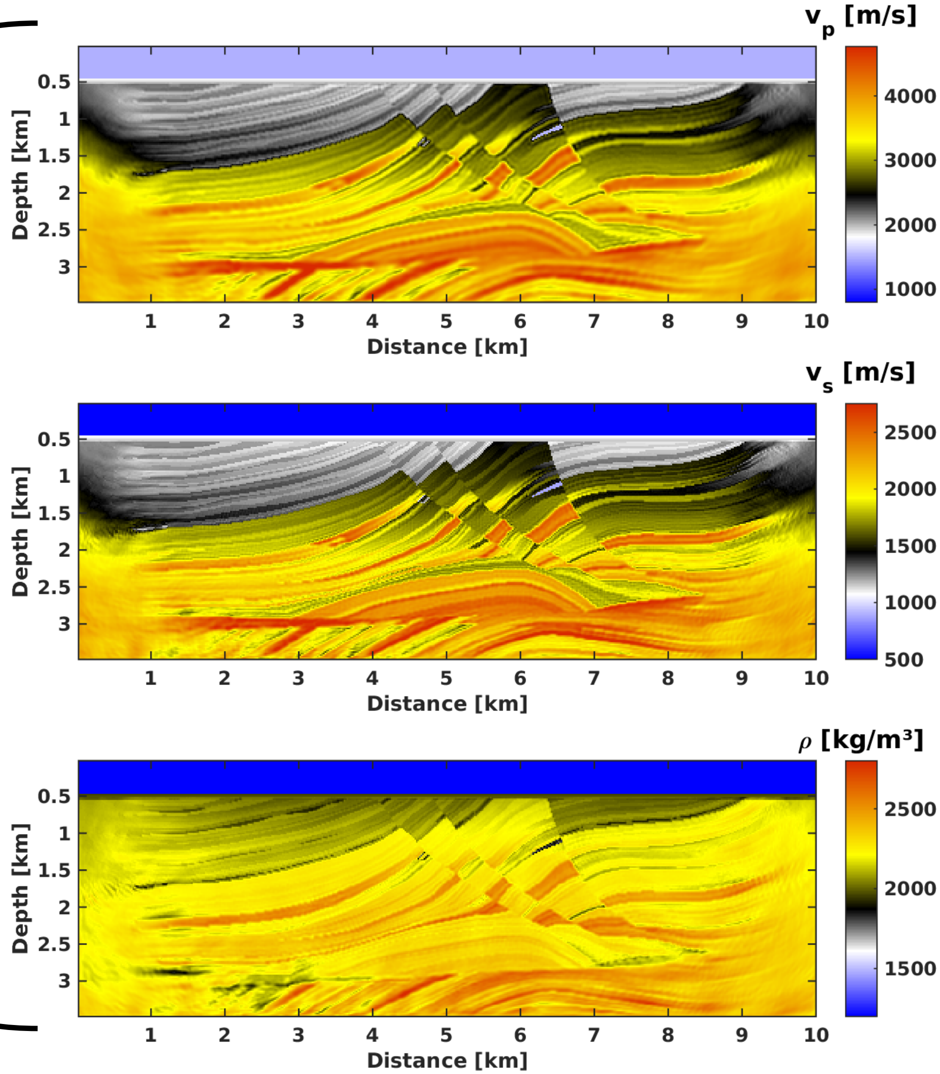
3.5 Hz data: combined Inversion

main structures resolved



0.5 Hz data: combined Inversion

high impact
of low frequency
content



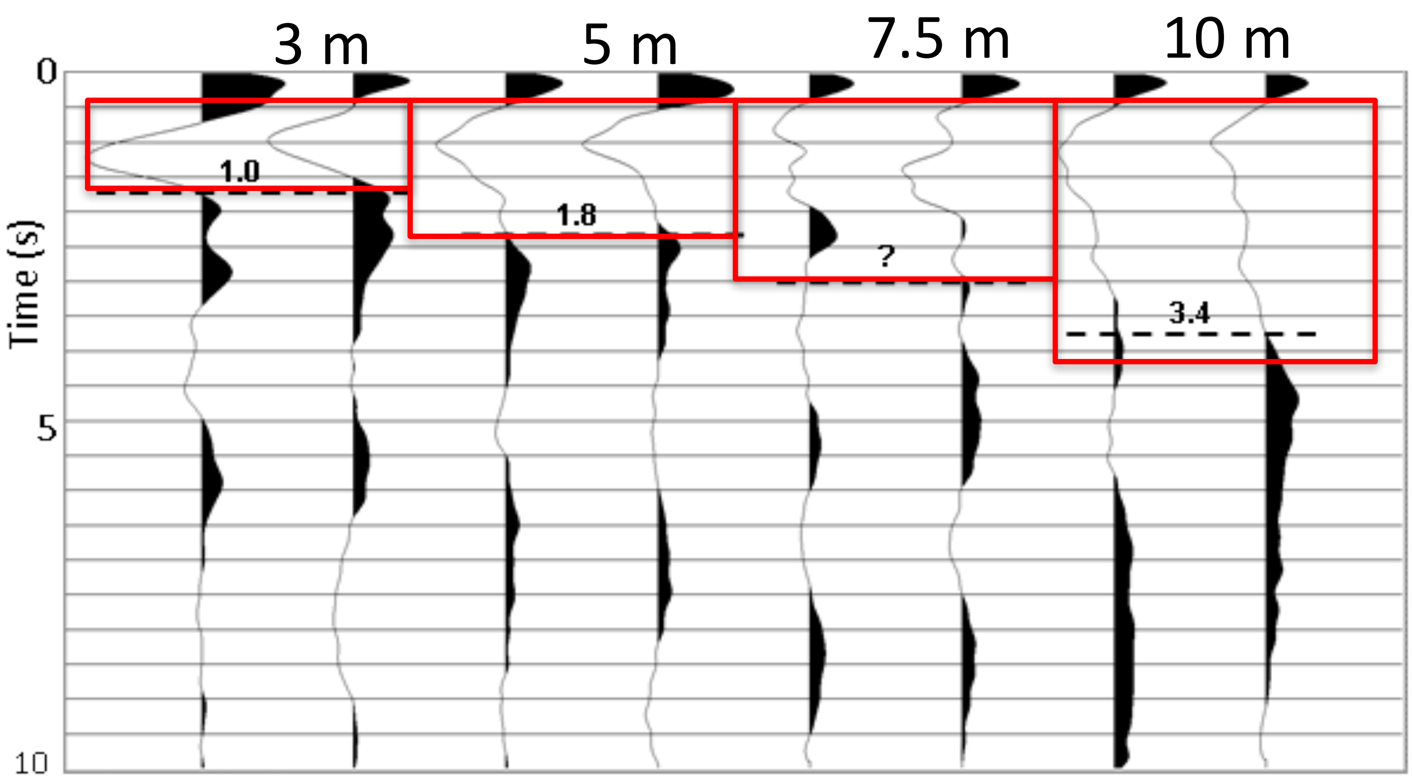
How can we handle the problem?

1. Combined Elastic Waveform and Gravity Inversion
2. Mechanism for low frequencies in seismic acquisition

Signal of rising Bubble?

