Upscaling for orthorhombic media

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Outline

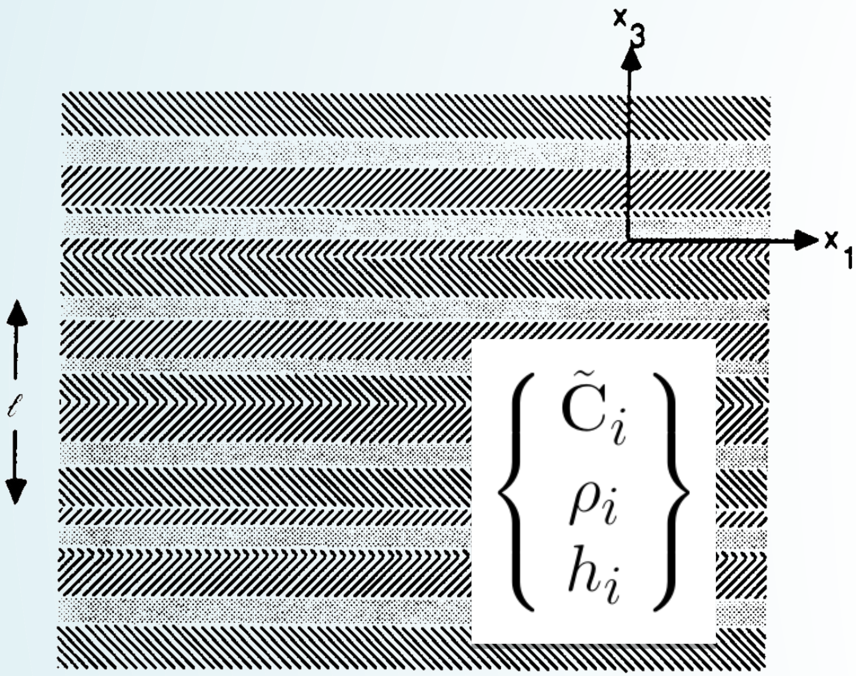
- Motivation
- Theory
- Considered models
  - ORT-az + ORT
  - VTI + ORT
- Conclusions

Motivation

http://imgkid.com/sedimentary-rock-layers.shtml

How does the seismic wave behave in the presence of thin (compared to wavelength) layering and/or fractures?

Elastic moduli of layered anisotropic media



(*Schoenberg and Muir, 1989*)

Schoenberg and Muir, 1989:

Assumptions:

1. Thickness of individual layer is smaller then a wavelength

$$h_i \ll \lambda$$

2. Stationarity

$$h_i = \text{const in } \forall \ell \ll \lambda$$

Elastic moduli of layered anisotropic media

$$\tilde{\mathbf{C}}^i_{\mathbf{NN}} = \begin{bmatrix} C^i_{33} & C^i_{34} & C^i_{35} \\ C^i_{34} & C^i_{44} & C^i_{45} \\ C^i_{35} & C^i_{45} & C^i_{55} \end{bmatrix}$$

$$\tilde{\mathbf{C}}^i_{\mathbf{TN}} = \begin{bmatrix} C^i_{13} & C^i_{14} & C^i_{15} \\ C^i_{23} & C^i_{24} & C^i_{25} \\ C^i_{36} & C^i_{46} & C^i_{56} \end{bmatrix}$$

$$\tilde{\mathbf{C}}^i_{\mathbf{TT}} = \begin{bmatrix} C^i_{11} & C^i_{12} & C^i_{16} \\ C^i_{12} & C^i_{22} & C^i_{26} \\ C^i_{16} & C^i_{26} & C^i_{66} \end{bmatrix}$$

$$\tilde{\mathbf{C}}^i_{\mathbf{NT}} = \tilde{\mathbf{C}}^i_{\mathbf{TN}}$$

$$\tilde{\mathrm{C}}_{\mathrm{NN}} = \left\langle \tilde{\mathrm{C}}^{-1}_{\mathrm{NN}} \right\rangle^{-1}$$

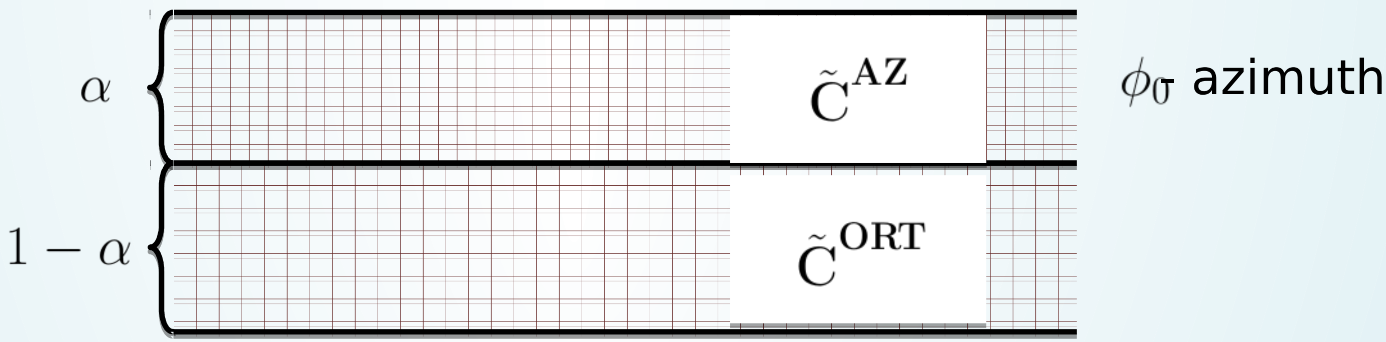
$$\tilde{\mathrm{C}}_{\mathrm{TN}} = \left\langle \tilde{\mathrm{C}}_{\mathrm{TN}} \tilde{\mathrm{C}}^{-1}_{\mathrm{NN}} \right\rangle \tilde{\mathrm{C}}_{\mathrm{NN}}$$

$$\tilde{\mathrm{C}}_{\mathrm{TT}} = \langle \tilde{\mathrm{C}}_{\mathrm{TT}} \rangle - \left\langle \tilde{\mathrm{C}}_{\mathrm{TN}} \tilde{\mathrm{C}}^{-1}_{\mathrm{NN}} \tilde{\mathrm{C}}_{\mathrm{NT}} \right\rangle + \left\langle \tilde{\mathrm{C}}_{\mathrm{TN}} \tilde{\mathrm{C}}^{-1}_{\mathrm{NN}} \right\rangle \tilde{\mathrm{C}}_{\mathrm{NN}} \left\langle \tilde{\mathrm{C}}^{-1}_{\mathrm{NN}} \tilde{\mathrm{C}}_{\mathrm{NT}} \right\rangle$$

- Valid for arbitrary anisotropy
- Does not assume periodicity of the layers
- Effective media symmetry order is equal to the lowest symmetry order of the individual layers

