Seismic wavepropagation concepts applied to the interpretation of marine CSEM data

Rune Mittet

Chief scientist EMGS

Adjunct Professor NTNU

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Outline

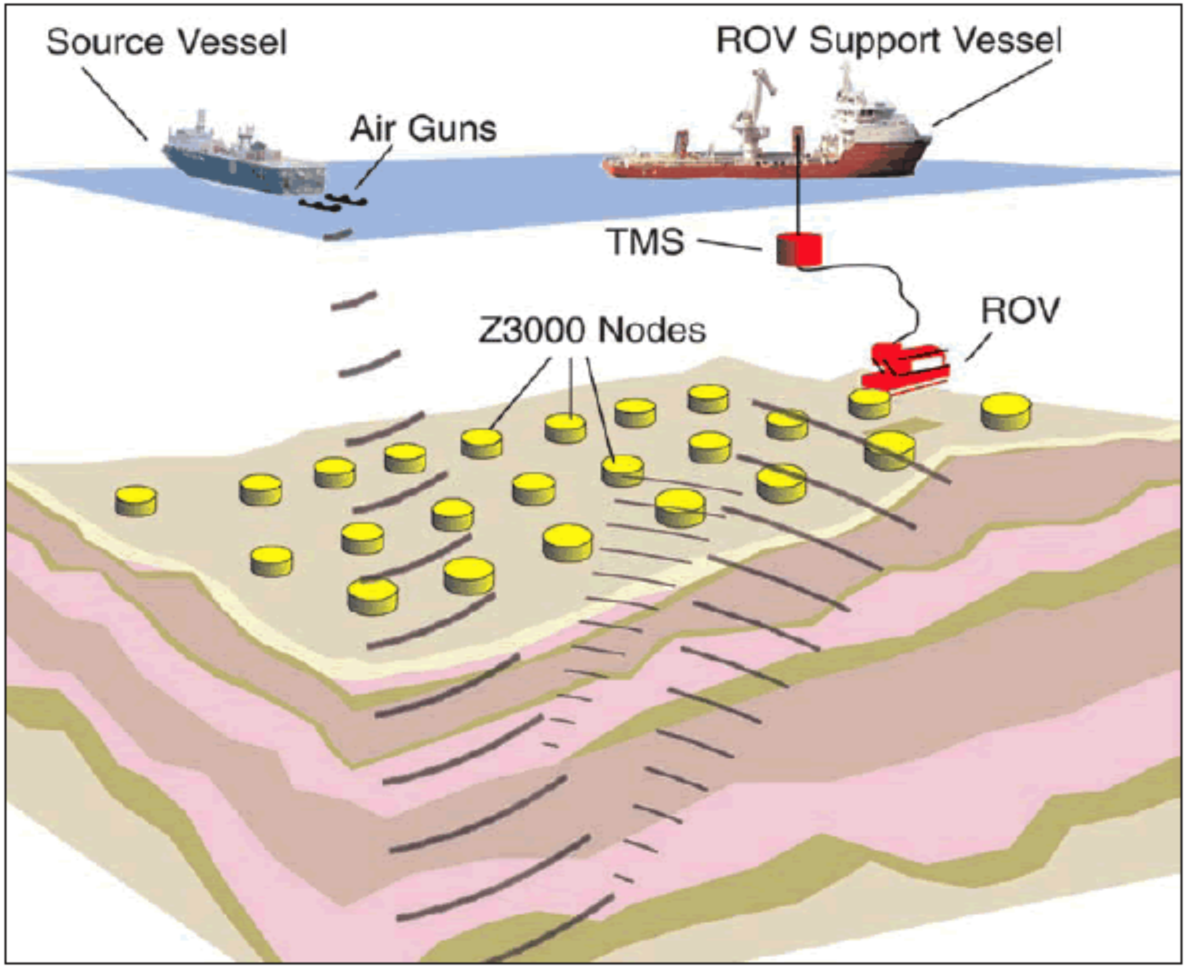
Introduction

Theory

Comparing seismic and controlled-source electromagnetic fields

Summary

Seabed seismic nodes