- Single 600 in^3 air gun at different depths, with hydrophone 20 m below
- Ormsby low pass filter (3 Hz)

- Signal of air gun: 1. Main impulse, 2. Bubble Oscillation, **3. Rising bubble**
- Period inreases with source depth



Source: Landrø and Amundsen, 2014

Signal of rising Sphere

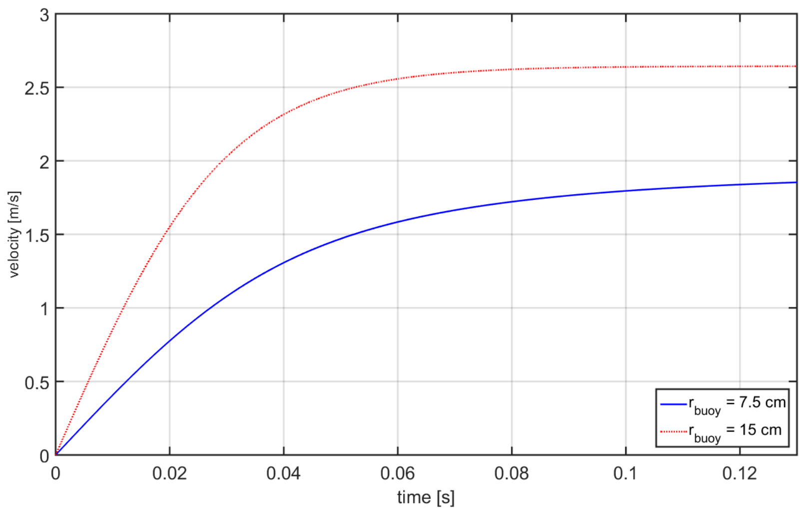
- Velocity of rising sphere

$$m\frac{dv}{dt} = F_B - F_D \quad \text{(force balance)}$$

$$\Rightarrow \quad v = \frac{1}{\beta}\tanh(\gamma t)$$

$$\beta = \sqrt{\frac{C_D A}{2Vg}}, \qquad \gamma = \frac{\rho}{m}\sqrt{\frac{C_D g A V}{2}}$$

C_D = drag coefficient , A = sph. cross-section, V = sph. volume

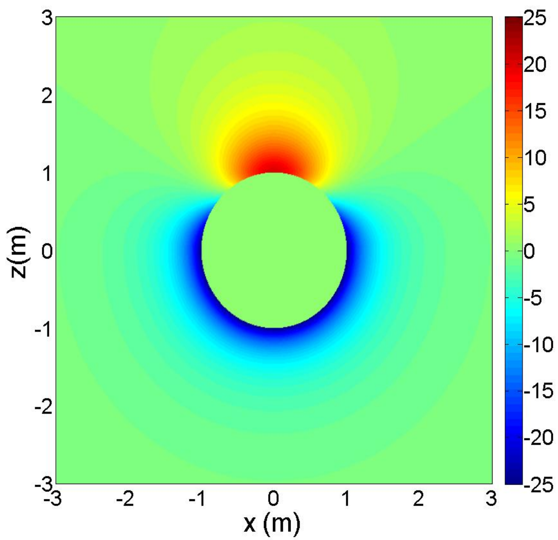


- Reynolds number

$$Re = \frac{v*D}{\mu} \quad \Rightarrow \quad Re \triangleq 10^3 - 10^5$$

D = diameter, μ = kinematic viscosity

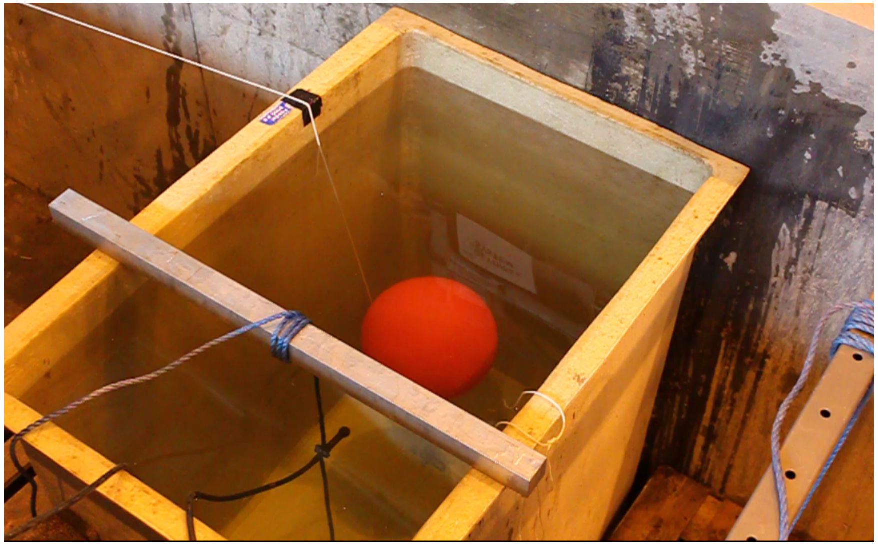
- Pressure distribution around sphere for high Reynolds numbers (by Achenbach, 1972)