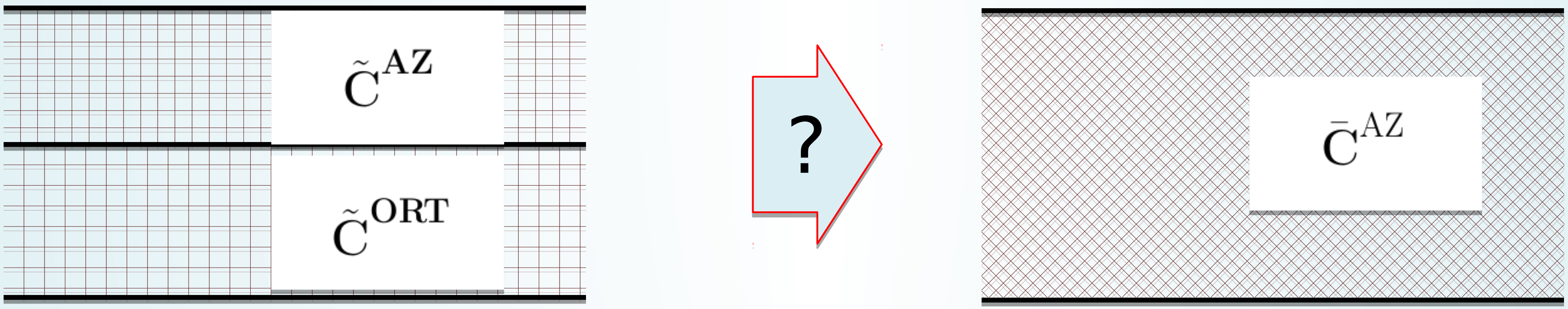
Effective anisotropic media: ORT-az + ORT

$$\tilde{\mathrm{C}}^{\mathbf{AZ}} = \begin{pmatrix} \hat{C}_{11} & \hat{C}_{12} & \hat{C}_{13} & 0 & 0 & \hat{C}_{16} \\ \hat{C}_{12} & \hat{C}_{22} & \hat{C}_{23} & 0 & 0 & \hat{C}_{26} \\ \hat{C}_{13} & \hat{C}_{23} & \hat{C}_{33} & 0 & 0 & \hat{C}_{36} \\ 0 & 0 & 0 & \hat{C}_{44} & \hat{C}_{45} & 0 \\ 0 & 0 & 0 & \hat{C}_{45} & \hat{C}_{55} & 0 \\ \hat{C}_{16} & \hat{C}_{26} & \hat{C}_{36} & 0 & 0 & \hat{C}_{66} \end{pmatrix}$$



Effective anisotropic media: ORT-az + ORT

Can **effective** (averaged) **orthorhombic** stiffness matrix be represented by some **azimuthally** rotated **orthorhombic** stiffness matrix?



Monoclini
c

Orthorhombic +
azimuth

Effective anisotropic media: ORT-az + ORT

Approach I

Can **effective** (averaged) **orthorhombic** stiffness matrix be represented by some **azimuthally** rotated **orthorhombic** stiffness matrix?

$$\tilde{\mathrm{C}}^{\mathbf{AV}} = \boldsymbol{M}\tilde{\mathrm{C}}^{\mathbf{ORT}}\boldsymbol{M}^{\mathrm{T}}$$

$$\boldsymbol{M}^{-1}\tilde{\mathrm{C}}^{\mathbf{AV}}\boldsymbol{M}^{-\mathrm{T}} = \tilde{\mathrm{C}}^{\mathbf{ORT}}$$

$$\boldsymbol{M} = \boldsymbol{M}_z(\phi^*) = \begin{pmatrix} \cos^2\phi^* & \sin^2\phi^* & 0 & 0 & 0 & -\sin 2\phi^* \\ \sin^2\phi^* & \cos^2\phi^* & 0 & 0 & 0 & \sin 2\phi^* \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\phi^* & \sin\phi^* & 0 \\ 0 & 0 & 0 & -\sin\phi^* & \cos\phi^* & 0 \\ \frac{\sin 2\phi^*}{2} & -\frac{\sin 2\phi^*}{2} & 0 & 0 & 0 & \cos 2\phi^* \end{pmatrix}$$

$$\begin{pmatrix} \hat{C}_{11} & \hat{C}_{12} & \hat{C}_{13} & 0 & 0 & \hat{C}_{16} \\ \hat{C}_{12} & \hat{C}_{22} & \hat{C}_{23} & 0 & 0 & \hat{C}_{26} \\ \hat{C}_{13} & \hat{C}_{23} & \hat{C}_{33} & 0 & 0 & \hat{C}_{36} \\ 0 & 0 & 0 & \hat{C}_{44} & \hat{C}_{45} & 0 \\ 0 & 0 & 0 & \hat{C}_{45} & \hat{C}_{55} & 0 \\ \hat{C}_{16} & \hat{C}_{26} & \hat{C}_{36} & 0 & 0 & \hat{C}_{66} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

$$\hat{C}_{16}, \hat{C}_{26}, \hat{C}_{36}, \hat{C}_{45} \rightarrow 0$$

Effective anisotropic media: ORT-az + ORT

$$\hat{C}_{16}, \hat{C}_{26}, \hat{C}_{36}, \hat{C}_{45} \rightarrow 0$$

$$\hat{C}_{16}, \hat{C}_{26}, \hat{C}_{36}, \hat{C}_{45} = \mathcal{F}(\phi_0, \phi^*)$$

ϕ^* - effective azimuth

$$\frac{1}{\bar{C}_{33}^2}\left[\hat{C}_{16}^2(\phi^*) + \hat{C}_{26}^2(\phi^*) + \hat{C}_{36}^2(\phi^*) + \hat{C}_{45}^2(\phi^*)\right] = f(\phi^*)$$

$$f(\phi^*) \rightarrow \min_{\phi \in [0;\pi/2]} f(\phi)$$

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
3.5	1.6	1.0	0.25	0.15	0.1	0.15	0.1	0.05	0.05	0.08	0.04	0.02

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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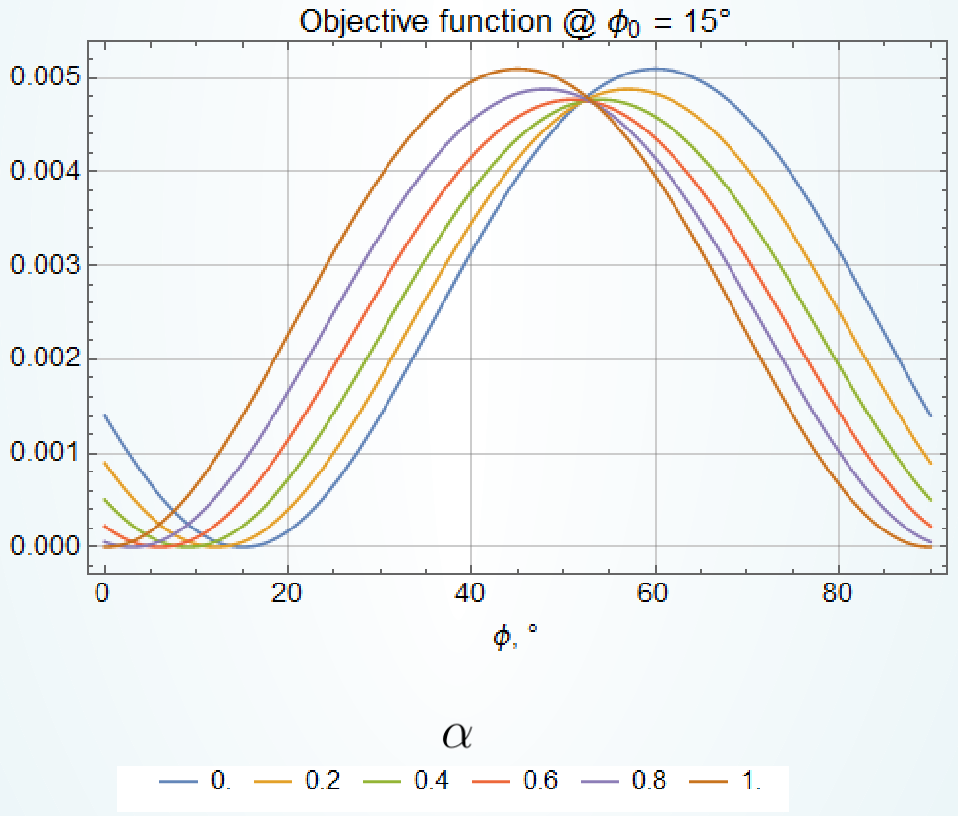