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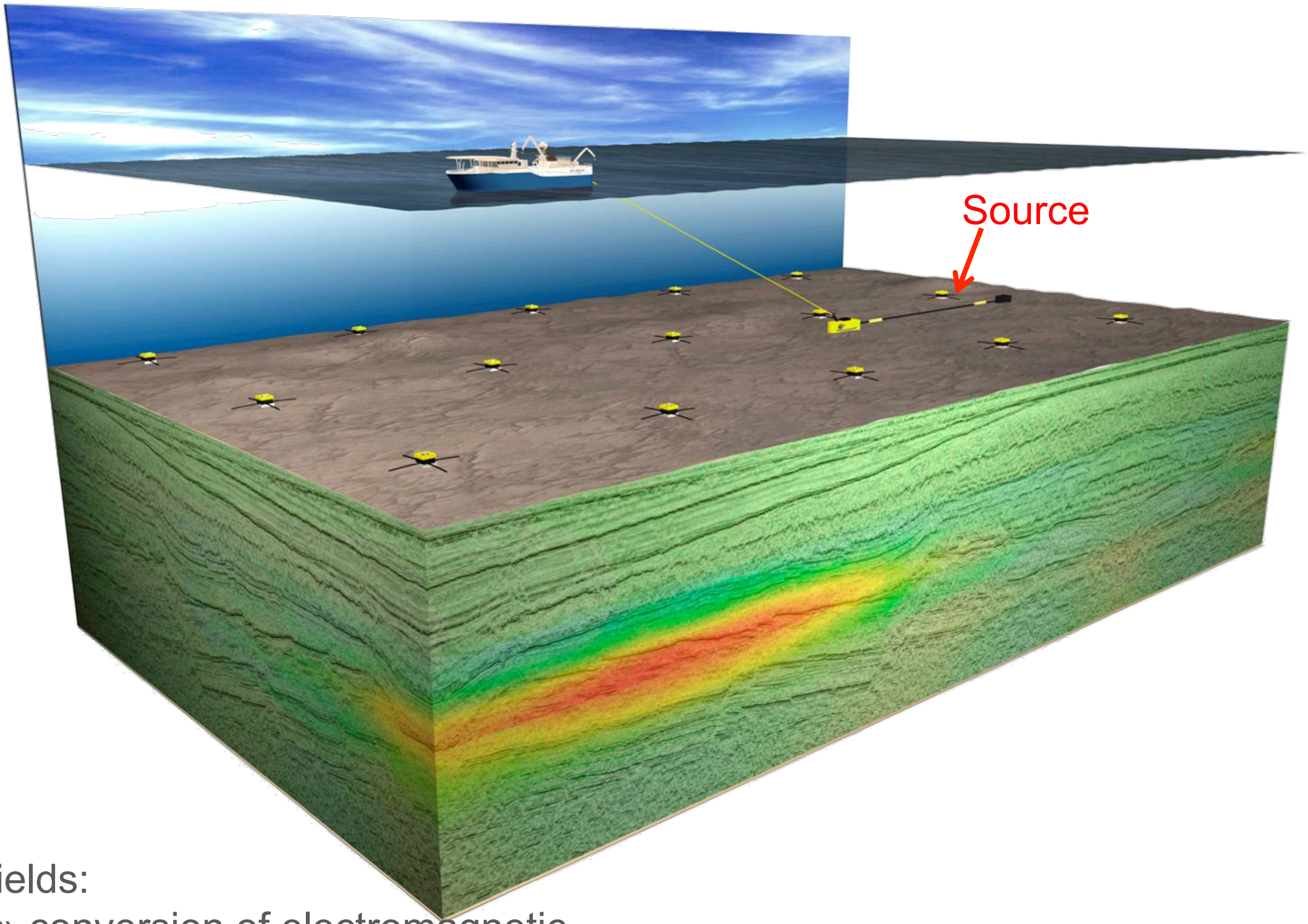
3D CSEM

Source: Electric dipole ~ surface tow or 40 m above seabed

Receivers: Voltmeters and coils at the seabed

Typical CSEM frequency range: 0.1 Hz – 5Hz



Diffusive fields:
«Effective» conversion of electromagnetic
energy to heat for typical subsurface resistivities (~1 Ωm)