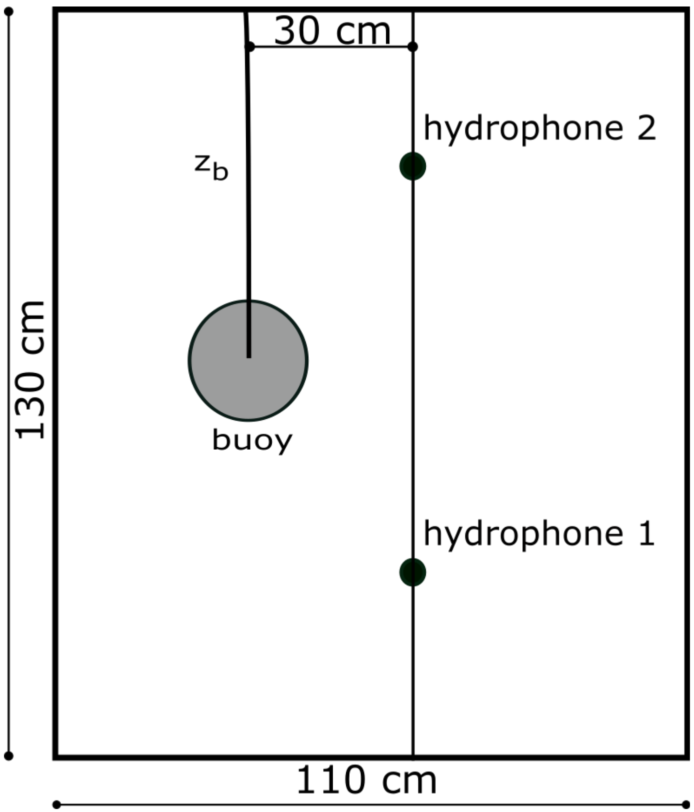


Source: Landrø and Amundsen, 2014

Tank experiment

- Release of buoy from different depth in small water tank

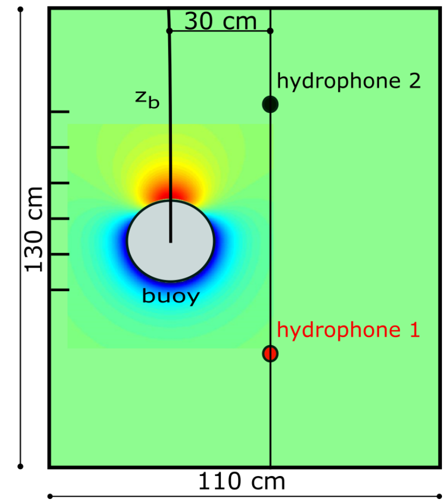


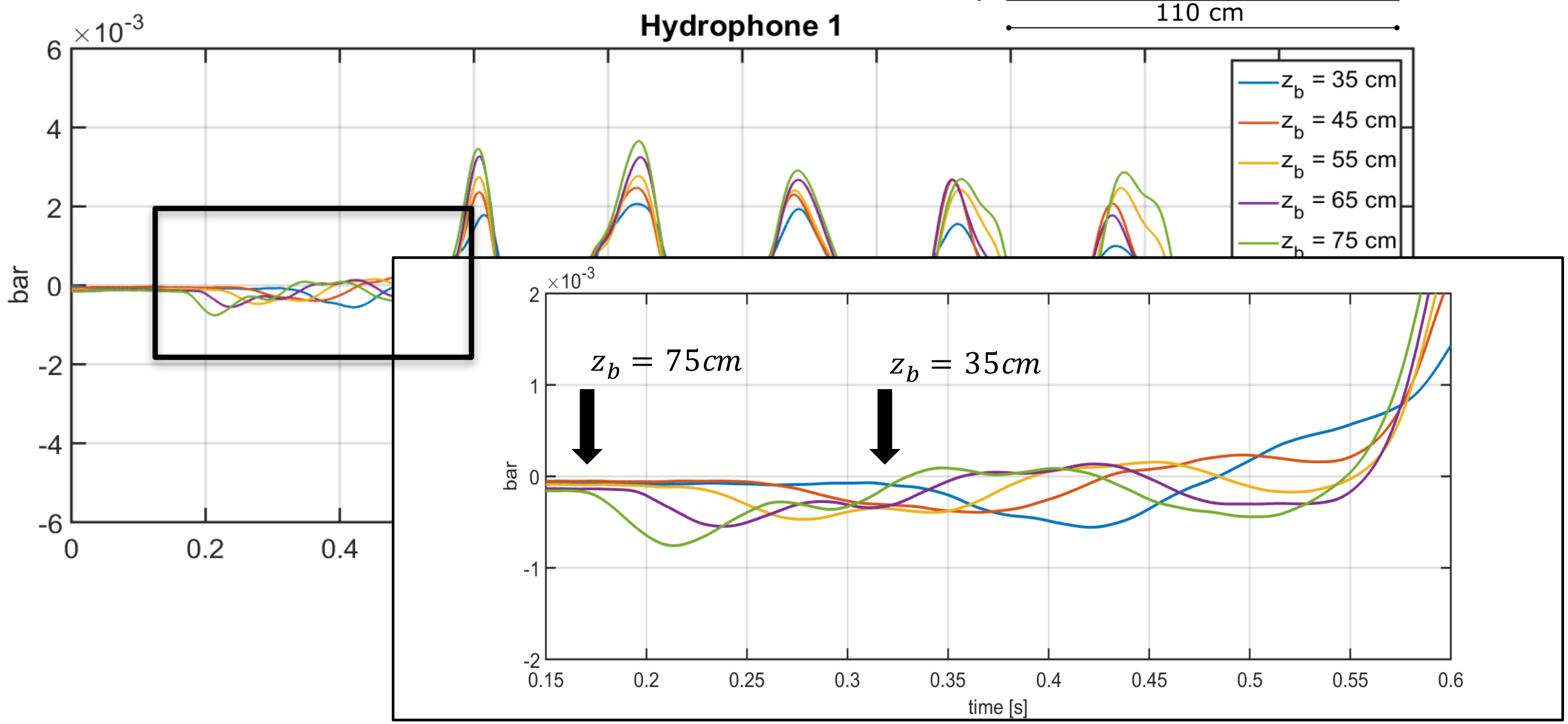


Signal of rising Buoy

- Signal at hydrophone 1
  - negative pressure below → rising buoy
  - duration of signal increases with depth
  - problem: effects due to tank size

Tank experiment

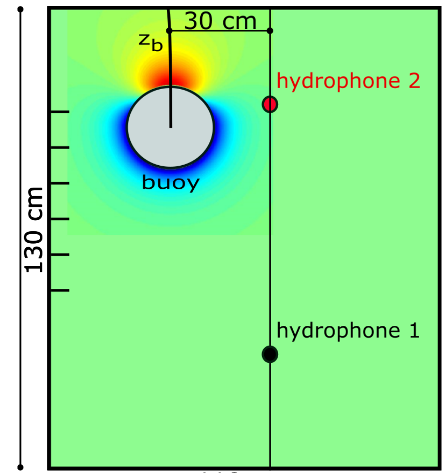


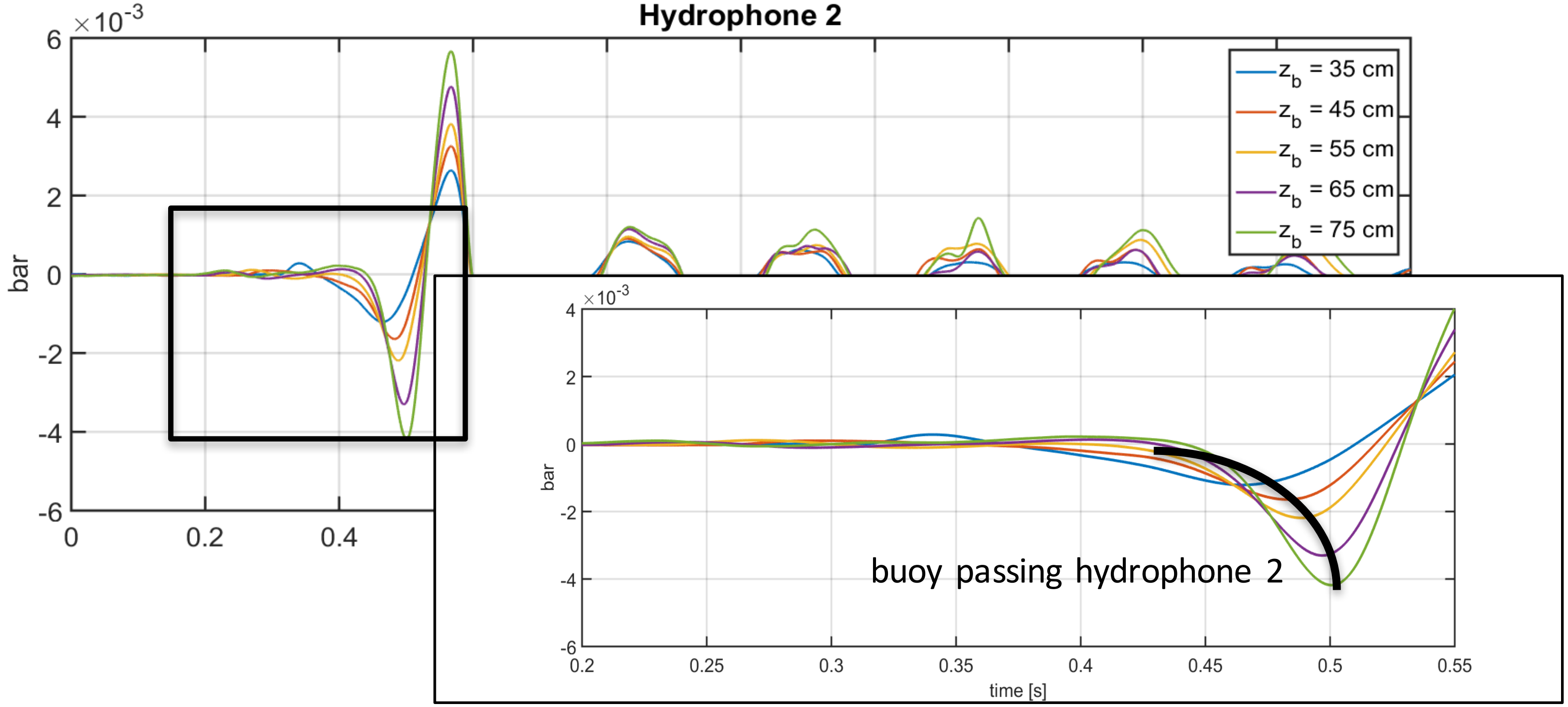


Signal of rising Buoy

- Signal at hydrophone 2
  - negative pressure when buoy passes hydrophone
  - amplitude related with rising velocity

