# Fracturing vs layering at low frequencies

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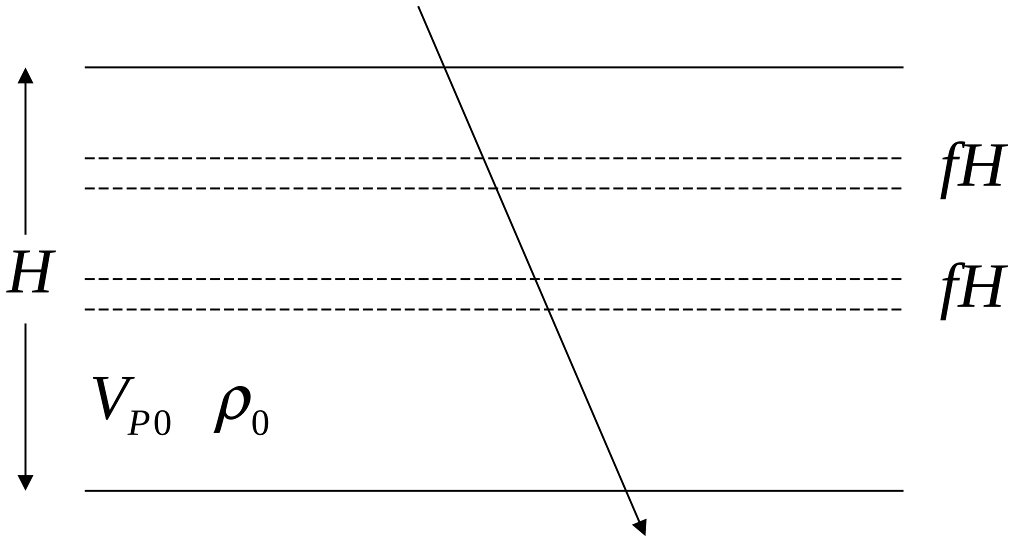
# OUTLINE

- Linear-slip model
- Fractures at low frequencies
- Fracturing vs layering
- Weak-contrast
- Conclusions

# Linear slip theory




$Fracture\ i:$

$Layer\ \ thickness\ \ fH$

$Stiffness\ \ coefficient\ \ \ fc_{pq,rs}^{(i)}$

$Density\ \ \ f\rho^{(i)}$

$(with\ \ f \rightarrow 0)$

$Velocity: \sqrt{\dfrac{fc_{pq,rs}^{(i)}}{f\rho^{(i)}}}$

$Impedance: \ f\sqrt{c_{pq,rs}^{(i)}\rho^{(i)}}$

*Schoenberg, 1983*

# Fracture propagator

Propagator

$$\mathbf{Q}_i = \lim_{f \to 0} \exp\left[ j\omega f H \, \mathbf{F}_i \right]$$

$$\lim_{f \to 0} \left( f\mathbf{F}_i \right) = \begin{pmatrix} \mathbf{0} & \mathbf{C}_{33}^{(i)^{-1}} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

System matrix

$$\mathbf{F}_i = \begin{pmatrix} f\mathbf{A}_i & f^{-1}\mathbf{C}_{33}^{(i)^{-1}} \\ f\mathbf{B}_i & f\mathbf{A}_i^T \end{pmatrix}$$

$$\boxed{\mathbf{Q}_i = \begin{pmatrix} \mathbf{I} & j\omega H \mathbf{C}_{33}^{(i)^{-1}} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}}$$

$$\mathbf{A}_i = p\mathbf{C}_{31}^{(i)}, \ \ \mathbf{B}_i = p^2 \mathbf{C}_{13}^{(i)} \mathbf{C}_{33}^{(i)^{-1}} \mathbf{C}_{31}^{(i)} + \rho_i, \ \ \mathbf{C}_{pr}^{(i)}\left( q, s \right) = c_{pq,rs}^{(i)}$$

# Fracture properties

No change in stress
and displacement velocity is changed

$$\left( \mathbf{u}, \boldsymbol{\tau} \right)^T \longrightarrow \Delta \mathbf{u} = j\omega H \mathbf{C}_{33}^{(i)^{-1}} \mathbf{u} \quad \text{(slip)}$$

For fractures with radial symmetry, the compliance matrix is diagonal

$$\mathbf{C}_{33}^{(i)^{-1}} = \mathrm{diag}\left( K_N^{(i)}, K_T^{(i)}, K_T^{(i)} \right)$$

$$\Delta_N^{(i)} = \frac{\left( \lambda + 2\mu \right) K_N^{(i)}}{1 + \left( \lambda + 2\mu \right) K_N^{(i)}}, \;\; \Delta_T^{(i)} = \frac{\mu K_T^{(i)}}{1 + \mu K_T^{(i)}}$$

$$0 \le \Delta_N^{(i)}, \Delta_T^{(i)} < 1$$

$\Delta_N$ and $\Delta_T$ are the weaknesses

*Bakulin et al., 2000*

# Wave propagation in multi-layered/fractured medium

$Z_0 = V_{P0}\rho_0 \quad t_i = h_i / V_{P0}$

$\mathbf{P}_i$

$\mathbf{Q}_i$

$$\mathbf{P}(\omega) = \mathbf{P}_n \mathbf{Q}_n \ldots \mathbf{P}_1 \mathbf{Q}_1$$

$t_0 = t_1 + .. + t_n$

$$\mathbf{P}_i = \begin{pmatrix} \cos \omega t_i & jZ_0^{-1} \sin \omega t_i \\ jZ_0 \sin \omega t_i & \cos \omega t_i \end{pmatrix}$$

$$\mathbf{Q}_i = \begin{pmatrix} 1 & j\omega t_0 K_N^{(i)} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & j\omega t_0 d_i Z_0^{-1} \\ 0 & 1 \end{pmatrix}$$

$$d_i = \frac{\Delta_N^{(i)}}{1 - \Delta_N^{(i)}}$$

*fracture intensity*

# Low-frequency dispersion

$$t^2(\omega) = \tau_0 + \tau_2\omega^2 + O(\omega^4)$$

$$\tau_0 = a_0 b_0, \quad \tau_2 = \frac{1}{3}a_0^2 b_0^2 - a_0 g_2 - b_0 f_2 - a_1^2 - a_0 b_0 \sum_{k=1}^{2n} t_k^2 + \frac{1}{3}\left( a_0 \sum_{k=1}^{2n} \frac{t_k^3}{Z_k} + b_0 \sum_{k=1}^{2n} t_k^3 Z_k \right)$$

$$a_0 = \sum_{k=1}^{2n} t_k Z_k, \;\; b_0 = \sum_{k=1}^{2n} \frac{t_k}{Z_k}, \;\; a_1 = \frac{1}{2} \sum_{2n \ge k_2 > k_1 \ge 1} t_{k1} t_{k2} \left( \frac{Z_{k2}}{Z_{k1}} - \frac{Z_{k1}}{Z_{k2}} \right),$$

$$f_2 = \sum_{2n \ge k_3 > k_2 > k_1 \ge 1} t_{k3} t_{k2} t_{k1} \frac{Z_{k3} Z_{k1}}{Z_{k2}}, \;\; g_2 = \sum_{2n \ge k_3 > k_2 > k_1 \ge 1} t_{k3} t_{k2} t_{k1} \frac{Z_{k2}}{Z_{k3} Z_{k1}}.$$