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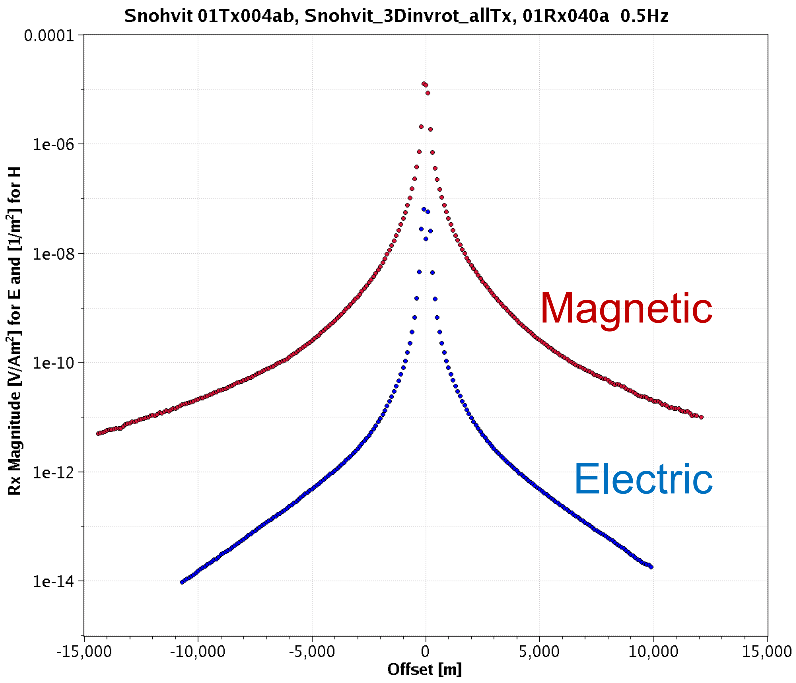
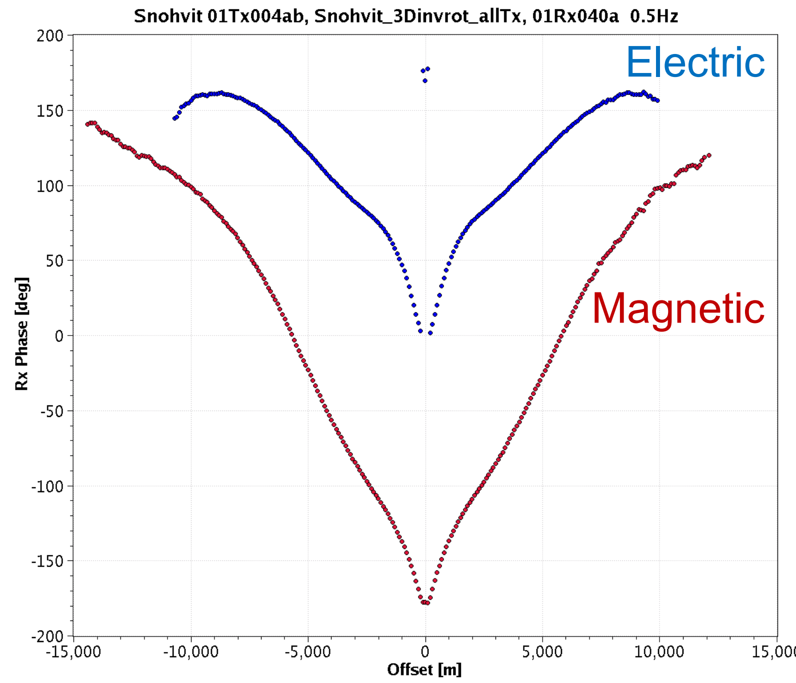
Measure horizontal electric and magnetic fields $E_x(x,\omega), E_y(x,\omega), H_x(x,\omega), H_y(x,\omega)$

$$E_x(x,\omega) = |E_x(x,\omega)|\, e^{i\phi(x,\omega)}$$

Amplitude (Log scale)



Phase



GEOPHYSICS, VOL. 48, NO. 8 (AUGUST 1983); P. 1063–1081, 3 FIGS.