Tank experiment





Outlook

- rising Bubble ←→ rising Sphere ←→ rising Buoy ?
  - promising, but has to be verified
- Simple, but could explain mechanism for low frequencies in air gun signal
  - low frequencies related to rising time (depends on velocity and depth of buoy)
  - not account for: bubble oscillation, bubble-size depth dependency, …
- Upscaling pressure to bigger radius (Gilmore, 1952; Davies and Taylor, 1950)
  - estimated with: $p - p_h \approx \rho \frac{Rv_b^2}{r} = \frac{4\rho g R^2}{9r}$ $, with$ $v_b = \frac{2}{3}\sqrt{gR}$

Radius (m)	Calculated pressure (mbar-m)	Measured pressure (mbar-m)
0.075	0.25	0.5
0.15	1	2
1.0	44	88 ?

- If mechanisms are the same, an optimal depth could be found with
  - biggest possible radius + required distance to reach terminal velocity
  - depends on favored frequency $\mathrm{f} = \frac{v_b}{z_b}$ (z_b= source depth)

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WAVES

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Thank you very much for your attention

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References

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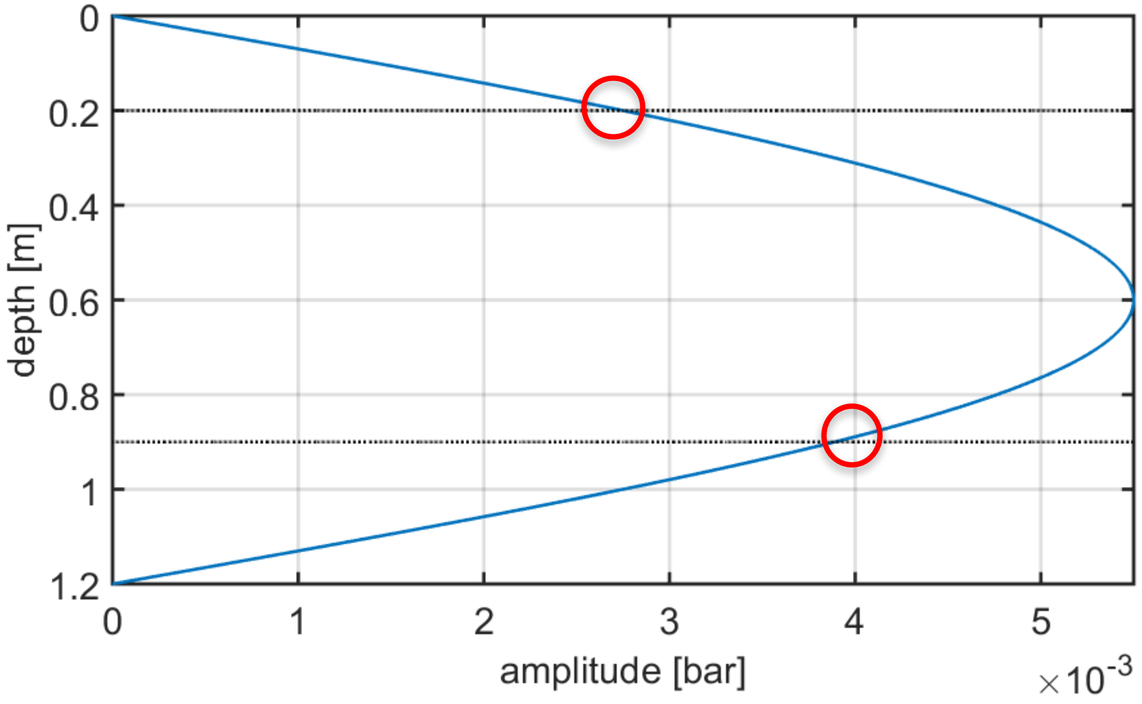
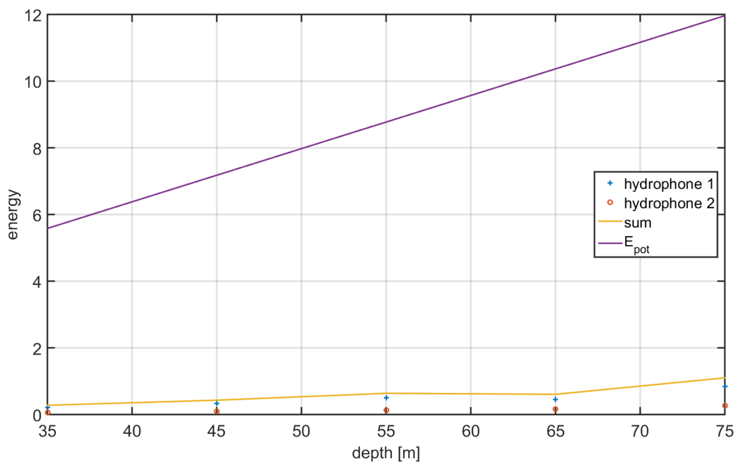
Standing wave

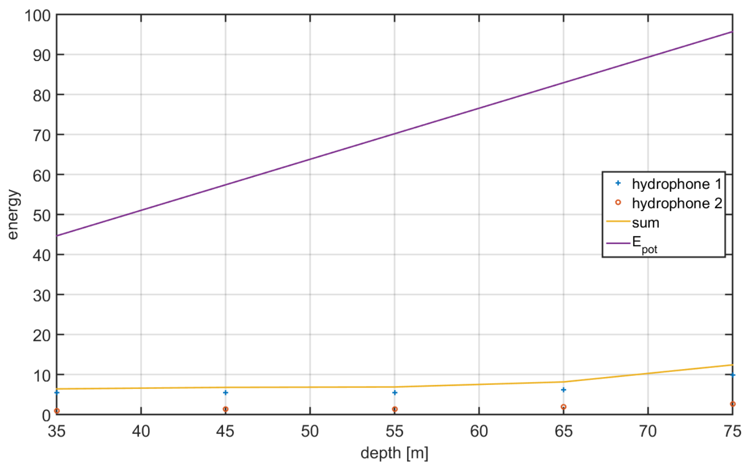
- Difference between amplitudes of hydrophone 1 (deep) and 2 (shallow)
- Frequency $f = \frac{v}{\lambda}$
  - $v = \sqrt{gd} \approx 3.5 \frac{m}{s}$ (for shallow water)
  - $\lambda = 2.4\ m\ , 1.2\ m$ (regarding to size of tank)