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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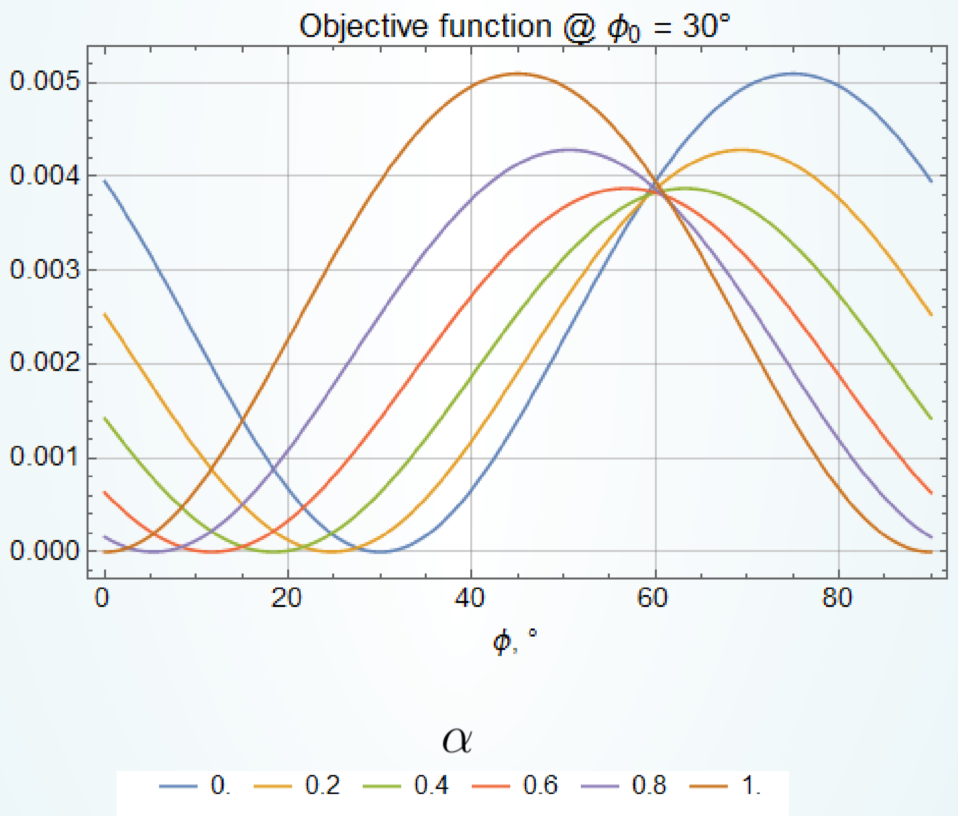

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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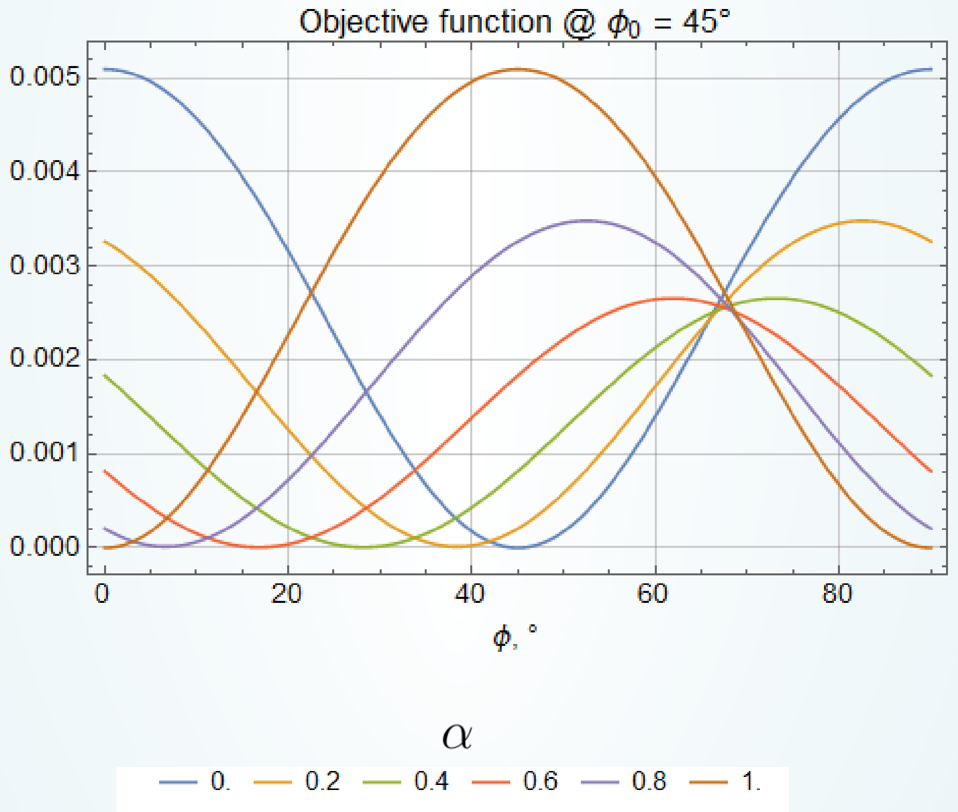