Reflection coefficients
Wave propagation
Conductivity for EM (diffusive fields/waves)

Review of elastic and electromagnetic wave propagation in horizontally layered media

Bjørn Ursin*

ABSTRACT

The objective of this paper is to provide a unified treatment of elastic and electromagnetic (EM) wave propagation in horizontally layered media for which the parameters in the partial differential equations are piece-wise continuous functions of only one spatial variable. By applying a combination of Fourier, Laplace, and Bessel transforms to the partial differential equations describing the elastic or EM wave propagation I obtain a system of $2n$ linear ordinary differential equations. The $2n \times 2n$ coefficient matrix is partitioned into 4 $n \times n$ submatrices. By a proper choice of variables, the diagonal submatrices are zero and the off-diagonal submatrices are symmetric.

mission matrices for two inhomogeneous layers is done by Redheffer's star product. This composition rule has been derived for *P-SV* waves by Kennett. The inverse of the star product, apparently unknown in seismology, is also given. This is a rule which may be used to remove the effect of an inhomogeneous layer at the top or bottom of a stack of layers. Such layer stripping techniques have possible applications in general inversion schemes. It is also shown that the reflection and transmission matrices of an inhomogeneous medium can be found by solving a matrix Riccati equation.

For a stack of inhomogeneous layers bounded above by a

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Geophysical Journal International

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On the electromagnetic fields produced by marine frequency domain controlled sources

A. D. Chave

Forced oscillations of energy in a diffusive medium such as (12) and (13) describe are sometimes called diffusion waves (Mandelis 2006), and it is common in the geophysical literature to refer to CSEM fields using wave equation terminology. However, both (12) and (13) are parabolic diffusion equations rather than hyperbolic wave equations. In one dimension, a parabolic equation has only a single family of characteristic curves (lines of constant t), whereas a hyperbolic equation has two such families (lines of constant $x \pm ct$, where c is the phase velocity). Because they are not invariant under the transformation $t \rightarrow -t$, solutions to parabolic equations evolve unidirectionally forward in time simultaneously at all points away from a source. This set of traits precludes the existence of reflection (and concomitantly, refraction) at interfaces, as well as the use of ray physics. Mandelis *et al.* (2001) summarize the arguments. Consequently, terminology from and analogies to wave phenomena will be avoided in the sequel.

Seismic wavepropagation concepts are useless!

Or maybe not?

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The acoustic wave equation

$\rho \partial_t \boldsymbol{v} = -\nabla P + \boldsymbol{f}^S$ Newton's second law

$\kappa \partial_t P = -\nabla \cdot \boldsymbol{v}$ Hooke's law for acoustic medium

Pressure: P
Particle velocity: $\boldsymbol{v}$
Density: ρ
Compliance: $\kappa = 1/M$
Source force density: $\boldsymbol{f}^S$

These two equations can be combined and give a wave equation.

Assuming constant density:

$$\nabla^2 P(\boldsymbol{x},t) - \rho_0 \kappa(\boldsymbol{x}) \partial_t^2 P(\boldsymbol{x},t) = \nabla \boldsymbol{f}^S(\boldsymbol{x},t)$$

The electromagnetic (Maxwell) field equations

$\varepsilon \partial_t \boldsymbol{E} + \sigma \boldsymbol{E} = \nabla \times \boldsymbol{H} - \boldsymbol{J}^S$ Ampere's law

$\mu_0 \, \partial_t \boldsymbol{H} = -\nabla \times \boldsymbol{E}$ Faraday's law

Electric field: $\boldsymbol{E}$
Magnetic field: $\boldsymbol{H}$
Conductivity: σ
Resistivity: $\rho = 1/\sigma$
Electric permittivity ε
Magnetic permeabillity of vaccum: $\mu_0 = 4\pi \times 10^{-7}$ H/m
Source current density: $\boldsymbol{J}^S$

Induction current can be neglected for low frequency geophysical applications

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Source current density: $\boldsymbol{J}^S$

Induction current neglected for low frequency geophysical applications

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Solution by transforming PDE