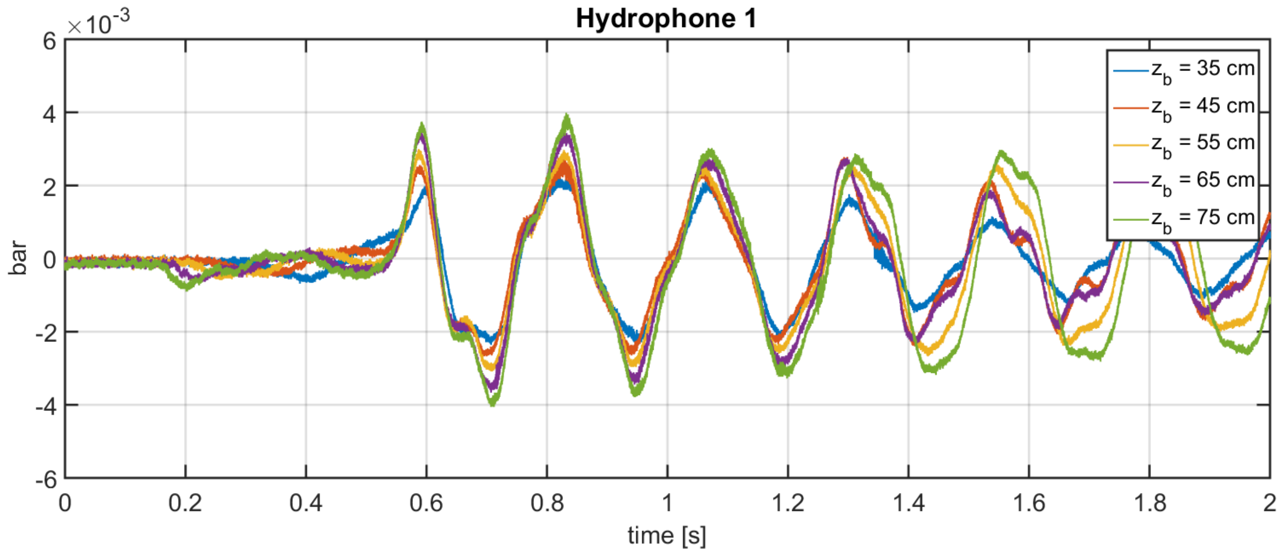
Mechanical to Acoustic Energy

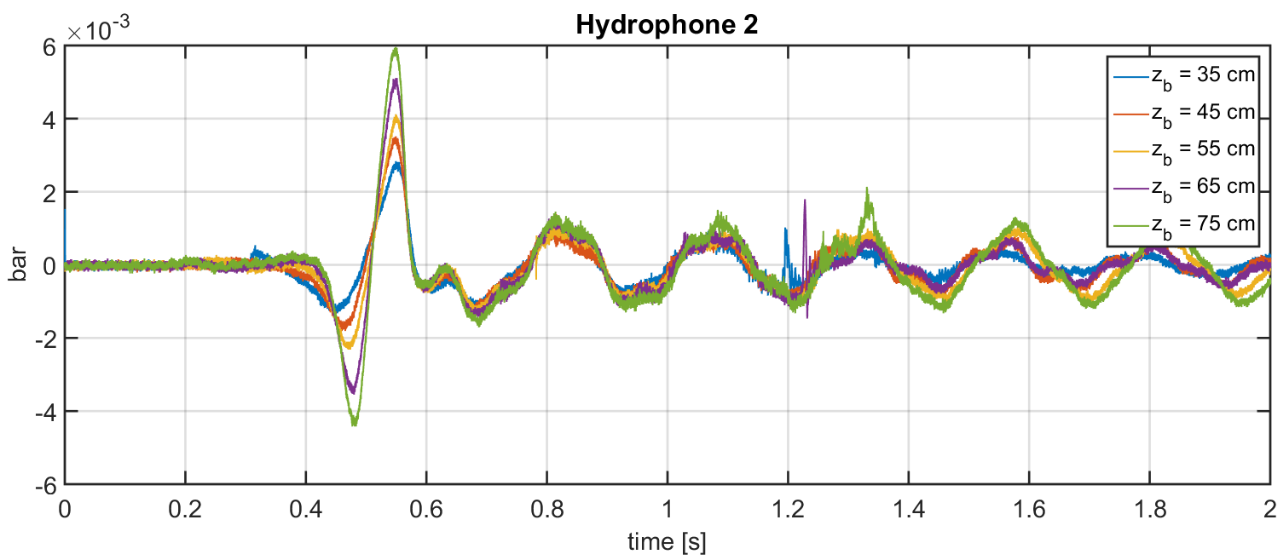
- How much energy of buoy is transfered to acoustic energy
  - Not reliable, because only to point measurements (bigger array required)