*Roganov and Stovas, 2012*

# Low-frequency dispersion

$$a_0 = t_0 Z_0,\ b_0 = \frac{t_0}{Z_0}\left(1 + d_1 + \dots + d_n\right),\ a_1 = \frac{t_0}{2}\sum_{i=1}^{n}\left(-t_1 - \dots t_{i-1} + t_i + \dots + t_n\right) d_i,$$

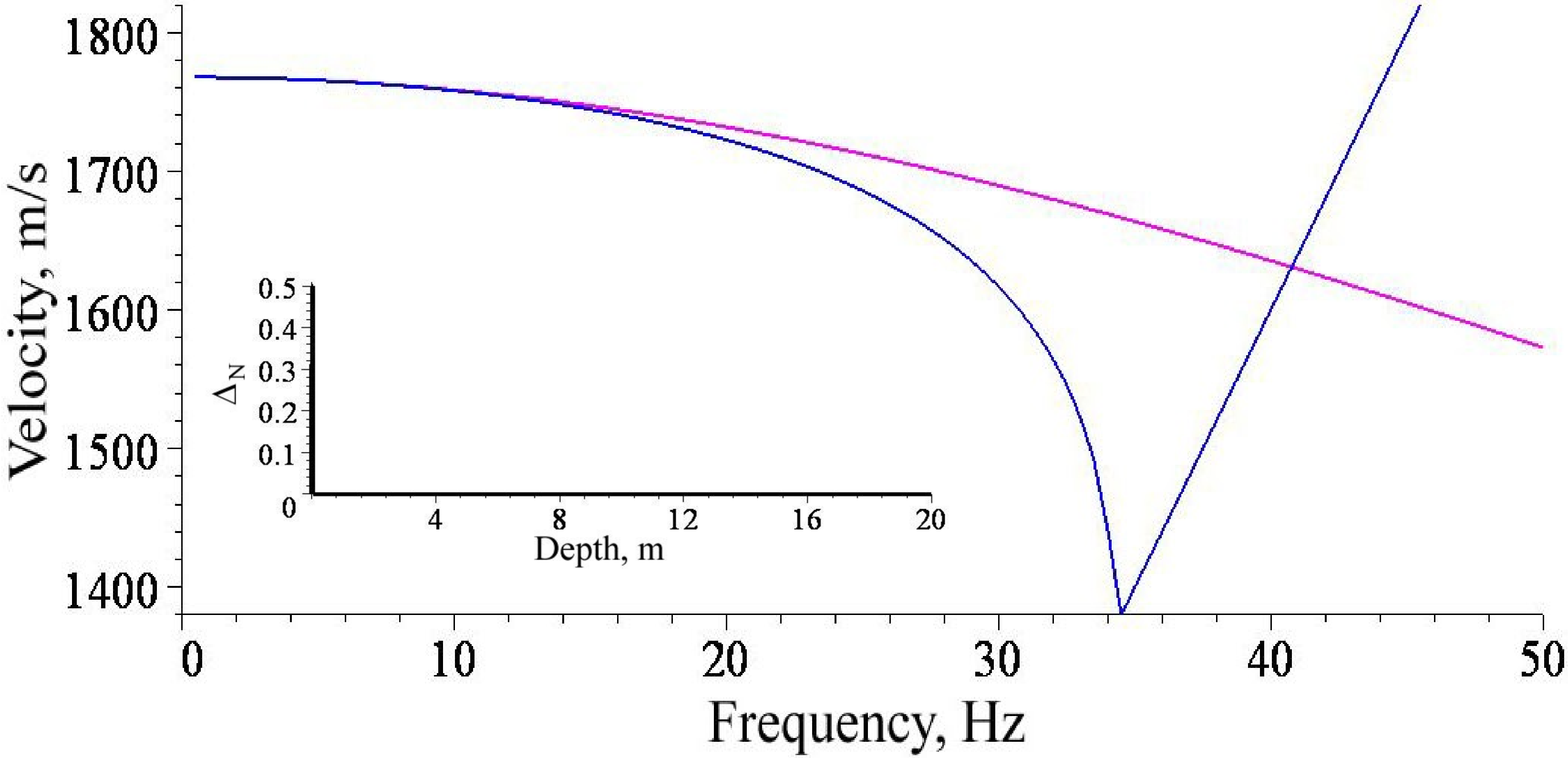
$$f_2 = t_0 Z_0 \sum_{i=2}^{n}\left(t_1 + \dots + t_{i-1}\right)\left(t_i + \dots + t_n\right) d_i,$$

$$g_2 = \frac{t_0}{Z_0}\sum_{i=3}^{n}\sum_{i \ge j_2 > j_1 \ge 1} t_{j2} t_{j1} d_i + \frac{t_0}{Z_0}\sum_{i=1}^{n-2}\sum_{i \ge j_2 > j_1 \ge i} t_{j2} t_{j1} d_i + \frac{t_0^2}{Z_0}\sum_{i=1}^{n}\sum_{n \ge j > i}\left(t_i + \dots + t_{j-1}\right) d_i d_j.$$