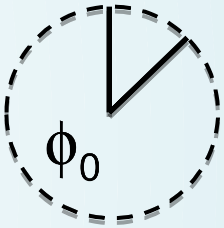

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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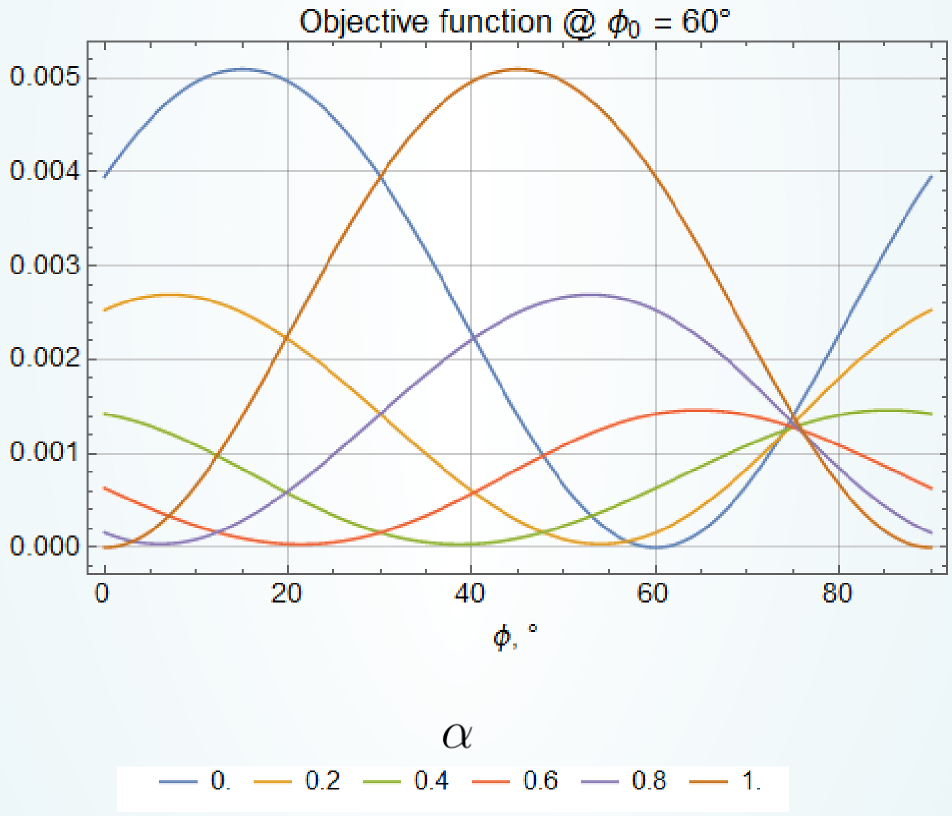

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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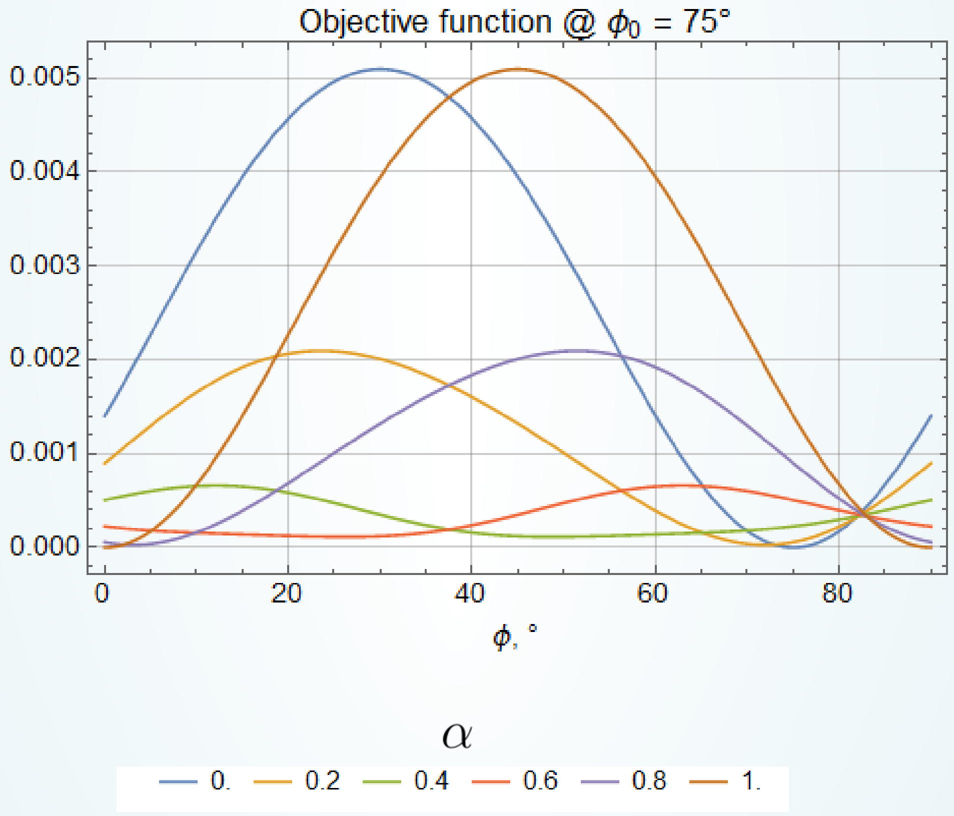

Effective anisotropic media: ORT-az + ORT

V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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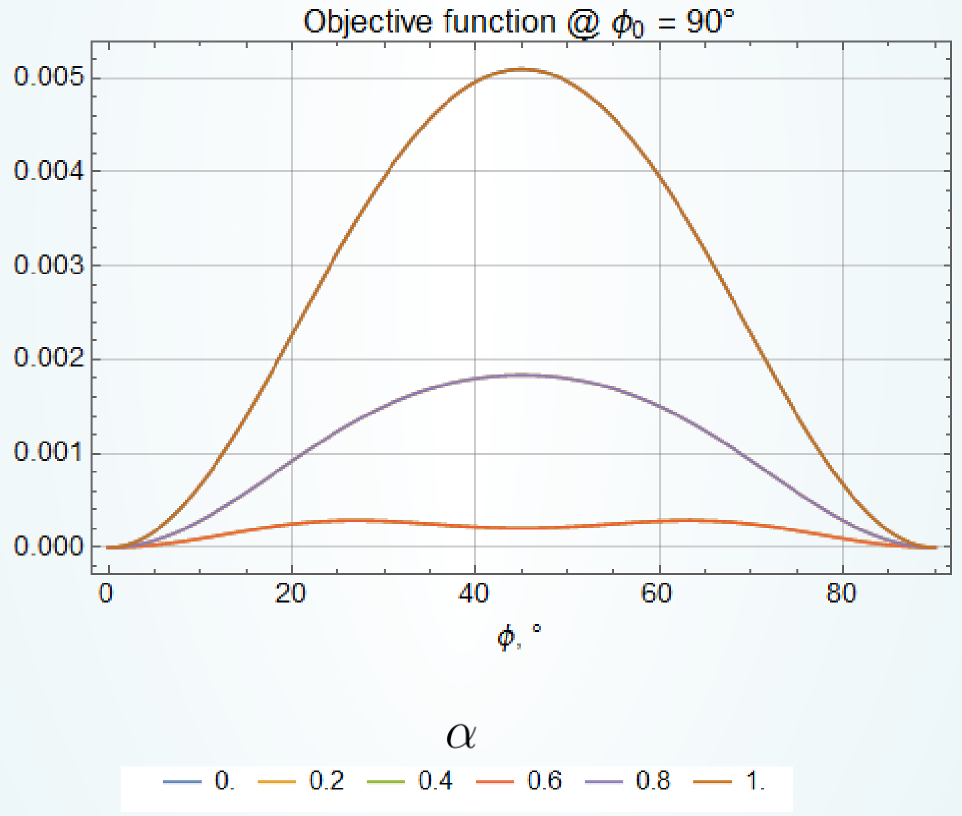
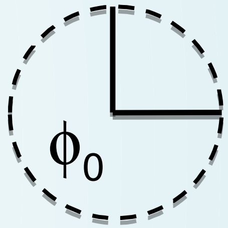
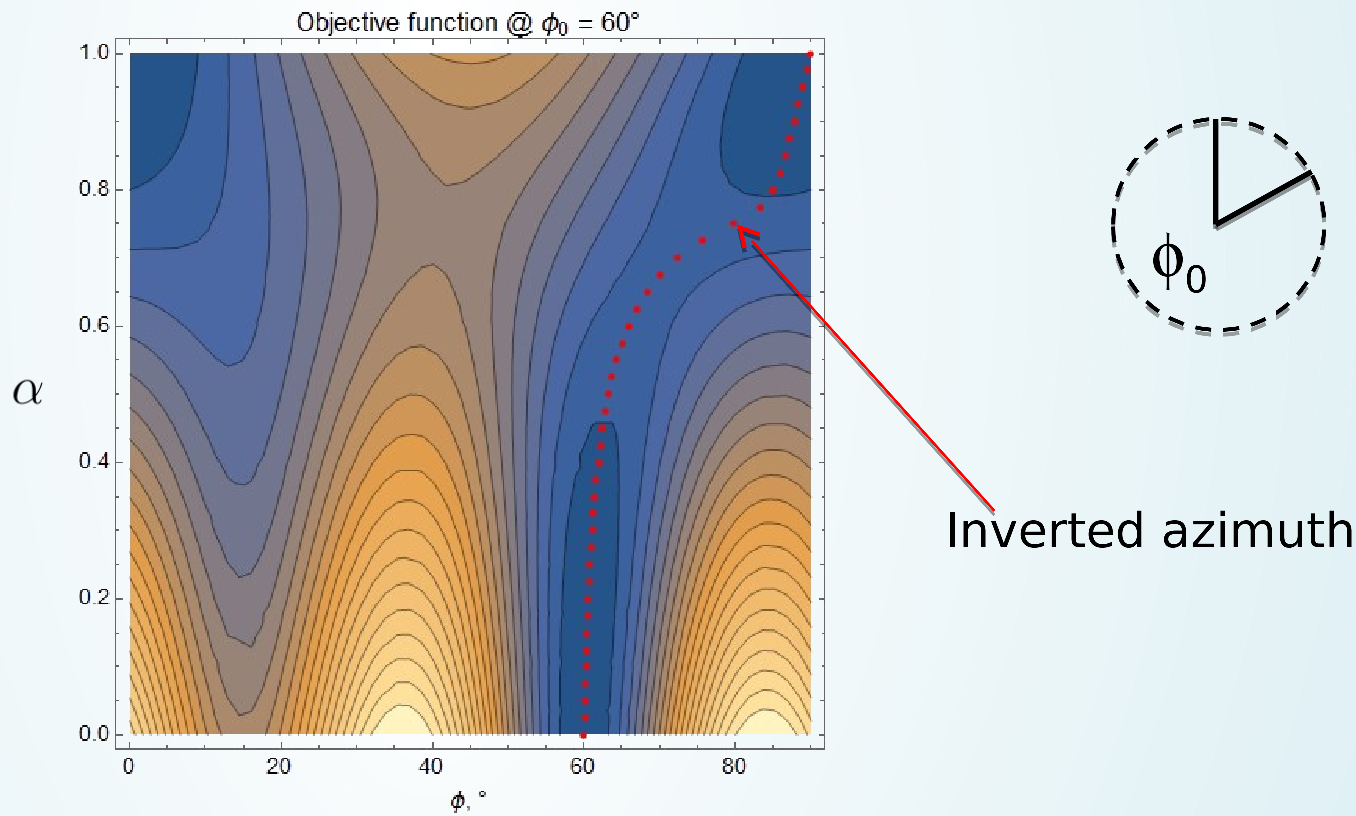