Analytical transform

$$\sigma(\boldsymbol{x})\boldsymbol{E}(\boldsymbol{x},t)=\nabla\times\boldsymbol{H}(\boldsymbol{x},t)-\boldsymbol{J}^{s}(\boldsymbol{x},t)$$

$$\mu_{0}\,\partial_{t}\boldsymbol{H}(\boldsymbol{x},t)=-\nabla\times\boldsymbol{E}(\boldsymbol{x},t)$$

$$\varepsilon'(\boldsymbol{x})\,\partial_{t'}\boldsymbol{E}'(\boldsymbol{x},t')=\nabla\times\boldsymbol{H}'(\boldsymbol{x},t')-\boldsymbol{J}'^{s}(\boldsymbol{x},t')$$

$$\mu_{0}\,\partial_{t'}\boldsymbol{H}'(\boldsymbol{x},t')=-\nabla\times\boldsymbol{E}'(\boldsymbol{x},t')$$

$$\varepsilon'(\boldsymbol{x})=\sigma(\boldsymbol{x})/2\omega_{0} \qquad \omega_{0}>0$$

Fields and parameters for fictitious (transformed) time domain is primed.

Transform back to real time domain given by:

$$\boldsymbol{E}(\boldsymbol{x},t)=\int_{0}^{T}dt'\,W(t,t',\omega_{0})\,\boldsymbol{E}'(\boldsymbol{x},t')\,H(t)$$

$$W(t,t',\omega_{0})=-\frac{1}{2\omega_{0}}\,\partial_{t'}\left(\frac{1}{2}\sqrt{\frac{2\omega_{0}}{\pi}}\,\frac{t'}{t^{3/2}}\,e^{-2\omega_{0}t'^{2}/4t}\right)$$

Solution by transforming PDE

$$\varepsilon'(\boldsymbol{x})\partial_{t'} \boldsymbol{E}'(\boldsymbol{x},t') = \nabla\times\boldsymbol{H}'(\boldsymbol{x},t') - \boldsymbol{J}'^{s}(\boldsymbol{x},t')$$

$$\varepsilon'(\boldsymbol{x}) = \sigma(\boldsymbol{x})/2\omega_0$$

$$\mu_0 \partial_{t'} \boldsymbol{H}'(\boldsymbol{x},t') = -\nabla\times\boldsymbol{E}'(\boldsymbol{x},t')$$

$$c(\boldsymbol{x}) = \sqrt{1/\mu_0 \varepsilon'(\boldsymbol{x})}$$

Propagation velocity:

$$c(\boldsymbol{x}) = \sqrt{2\omega_0 \rho(\boldsymbol{x})/\mu_0}$$

Transform from fictitous time to real time.

$$\boldsymbol{E}(\boldsymbol{x},t) = \int_0^T dt' \, W(t,t',\omega_0)\boldsymbol{E}'(\boldsymbol{x},t')H(t)$$

Transform is simple going from fictitous time to «real» frequency.

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \int_0^T dt' \, \boldsymbol{E}'(\boldsymbol{x},t') \, e^{-\sqrt{\omega\omega_0}\, t'} e^{i\sqrt{\omega\omega_0}\, t'}$$

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A general correspondence principle for time-domain electromagnetic wave and diffusion fields

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To obtain the time-domain expressions for each $\mathbf{\Gamma}$, the Schouten–Van der Pol theorem in the theory of Laplace transformation is applied. This theorem relates time-domain results that are associated with the replacement of the Laplace-transform parameter s by a function of s, subject to some restrictions. For the present case, the result for the replacement of s by $(\alpha s)^{1/2}$ is needed. Using eqs (A6), (A9) and (A10) from Appendix A, it is found that

$$\mathbf{\Gamma}^{E,H|J,K}(\mathbf{r},\mathbf{r}',t) = \left[\int_{\tau=0}^{\infty} W^{E,H|J,K}(t,\tau,\alpha)\,\mathscr{G}^{E,H|J,K}(\mathbf{r},\mathbf{r}',\tau)\,d\tau\right]H(t), \tag{24}$$

where the intervening kernel functions $W^{E,H|J,K}$ are given by

$$W^{E,J} = \frac{1}{2}\left(\frac{1}{\alpha\pi}\right)^{1/2}\frac{1}{t^{3/2}}\left(\frac{\alpha\tau^2}{2t}-1\right)\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{25}$$

$$W^{H,J} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2}\frac{\tau}{t^{3/2}}\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{26}$$

$$W^{E,K} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2}\frac{\tau}{t^{3/2}}\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{27}$$

$$W^{H,K} = \left(\frac{\alpha}{\pi t}\right)^{1/2}\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{28}$$