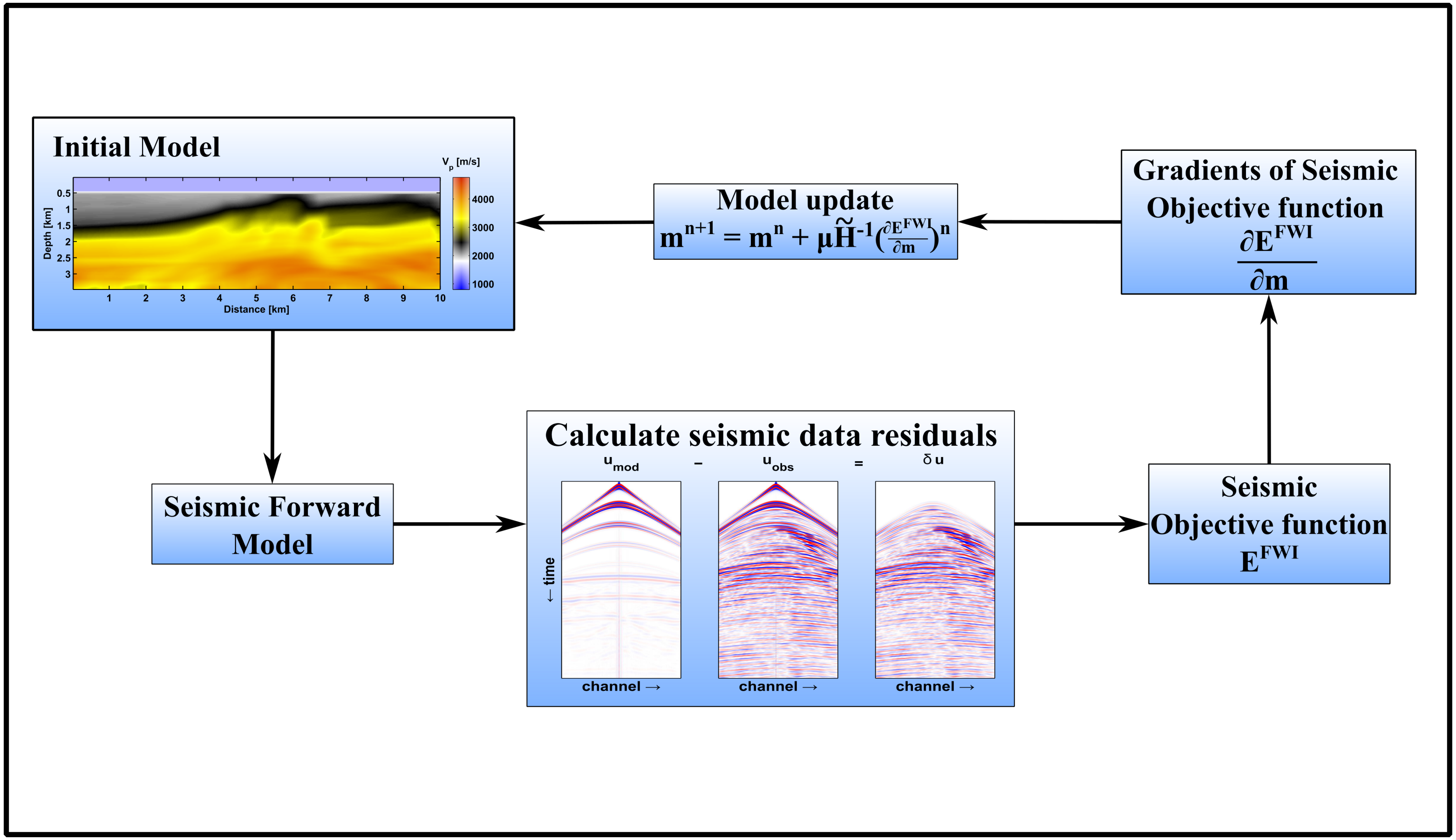


Unfiltered Signal of buoy

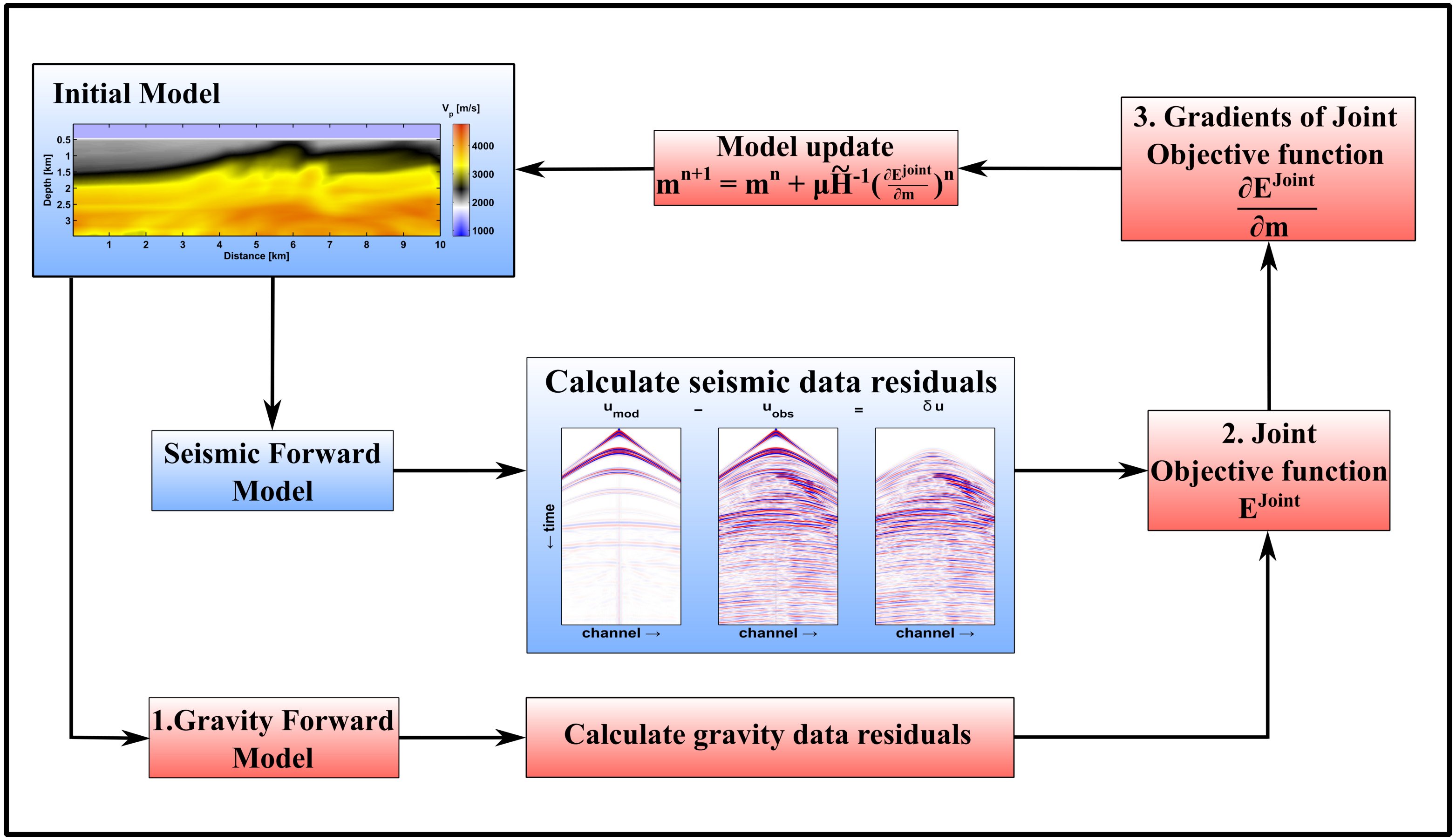




FWI workflow



FWI workflow with GRAVITY

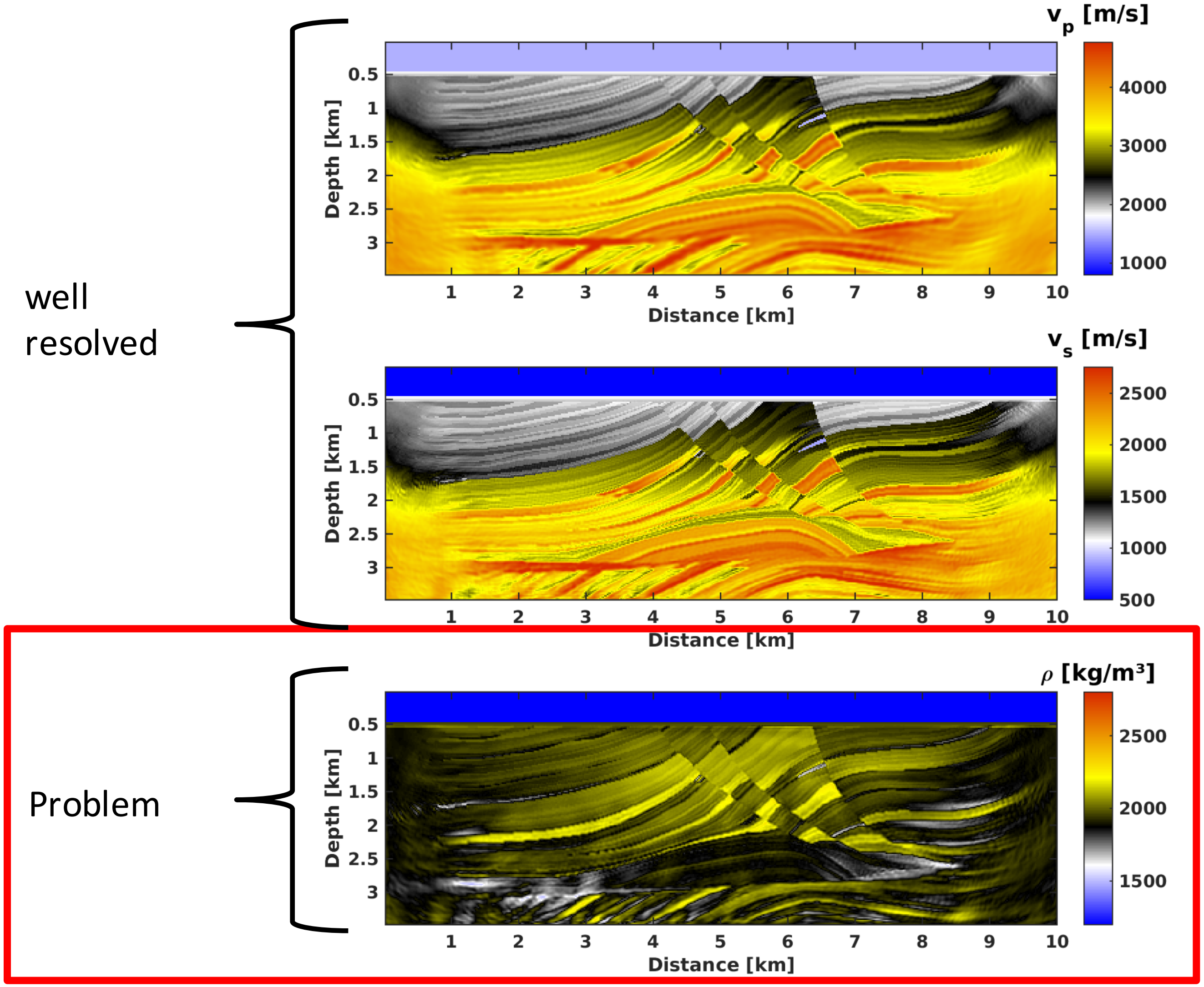


Workflow

- Pure FWI
  - Seismic frequency content down to 3 Hz
  - sequential inversion of different frequency bands (2.5, 5, 10, 20 Hz)
  - invert for all parameters (v_p, v_s, ρ) simultaneously

- Combined Inversion
  - Seismic frequency content down to 3 Hz