Effective anisotropic media: ORT-az + ORT

Layer	V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
I	3.5	1.6	1.0	0.25	0.15	0.1	0.15	0.1	0.05	0.05	0.08	0.04	0.02
II	4.0	2.0	1.0	0.15	-0.1	0.05	0.2	-0.15	0.1	-0.2	0.31	0.5	0.27



Effective anisotropic media: ORT-az + ORT

Layer	V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
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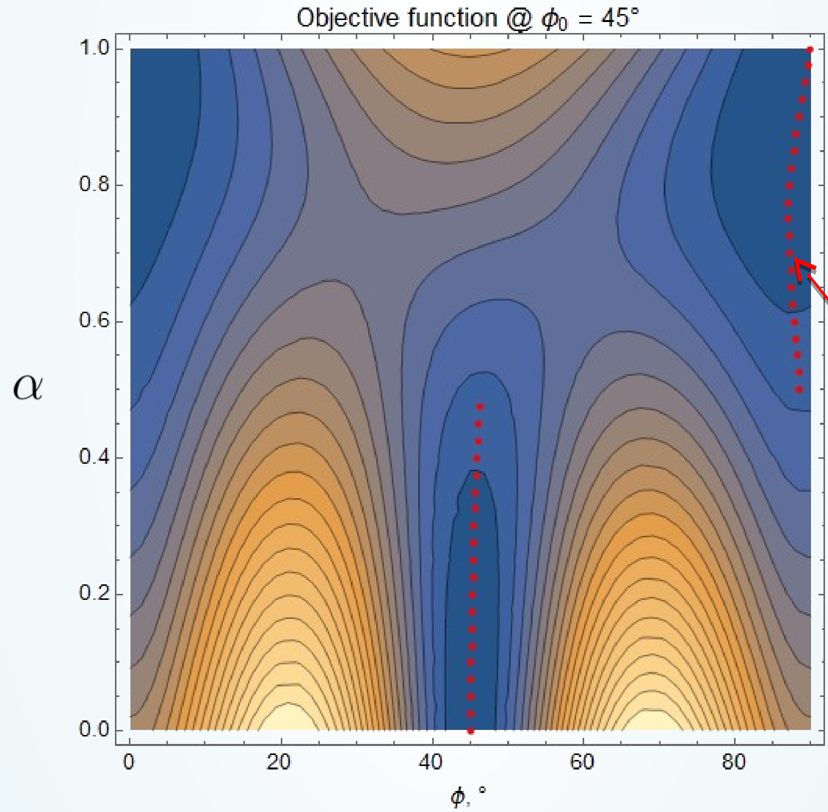
II	4.0	2.0	1.0	0.15	-0.1	0.05	0.2	-0.15	0.1	-0.2	0.31	0.5	0.27



Inverted azimuth

Effective anisotropic media: ORT-az + ORT

Approach II

Cowin S.C., Mehrabadi M.M., 1987:
On the identification of material symmetry for anisotropic elastic materials.

$$\underset{\sim}{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix}$$

Effective anisotropic media: ORT-az + ORT

Dilatational stiffness tensor:

$$c_{ijkk} = \begin{bmatrix} c_{11}+c_{12}+c_{13} & c_{16}+c_{26}+c_{36} & c_{15}+c_{25}+c_{35} \\ c_{16}+c_{26}+c_{36} & c_{12}+c_{22}+c_{23} & c_{14}+c_{24}+c_{34} \\ c_{15}+c_{25}+c_{35} & c_{14}+c_{24}+c_{34} & c_{13}+c_{23}+c_{33} \end{bmatrix}$$

Voight stiffness tensor:

$$c_{ikkj} = \begin{bmatrix} c_{11}+c_{55}+c_{66} & c_{16}+c_{26}+c_{45} & c_{15}+c_{46}+c_{35} \\ c_{16}+c_{26}+c_{45} & c_{22}+c_{44}+c_{66} & c_{24}+c_{34}+c_{56} \\ c_{15}+c_{46}+c_{35} & c_{24}+c_{34}+c_{56} & c_{33}+c_{44}+c_{55} \end{bmatrix}$$