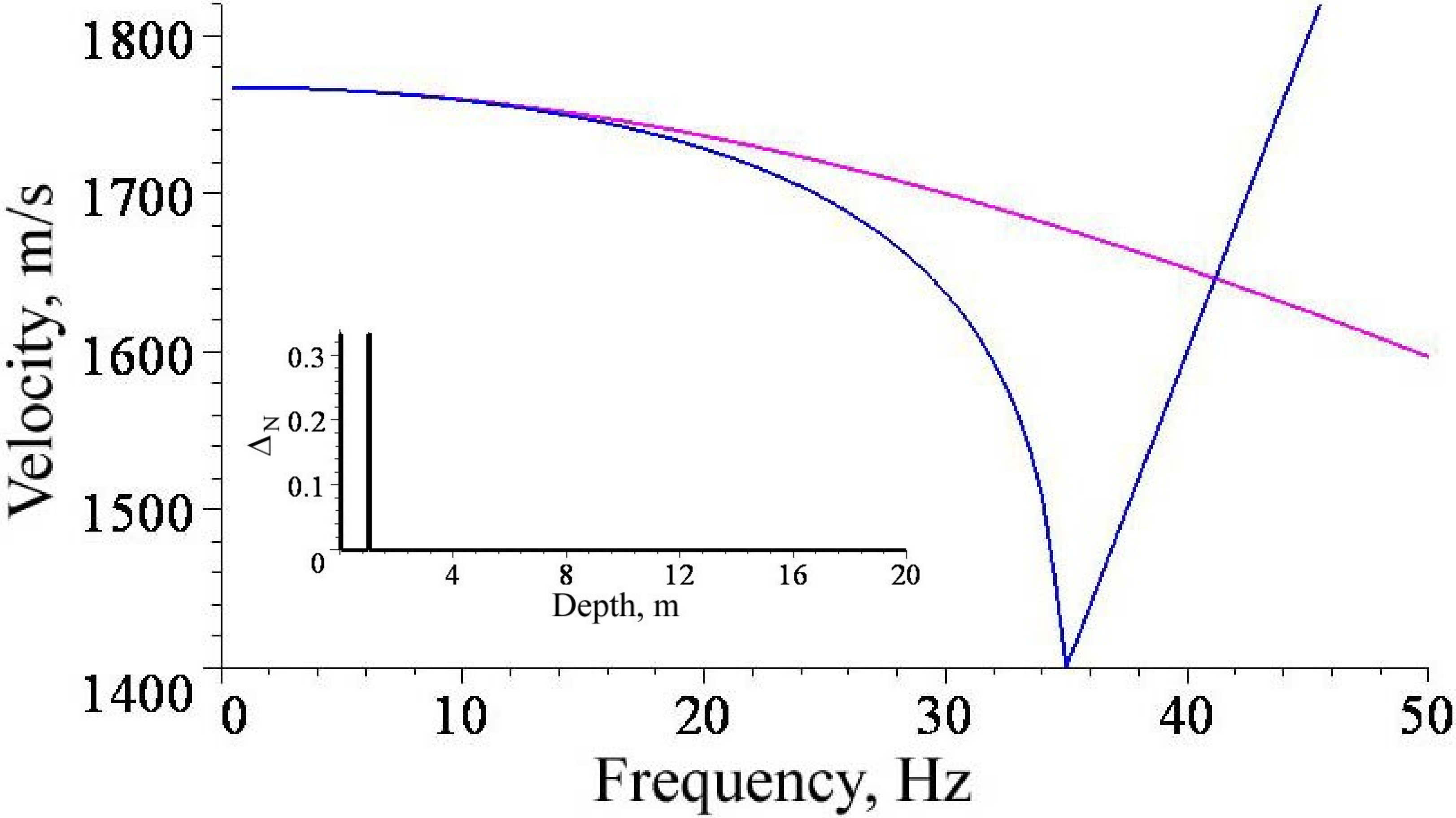
$$\tau_2 = \frac{t_0^2}{12}\left(d_1 + \dots + d_n\right)^2 - \frac{1}{2} t_0^2 \sum_{i,j=1}^{n} s_{ij} d_i d_j$$

$$s_{ij} = s_{ji},\ s_{ii} = 0 \qquad s_{ij} = \left(t_i + \dots + t_{j-1}\right)\left((t_1 + \dots + t_{i-1}) + (t_j + \dots + t_n)\right)$$