  - sequential inversion of different frequency bands (2.5, 5, 10, 20 Hz)
  - 1 step: invert for density only
  - 2 step: invert for all paramters (v_p, v_s, ρ) simultaneously

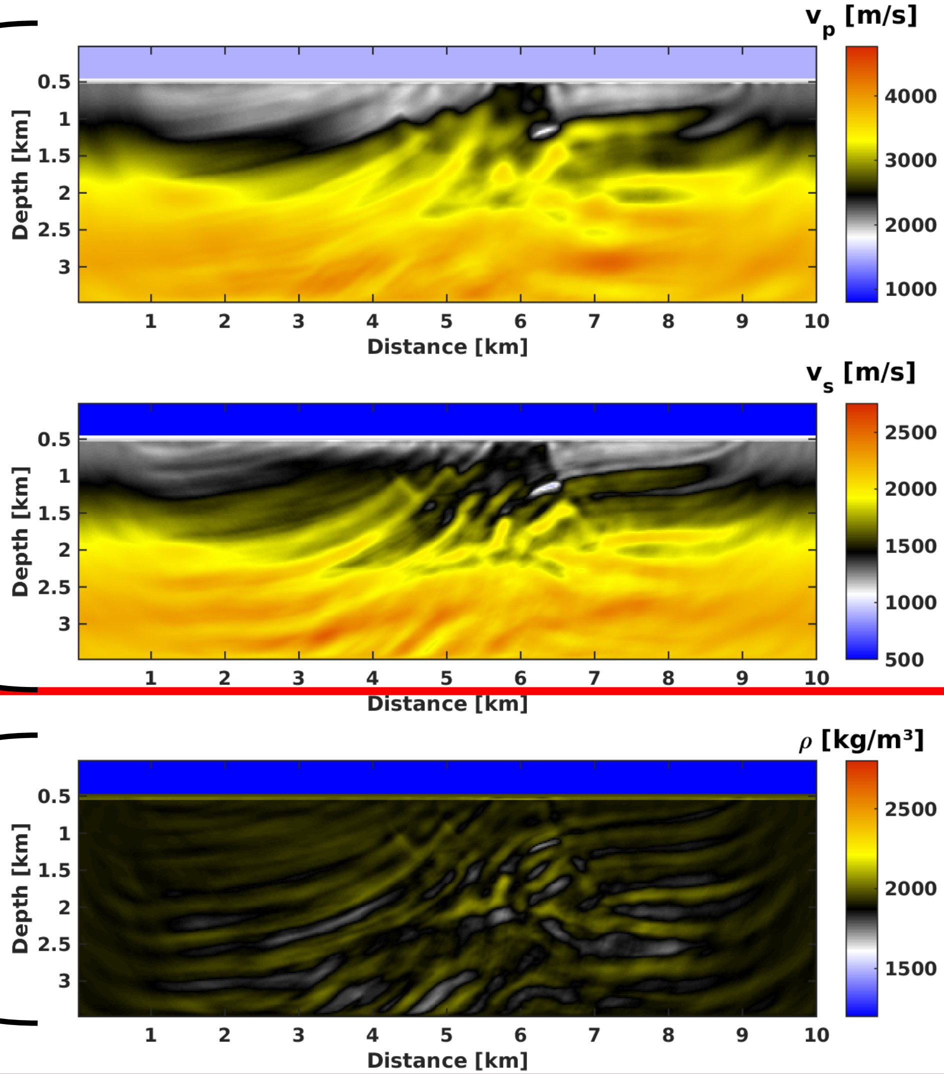
Results: pure FWI



Results: pure FWI

main structures resolved

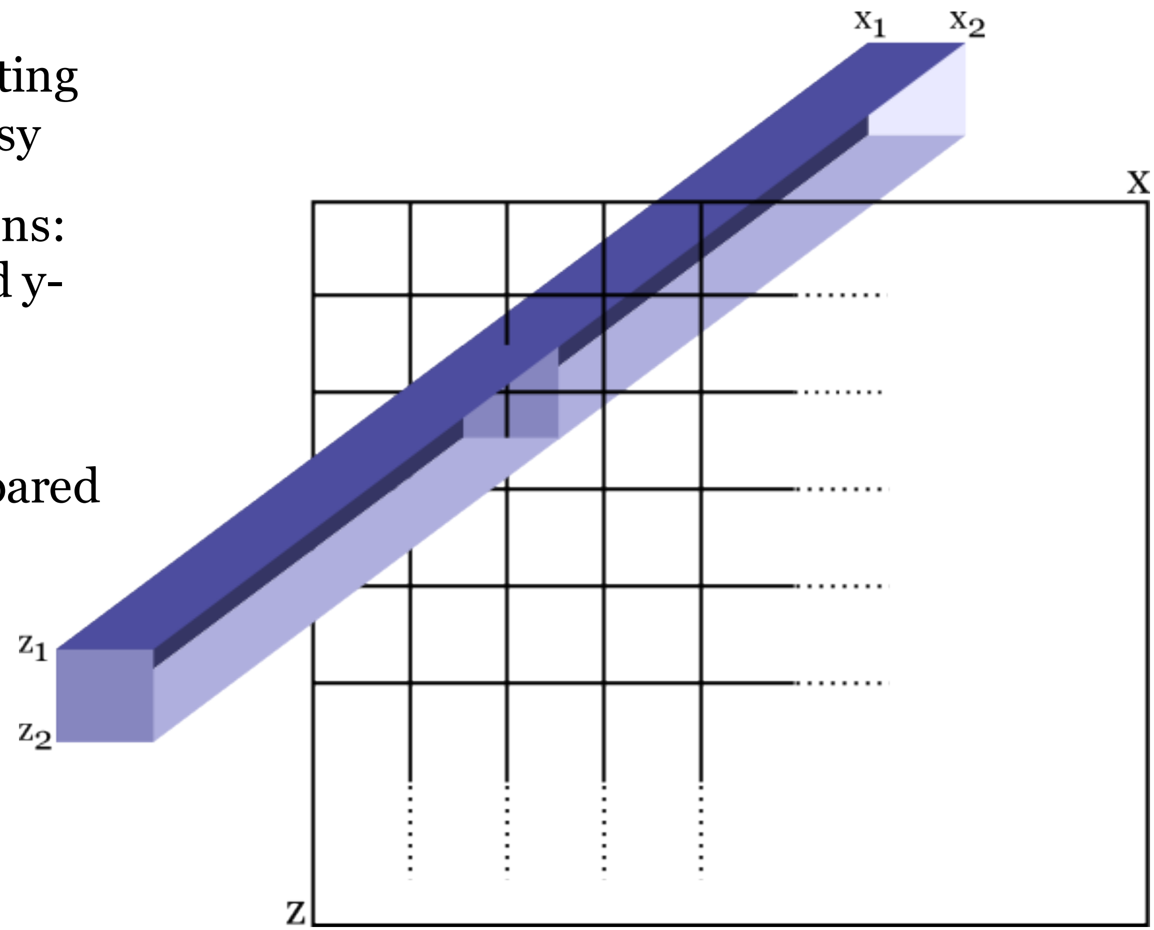
Problem



Gravity modeling

gravity of prism $$g_z = G\rho \int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} \frac{z}{r^3}\,dxdydz$$