Effective anisotropic media: ORT-az + ORT

$$\mathbf{C}_{ikkj} = \mathbf{Q}_1 \mathbf{\Lambda}_1 \mathbf{Q}_1^{-1}$$

Voight stiffness tensor

$$\mathbf{C}_{ijkk} = \mathbf{Q}_2 \mathbf{\Lambda}_2 \mathbf{Q}_2^{-1}$$

Dilatational stiffness tensor

⇨ Number of distinct eigenvalues can be used to deduce number of symmetry planes

⇨ $\mathbf{Q} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ - transformation matrix

$$\mathbf{C}_{ijkl} = \mathbf{Q}_{ii'} \mathbf{Q}_{jj'} \mathbf{Q}_{kk'} \mathbf{Q}_{ll'} \mathbf{C}_{i'j'k'l'}$$

$\mathrm{Q}_1 = \mathrm{Q}_2$ (pure orthorhombic)

$\mathrm{Q}_1 \neq \mathrm{Q}_2$ (monoclinic)

Effective anisotropic media: ORT-az + ORT

$$\mathbf{C}_{ikkj} = \mathbf{Q}_1 \mathbf{\Lambda}_1 \mathbf{Q}_1^{-1}$$

Voight stiffness tensor

$$\mathbf{C}_{ijkk} = \mathbf{Q}_2 \mathbf{\Lambda}_2 \mathbf{Q}_2^{-1}$$

Dilatational stiffness tensor

⇒

$\mathrm{Q}_1 \neq \mathrm{Q}_2$ (monoclinic)

$$\mathbf{Q}_1 = \begin{pmatrix} \cos\phi_1^* & \sin\phi_1^* & 0 \\ \sin\phi_1^* & \cos\phi_1^* & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Q}_2 = \begin{pmatrix} \cos\phi_2^* & \sin\phi_2^* & 0 \\ \sin\phi_2^* & \cos\phi_2^* & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

? ⇒ $\langle \phi_1^*, \phi_2^* \rangle$

Browayes et al., 2004

Effective anisotropic media: ORT-az + ORT

Layer	V_{P0}	V_{S0}	ρ	$\epsilon^{(1)}$	$\delta^{(1)}$	$\gamma^{(1)}$	$\epsilon^{(2)}$	$\delta^{(2)}$	$\gamma^{(2)}$	$\delta^{(3)}$	$\eta^{(1)}$	$\eta^{(2)}$	$\eta^{(3)}$
I	3.5	1.6	1.0	0.25	0.15	0.1	0.15	0.1	0.05	0.05	0.08	0.04	0.02
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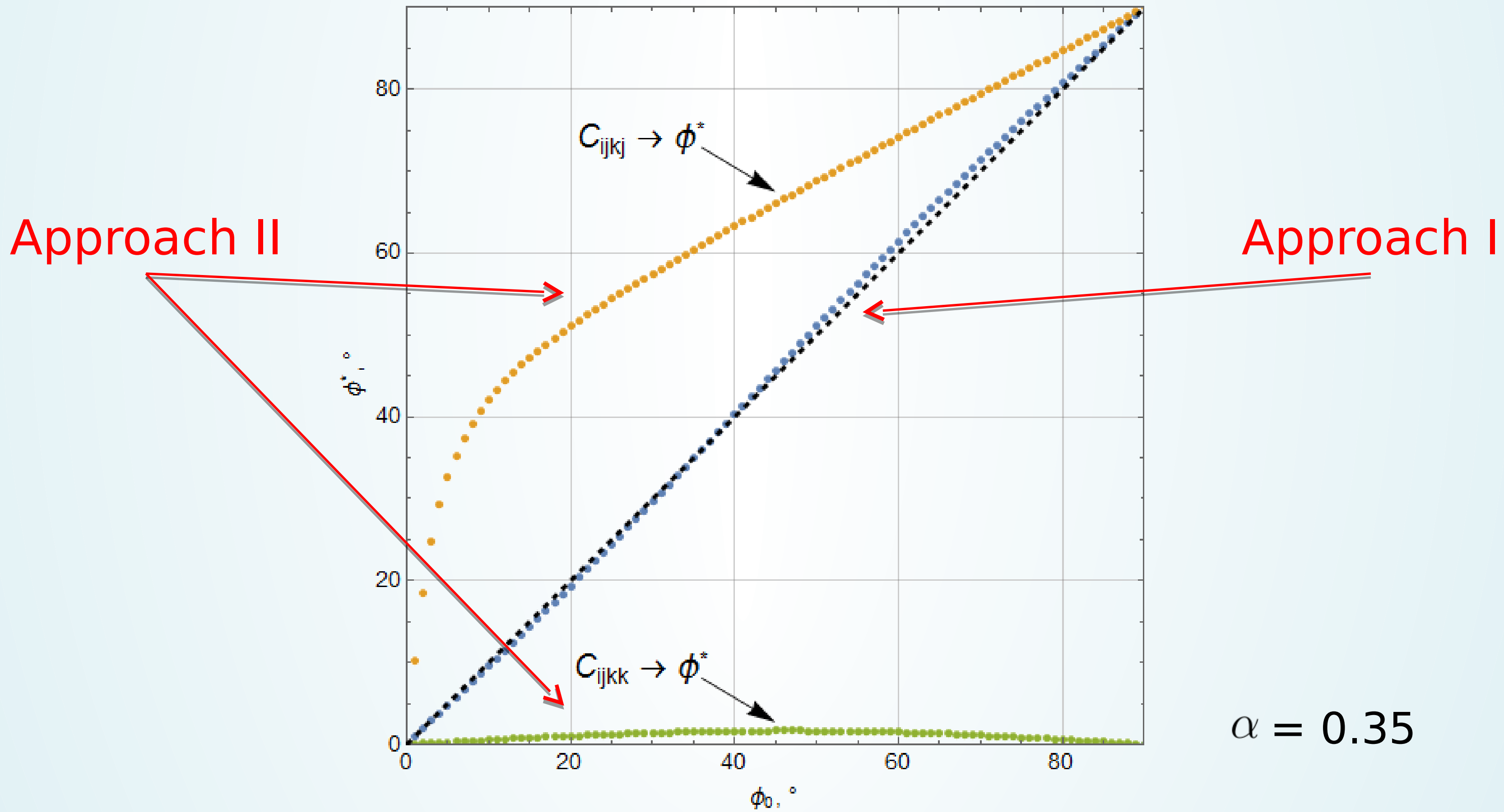
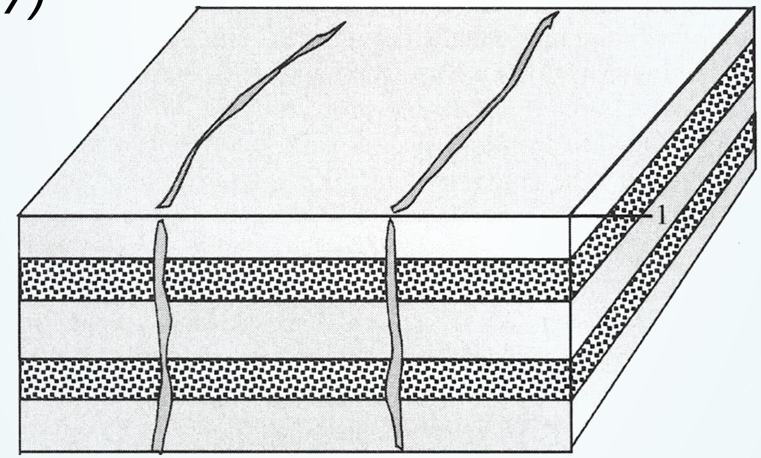