# Single fracture in a homogeneous medium



# Two fractures in a homogeneous medium



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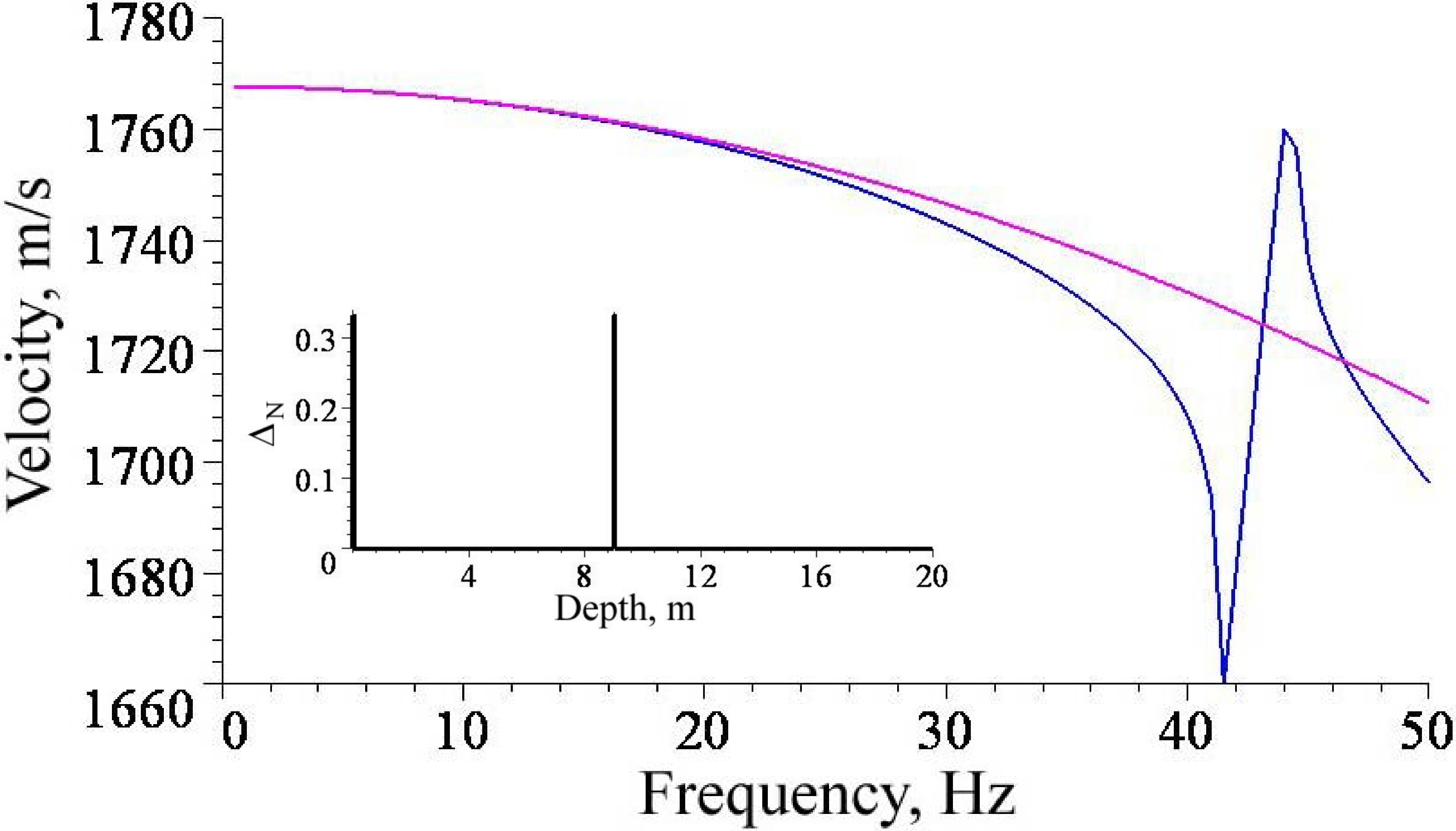
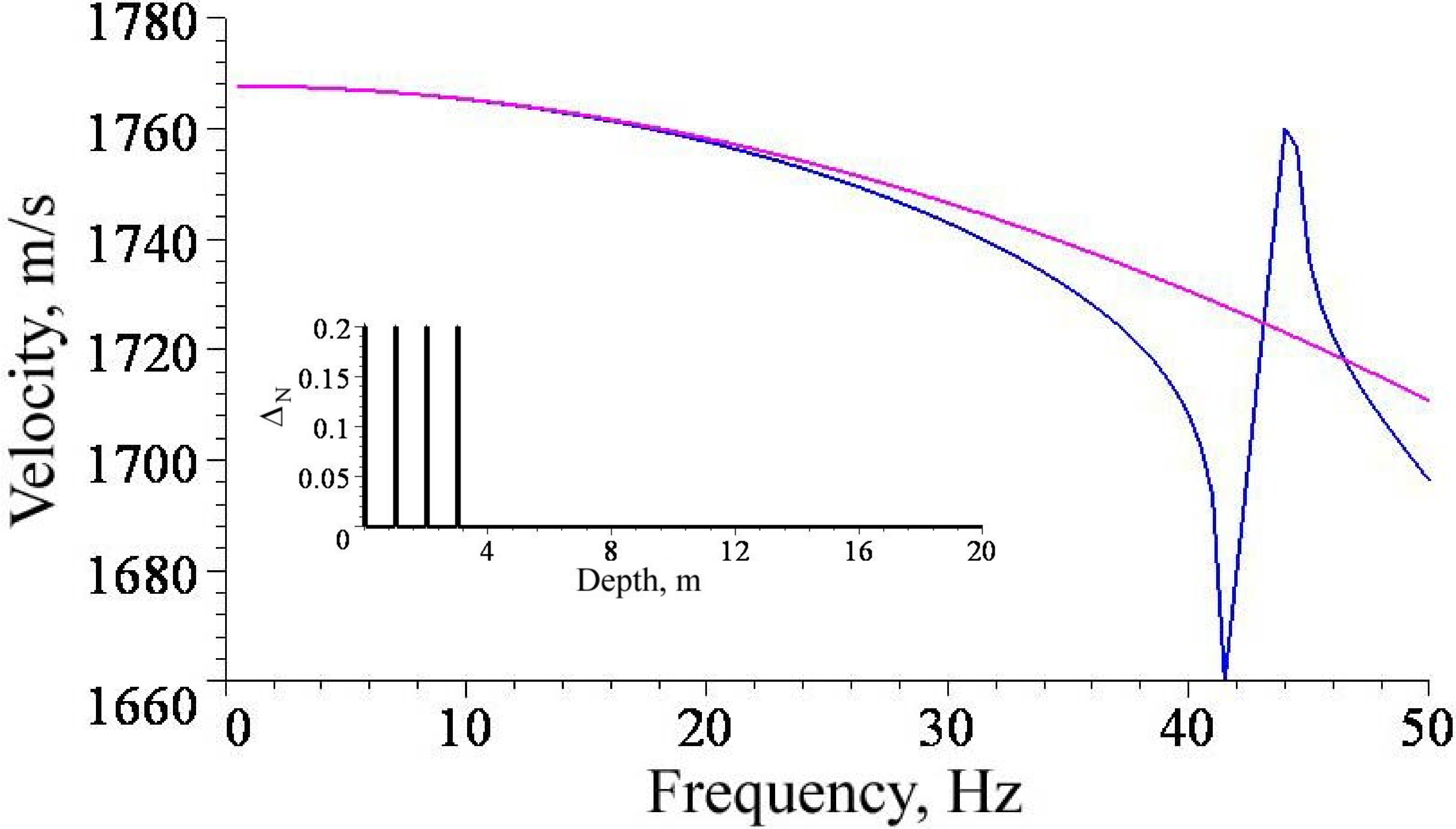
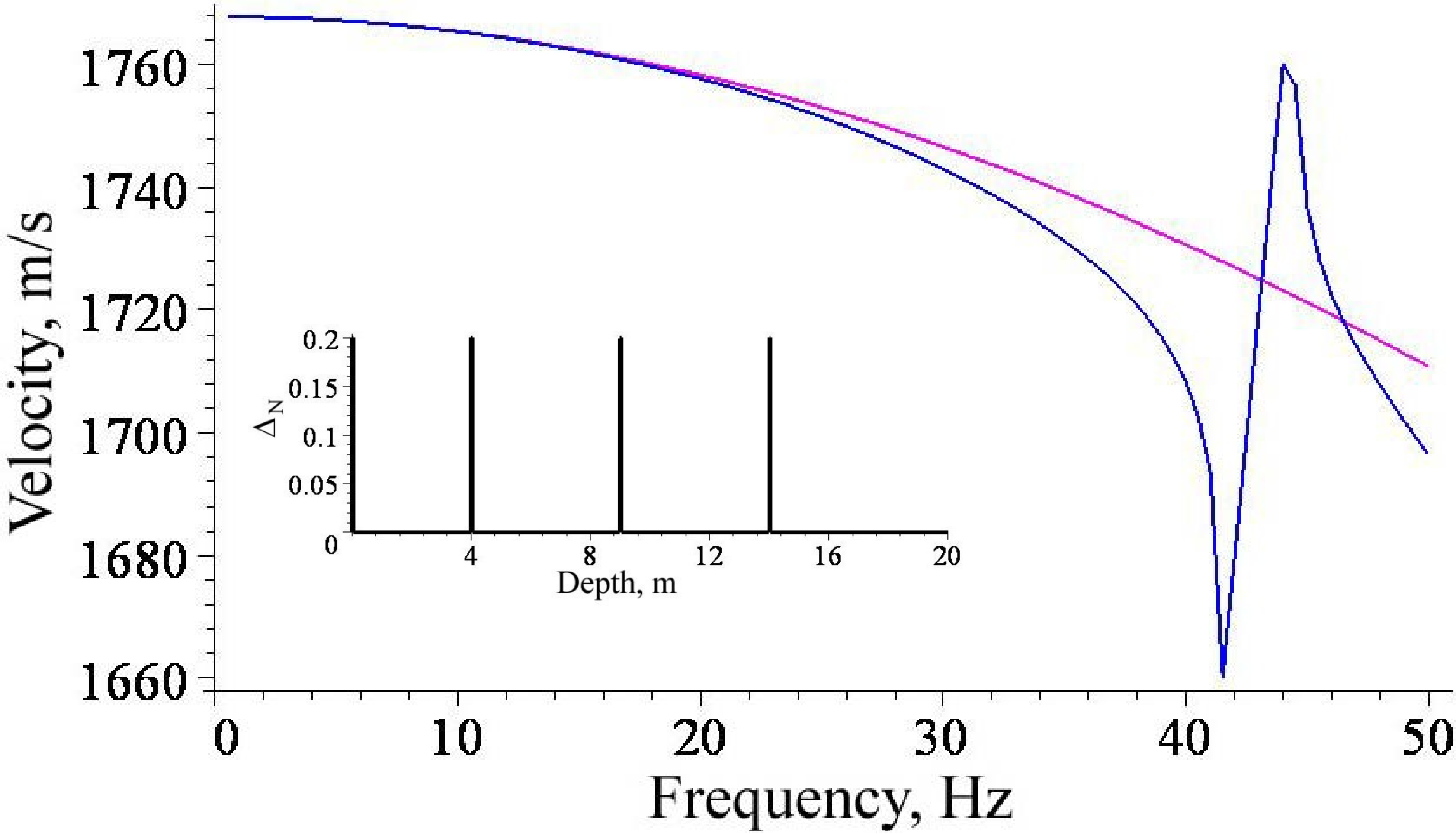