- Integration in existing FWI FD-Grid is easy
- Boundary conditions: extension in x- and y-direction
- Cost effective computation compared to FWI



Objective Function

- Objective function for FWI

$$E^{FWI} = \frac{1}{2}\delta u^T \delta u$$

- Minimizing the objective function by iteratively updating seismic velocities and densities with Quasi-Newton l-BFGS method (Nocedal & Wright, 2006; Brossier, 2011)

$$V_p^{n+1} = V_p^n - \mu^n H_n^{-1}\left(\frac{\delta E^{FWI}}{\delta V_p}\right)^n$$

$$V_s^{n+1} = V_s^n - \mu^n H_n^{-1}\left(\frac{\delta E^{FWI}}{\delta V_s}\right)^n$$

$$\rho^{n+1} = \rho^n - \mu^n H_n^{-1}\left(\frac{\delta E^{FWI}}{\delta \rho}\right)^n$$

Joint Objective Function

- Modified objective function for Joint Inversion

$$E^{JOINT} = \frac{1}{2}(\delta u^T \delta u + \lambda_1 \, \delta g_z^T \delta g_z) = E^{FWI} + \lambda_1 \, E^{GRAV}$$

- Minimizing the objective function by iteratively updating seismic velocities and densities with Quasi-Newton l-BFGS method (Nocedal & Wright, 2006; Brossier, 2011)

$$V_p^{n+1} = V_p^n - \mu^n H_n^{-1} \left(\frac{\delta E^{FWI}}{\delta V_p} \right)^n$$

$$V_s^{n+1} = V_s^n - \mu^n H_n^{-1} \left(\frac{\delta E^{FWI}}{\delta V_s} \right)^n$$

$$\rho^{n+1} = \rho^n - \mu^n H_n^{-1} \left(\frac{\delta E^{FWI}}{\delta \rho} + \lambda_1 \lambda_2 \, \frac{\delta E^{GRAV}}{\delta \rho} \right)^n$$

Parameter λ

Calculation of weighting parameter λ

- λ_1 (objective function)

$$\lambda_1 = \gamma \frac{E^{FWI}(1)}{E^{GRAV}(1)}$$

- λ_2 (gradients)

$$\lambda_2 = \gamma \frac{\max(\frac{\partial E^{FWI}}{\partial \rho})}{\max(\frac{\partial E^{GRAV}}{\partial \rho})} \lambda_1^{-1}$$

Gradient

- Gradient for the density (FWI) (Köhn et al., 2012)

$$\frac{\delta E^{FWI}}{\delta \rho} = \sum_{soures} \int dt \left(\frac{\delta^2 u_x}{\delta t^2} \psi_x + \frac{\delta^2 u_z}{\delta t^2} \psi_z \right)$$

Construction of the gradient by zero-lag correlation of forward wavefield u and backpropagated data residual wavefield ψ

- Gradient for the density (Gravity)

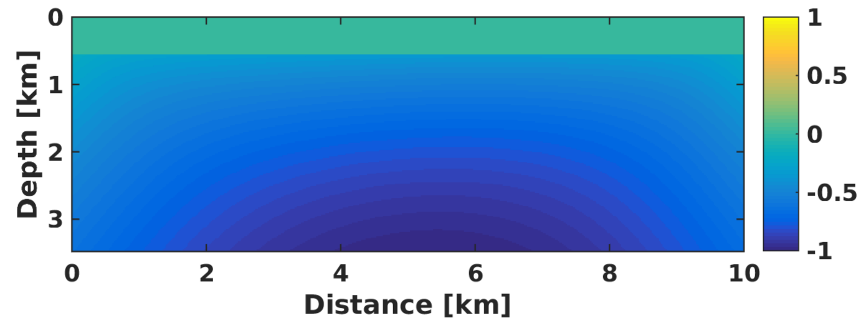
$$\frac{\delta E^{GRAV}}{\delta \rho} = G \int_S \delta g_z \boldsymbol{K} \, dS$$

$\boldsymbol{K}$ = geometrical kernel

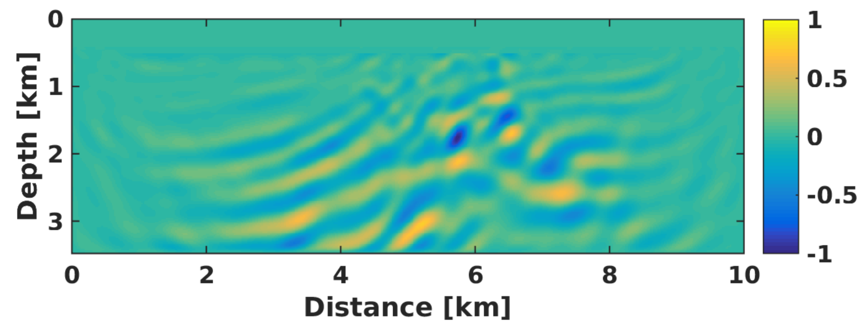
Wavenumber analysis

Gradient of first iteration step during combined inversion…

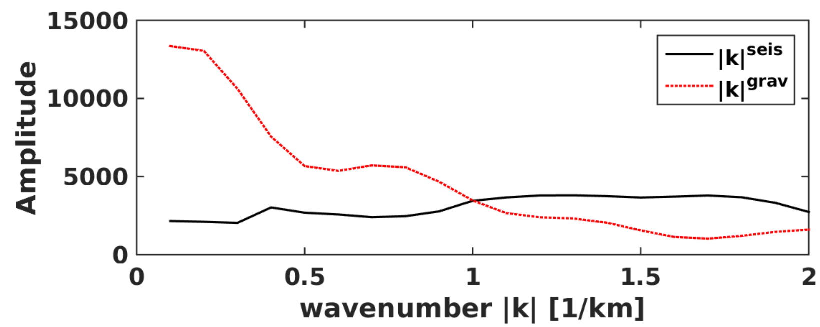
…for gravity data…



…and seismic data.
(low-pass filtered, 2 Hz)



average wavenumbers of gradients



Gravity contributes information to low frequencies

Further tasks

- apply combined inversion to salt/basalt model
  - use empirical relations, constrain velocities through gravity data
  - impedance inversion instead of velocities
  - acoustic or elastic modelling/inversion, 2D/3D modelling/inversion
- inversion of gravity gradient data $\frac{\partial g_z}{\partial z}$ → more sensitive to local structures
- reduce trade-off between attenuation and density in visco-elastic media by combined inversion
- impact of enhancement of low frequency seismic data

⇨ when is combined inversion necessary, if we have lower frequencies in seismic data