Effective anisotropic media: VTI + ORT

$$\tilde{C}^{\mathbf{ORT}^*} = \begin{pmatrix} C_{11b}(1-\delta_N) & C_{12b}(1-\delta_N) & C_{13b}(1-\delta_N) & 0 & 0 & 0 \\ C_{12b}(1-\delta_N) & C_{11b} - \delta_N \frac{C12_b^2}{C11_b} & C_{13b}\left(1-\delta_N \frac{C_{12b}}{C_{11b}}\right) & 0 & 0 & 0 \\ C_{13b}(1-\delta_N) & C_{13b}\left(1-\delta_N \frac{C_{12b}}{C_{11b}}\right) & C_{33b} - \delta_N \frac{C_{13b}^2}{C_{11b}} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44b} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44b}(1-\delta_V) & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66b}(1-\delta_H) \end{pmatrix}$$

C_{ijb} - background VTI medium parameters

$\delta_N, \delta_V, \delta_H$ - fracture weaknesses *(Schoenberg and Helbig, 1997)*

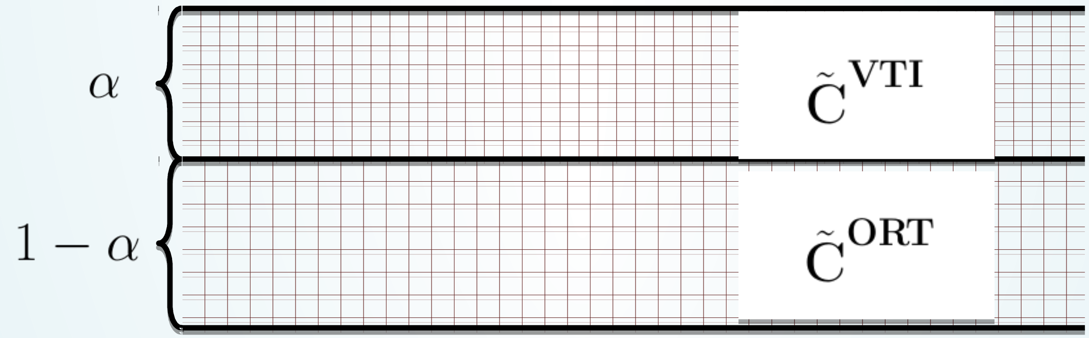


Effective anisotropic media: VTI + ORT

$$\tilde{C}^{\mathbf{ORT}^*} = \begin{pmatrix} C_{11b}(1-\delta_N) & C_{12b}(1-\delta_N) & C_{13b}(1-\delta_N) & 0 & 0 & 0 \\ C_{12b}(1-\delta_N) & C_{11b} - \delta_N \frac{C12_b^2}{C11_b} & C_{13b}\left(1-\delta_N \frac{C_{12b}}{C_{11b}}\right) & 0 & 0 & 0 \\ C_{13b}(1-\delta_N) & C_{13b}\left(1-\delta_N \frac{C_{12b}}{C_{11b}}\right) & C_{33b} - \delta_N \frac{C_{13b}^2}{C_{11b}} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44b} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44b}(1-\delta_V) & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66b}(1-\delta_H) \end{pmatrix}$$

C_{ijb} - background VTI medium parameters

$\delta_N, \delta_V, \delta_H$ - fracture weaknesses *(Schoenberg and Helbig, 1997)*



Effective anisotropic media: VTI + ORT

Following Bakulin et al. (2000):

$$\epsilon_b, \delta_b, \gamma_b, \delta_N, \delta_V, \delta_H \ll 1$$

$$\bar{V}_{P0} = V_{P0_b} - \frac{1}{2} V_{P0_b} \underline{(1-\alpha)\delta_N} (1-2g)^2,$$

$$\bar{V}^{(1)}_{\text{nmo}} = V_{\text{nmo}_b} - \frac{1}{2} V_{P0_b} (1-\alpha)\delta_N (1-2g)^2,$$

$$\bar{V}^{(2)}_{\text{nmo}} = V_{\text{nmo}_b} - \frac{1}{2} V_{P0_b} (1-\alpha)(4g\delta_V + (1-4g^2)\delta_N),$$

$$\bar{\eta}^{(1)} = \varepsilon_b - \delta_b,$$

$$\bar{\eta}^{(2)} = \varepsilon_b - \delta_b + 2g(1-\alpha)(\delta_V - g\delta_N),$$

$$\bar{\eta}^{(3)} = 2g(1-\alpha)(\delta_H - g\delta_N),$$

$$g = \frac{V_{S0}^2}{V_{P0}^2}$$

Effective anisotropic media: VTI + ORT

Following the approach of Bakulin et al. (2000), fracture parameters can be estimated (not resolvable if ORT/VTI ratio in the composite is unknown):

$$(1-\alpha)\delta_N = -\frac{\delta^{(2)} - \delta^{(1)} + \eta^{(2)} - \eta^{(1)}}{2g(1-g)}$$

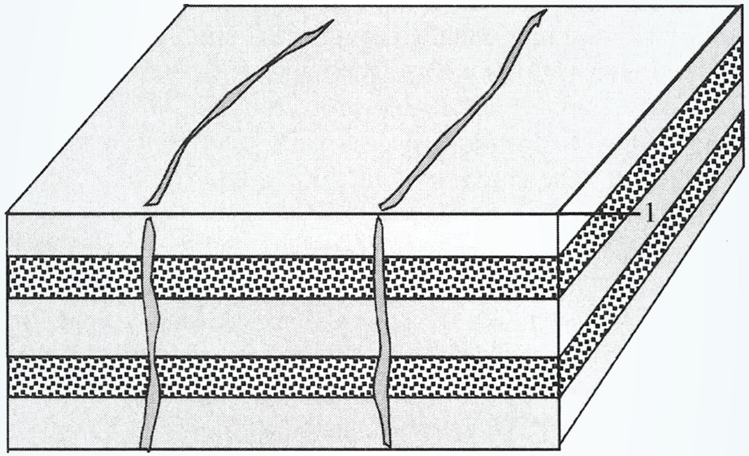
$$(1-\alpha)\delta_V = \frac{1}{2(1-g)}\left[\frac{1-2g}{g}\left(\eta^{(2)} - \eta^{(1)}\right) - \left(\delta^{(2)} - \delta^{(1)}\right)\right] \Rightarrow \frac{\delta_N}{\delta_V}$$