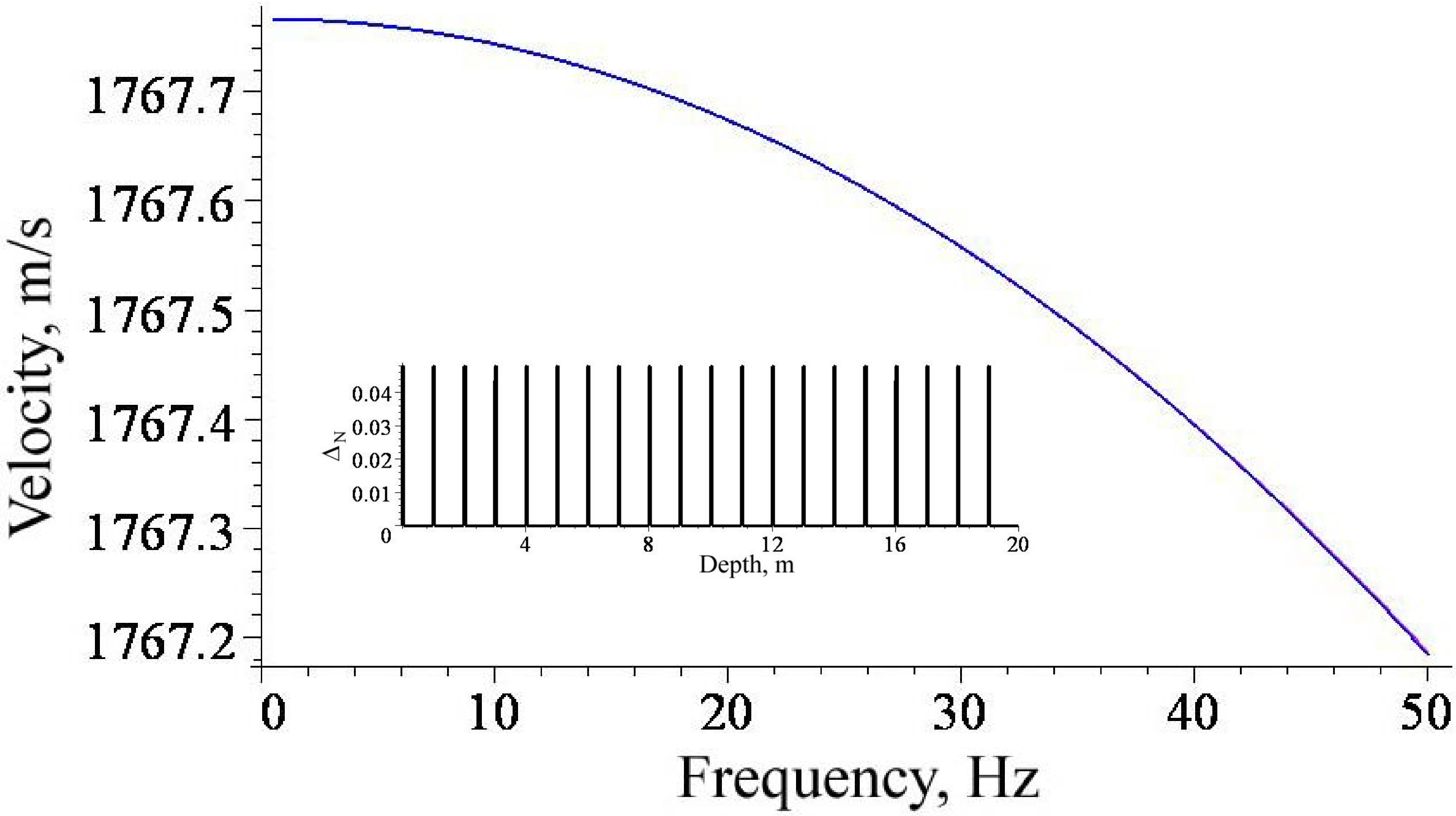
# Four fractures in a homogeneous medium



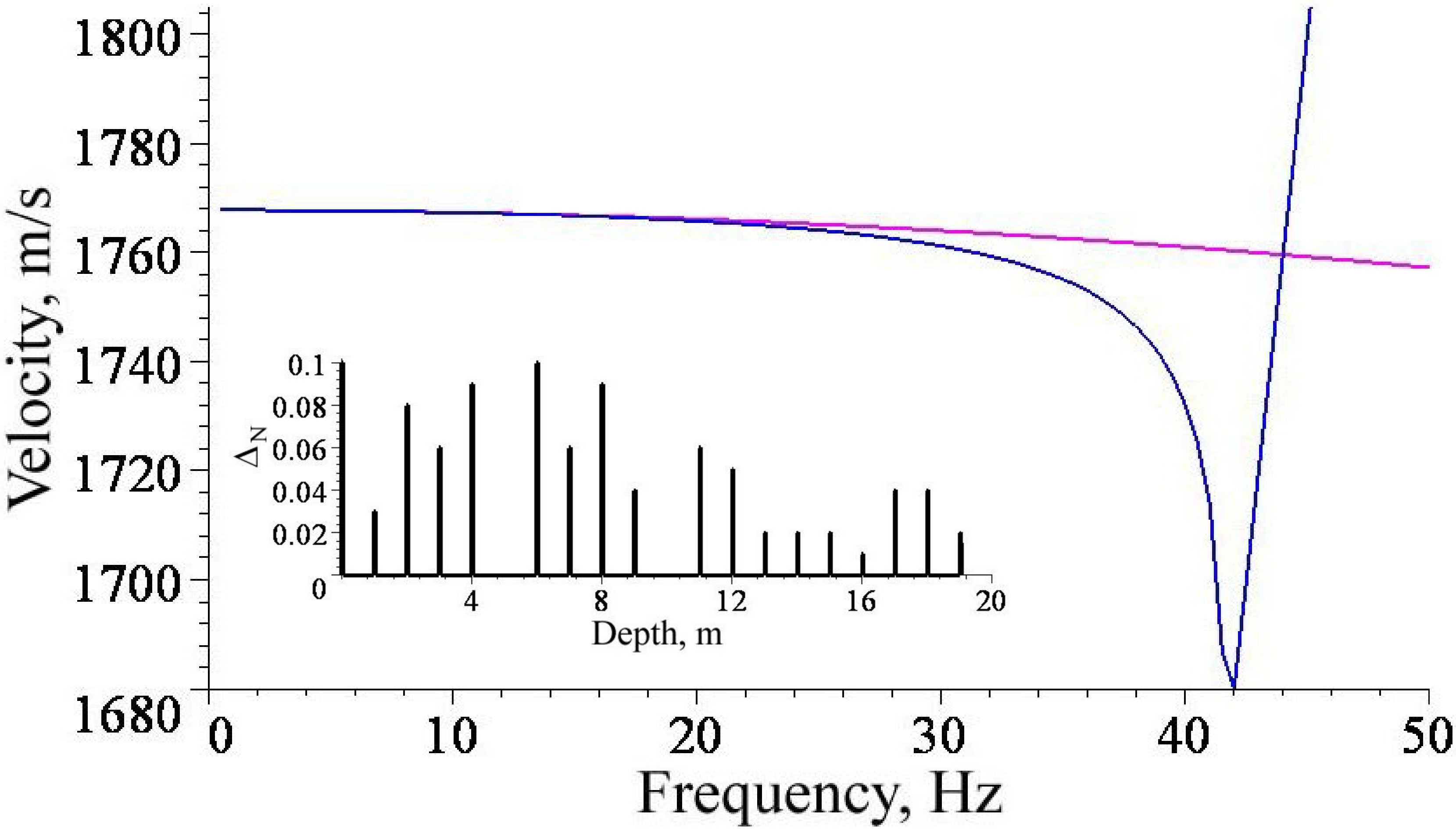
# Four fractures in a homogeneous medium