$$(1-\alpha)\delta_H = \frac{\eta^{(3)}}{2g} + g(1-\alpha)\delta_N$$

$$g = \frac{V_{S0}^2}{V_{P0}^2}$$

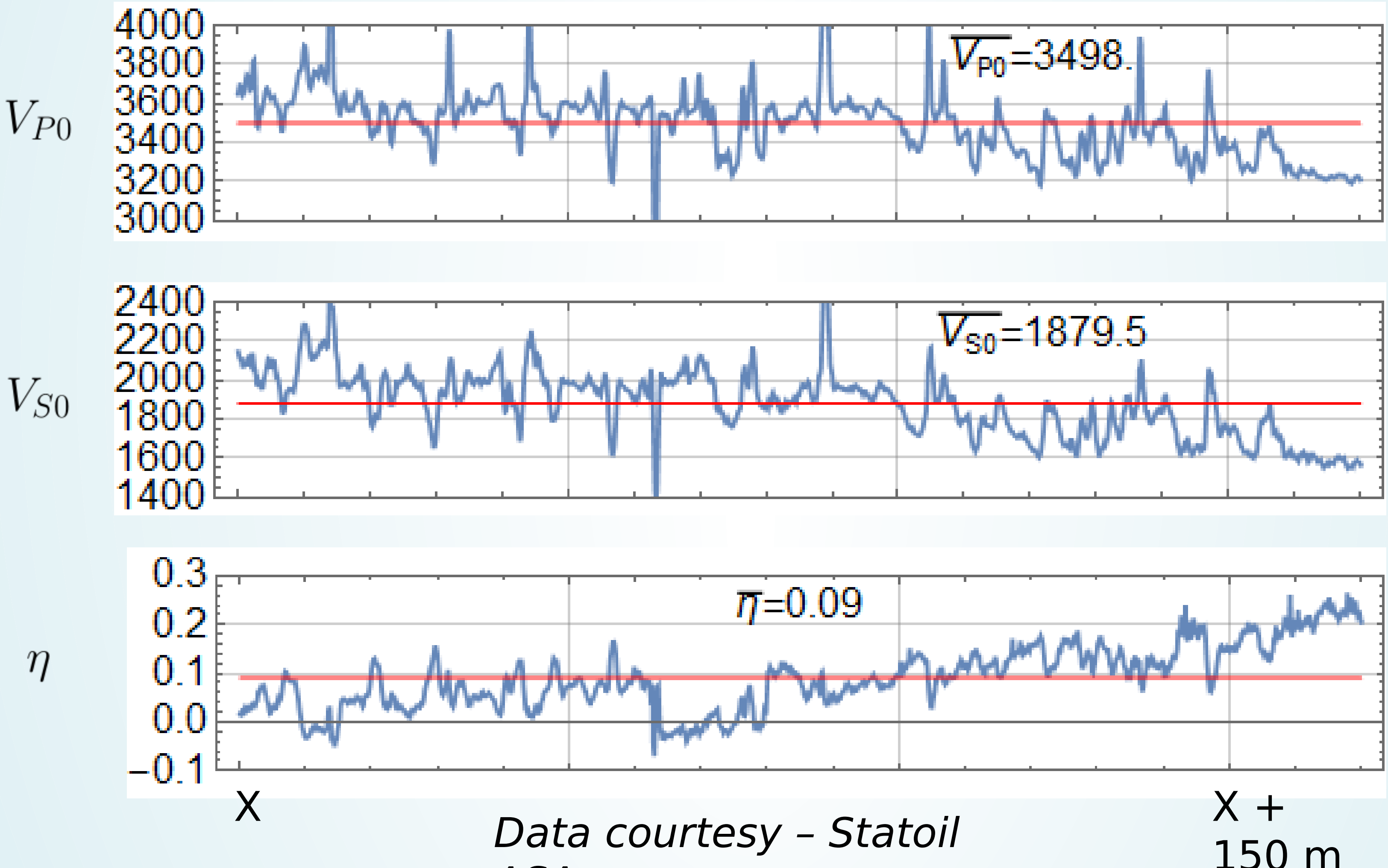
Effective anisotropic media: VTI + ORT

What if the logged interval is known to have fractures?



$$(V_{P0}, V_{S0}, \epsilon, \delta, \gamma) + (\delta_N, \delta_H, \delta_V) = (V^*_{P0}, V^*_{S0}, \epsilon^{(1)}, \epsilon^{(2)}, \delta^{(1)}, \delta^{(2)}, \delta^{(3)}, \gamma^{(1)}, \gamma^{(2)})$$

Effective anisotropic media: VTI + ORT



Data courtesy – Statoil ASA

Effective anisotropic media: VTI + ORT

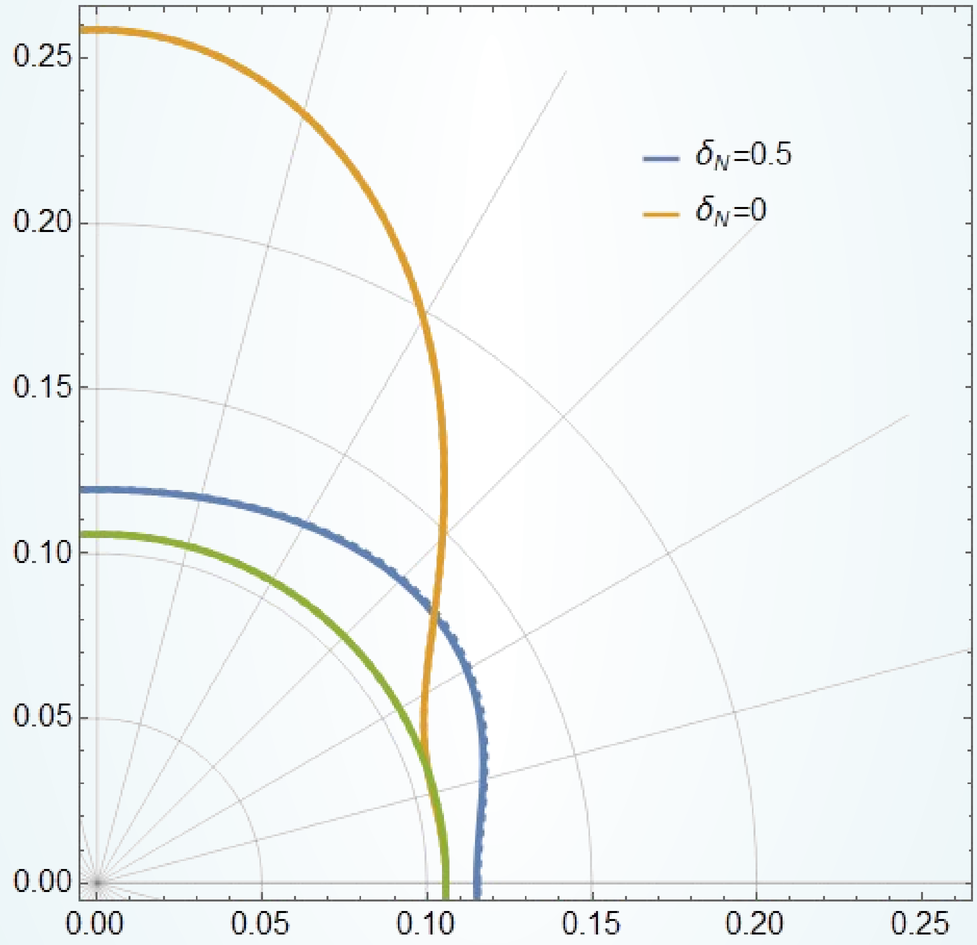
Following Bakulin et al. (2000):

$\delta_H = \delta_V = 0.2$

$\delta_N = 0$ - wet

$\delta_N = 0.5$ - dry

$\eta(\phi)$



Conclusions

- Averaged media behavior is studied for different types of constituent media:
  - ORT-az + ORT
  - VTI + ORT (vertically fractured VTI)
- Two approaches on approximation ORT-az + ORT composite with an effective ORT-az medium is shown
- Linearized expressions for anisotropy parameters for VTI + ORT composite are derived
- Demonstrated how fractures can be included in well-logging data