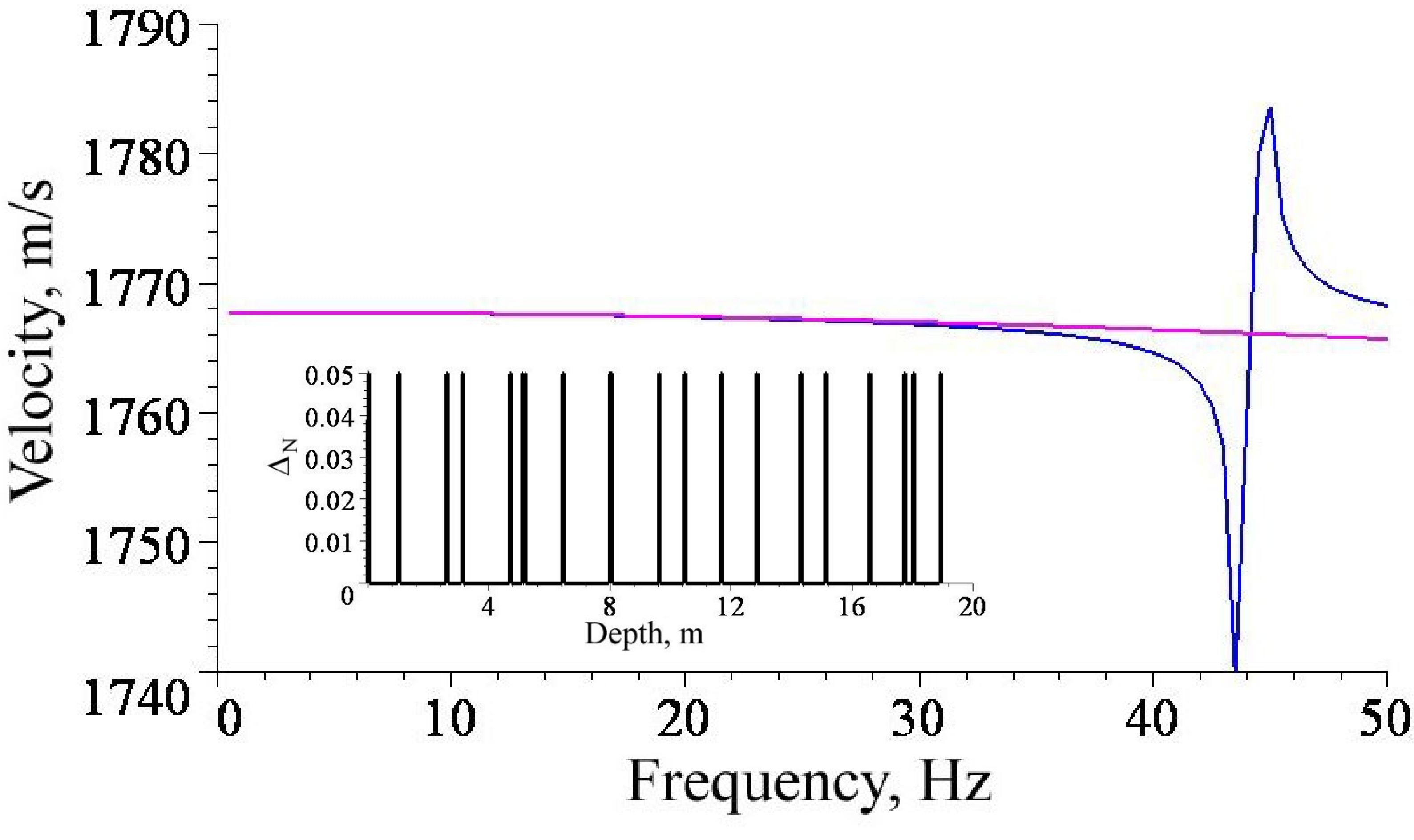
# Twenty fractures in a homogeneous medium



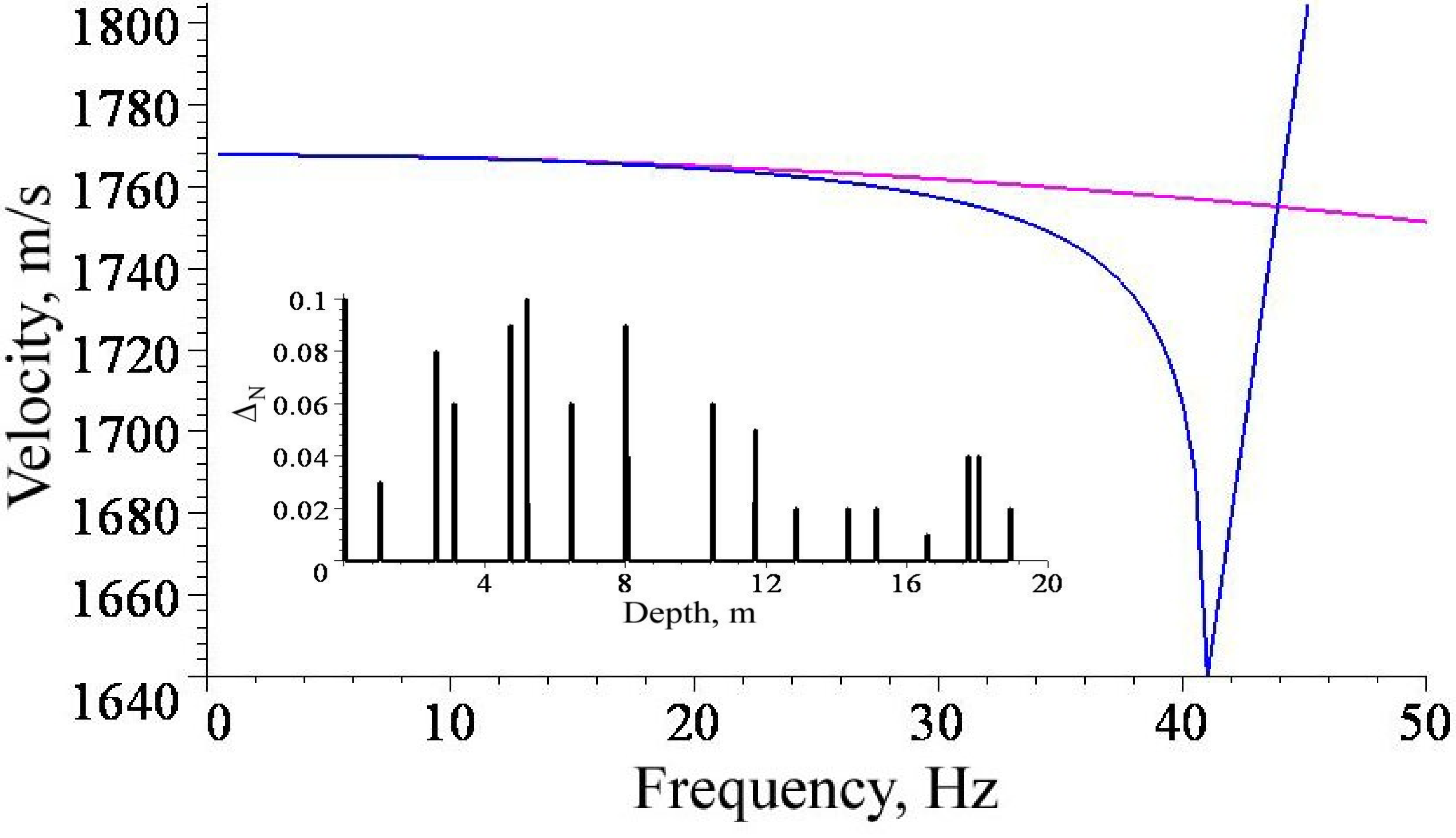
# Randomly intensive fractures in a homogeneous medium



# Randomly distributed fractures in a homogeneous medium



# Randomly intensive&distributed fractures in a homogeneous medium



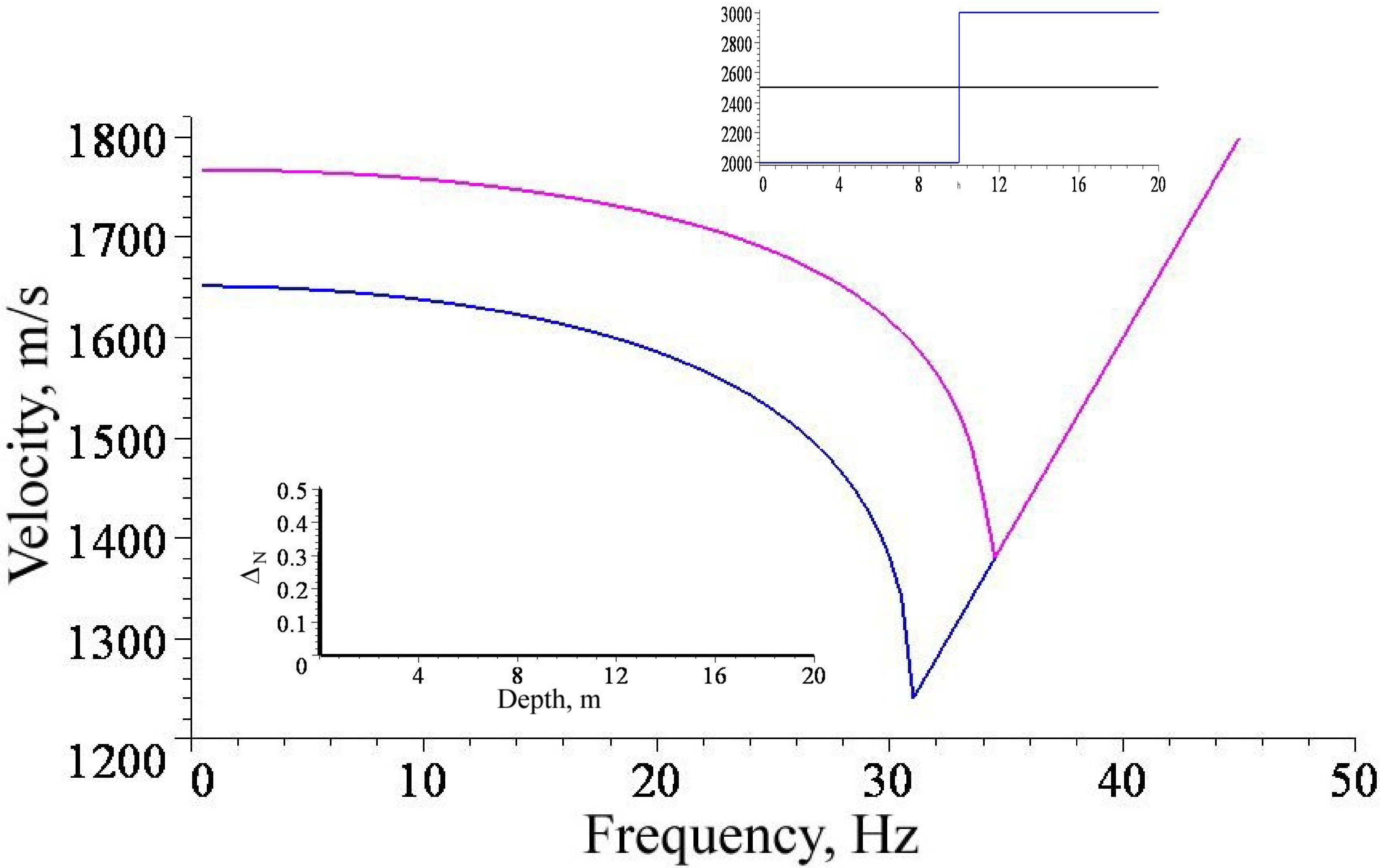
# Fracturing vs layering

## 2 layers and 2 fractures

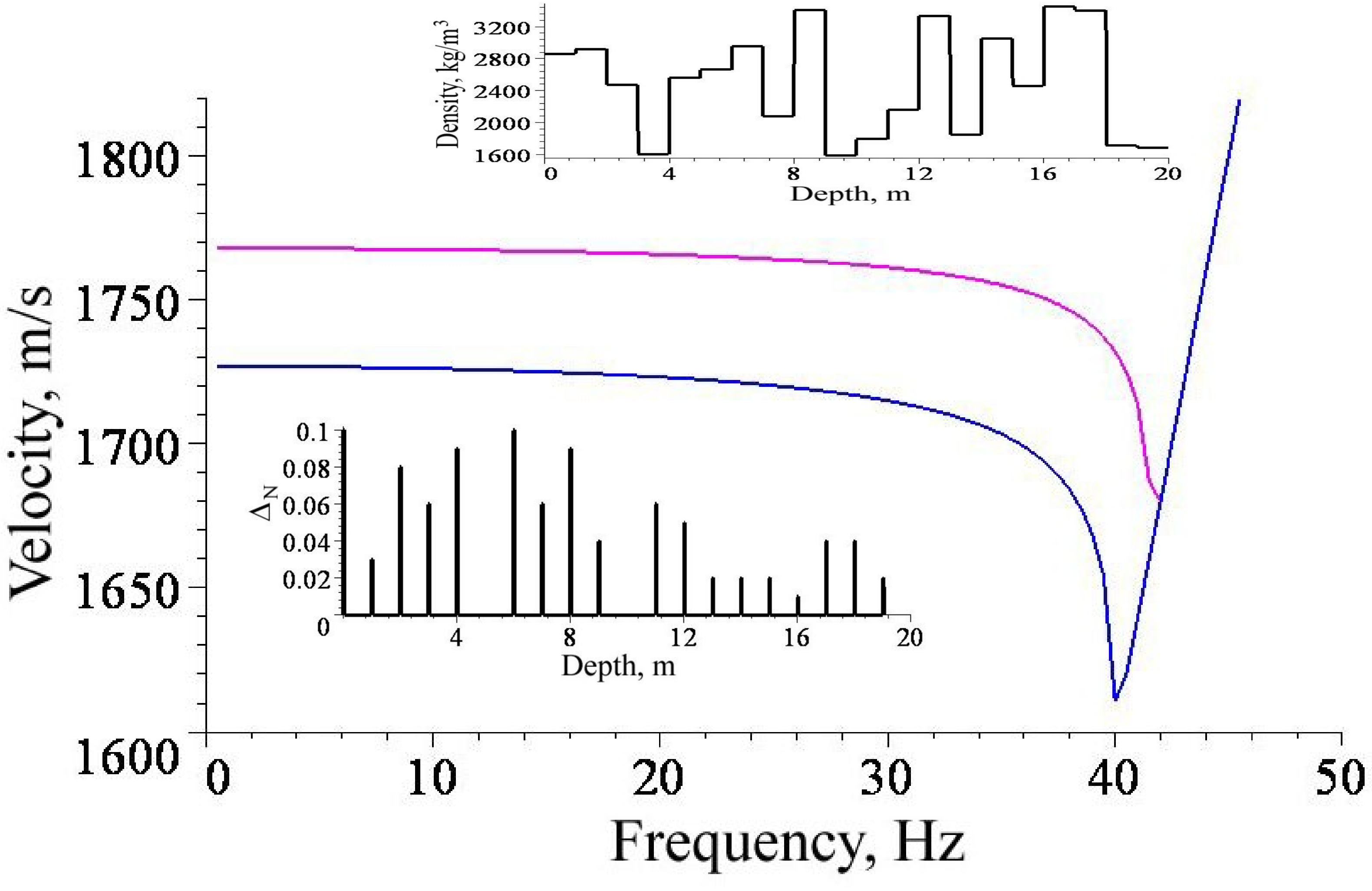
$$t_0 = t_1 + t_2,\ \ t_Z = t_1 Z_1 + t_2 Z_2,\ \ d_Z = d_1 / Z_1 + d_2 / Z_2$$

$$\tau_2 = \frac{1}{12} t_0^2 t_Z^2 d_Z^2 - t_0^2 t_1 t_2 d_1 d_2 + \frac{1}{6} t_0 t_1 t_2 \left( \frac{Z_1}{Z_2} - \frac{Z_2}{Z_1} \right) \left( t_1 Z_1 - t_2 Z_2 \right) d_Z + \frac{1}{12} t_1^2 t_2^2 \left( \frac{Z_1}{Z_2} - \frac{Z_2}{Z_1} \right)^2$$

# Fracturing vs layering



# Fracturing vs layering



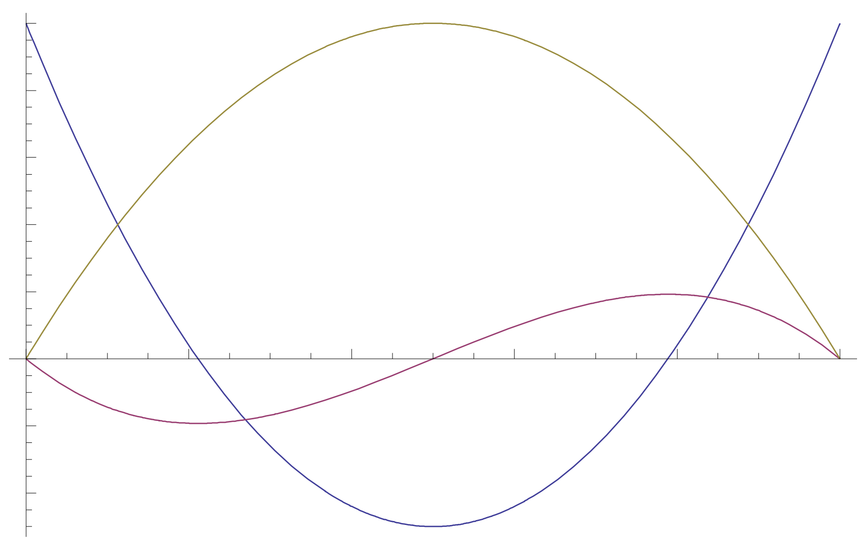
# Weak-contrast

$$\mathbf{v} = \left( d_1, d_2, \delta Z \right)^T \qquad \tau_2 = \mathbf{v}^T \mathbf{A} \mathbf{v}$$

$$\mathbf{A} = \frac{t_0^4}{12} \begin{pmatrix} 1 & 1-6\alpha(1-\alpha) & 2\alpha(1-\alpha)(2\alpha-1) \\ 1-6\alpha(1-\alpha) & 1 & 2\alpha(1-\alpha)(2\alpha-1) \\ 2\alpha(1-\alpha)(2\alpha-1) & 2\alpha(1-\alpha)(2\alpha-1) & 4\alpha(1-\alpha) \end{pmatrix}$$

$\alpha = t_1 / t_0$ *time fraction*

$$\boxed{\tau_2 \geq 0}$$



# Conclusions

- The propagator is defined for fractures set within the linear slip theory.
- The low-frequency dispersion is considered for set of fractures in homogeneous and layered medium.
- The sensitivity analysis shows the effect of fracture vs layering in a weak-contrast approximation.