Acoustic and electromagnetic wave equations

The acoustic wave equation

$$\nabla^2 P(\boldsymbol{x},t) - \frac{1}{c^2(\boldsymbol{x})}\partial_t^2 P(\boldsymbol{x},t) = \nabla \boldsymbol{f}^S(\boldsymbol{x},t)$$

$$c(\boldsymbol{x}) = \sqrt{\frac{1}{\rho_0\,\kappa(\boldsymbol{x})}}$$

Acoustic velocity

The electromagnetic wave equation

$$\nabla^2 \boldsymbol{E}'(\boldsymbol{x},t') - \nabla(\nabla\cdot\boldsymbol{E}'(\boldsymbol{x},t')) - \frac{1}{c^2(\boldsymbol{x})}\partial_{t'}^2 \boldsymbol{E}'(\boldsymbol{x},t') = \mu_0\,\partial_{t'} \boldsymbol{J}^S(\boldsymbol{x},t')$$

$$c(\boldsymbol{x}) = \sqrt{\frac{1}{\mu_0\,\varepsilon'(\boldsymbol{x})}}$$

Electromagnetic velocity

$$\boxed{c(\boldsymbol{x}) = \sqrt{\frac{2\omega_0\,\rho(\boldsymbol{x})}{\mu_0}}}$$

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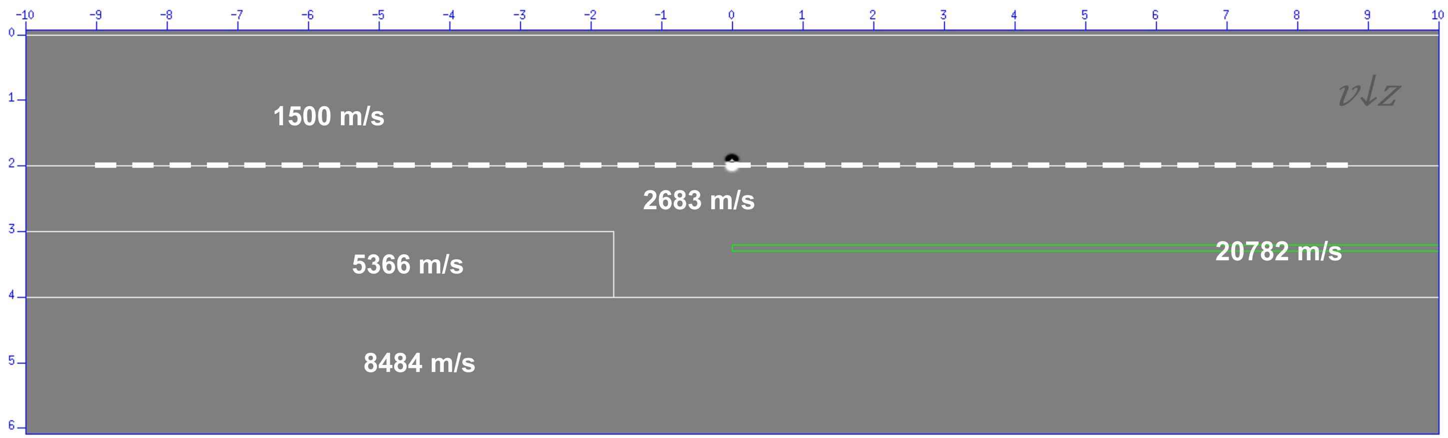
3D acoustic simulation

Waterdepth 2 km
«High velocity» subsurface (2 – 6 km)
Source 40 m above seabed (1.96 km)
Recording at seabed (2 km)



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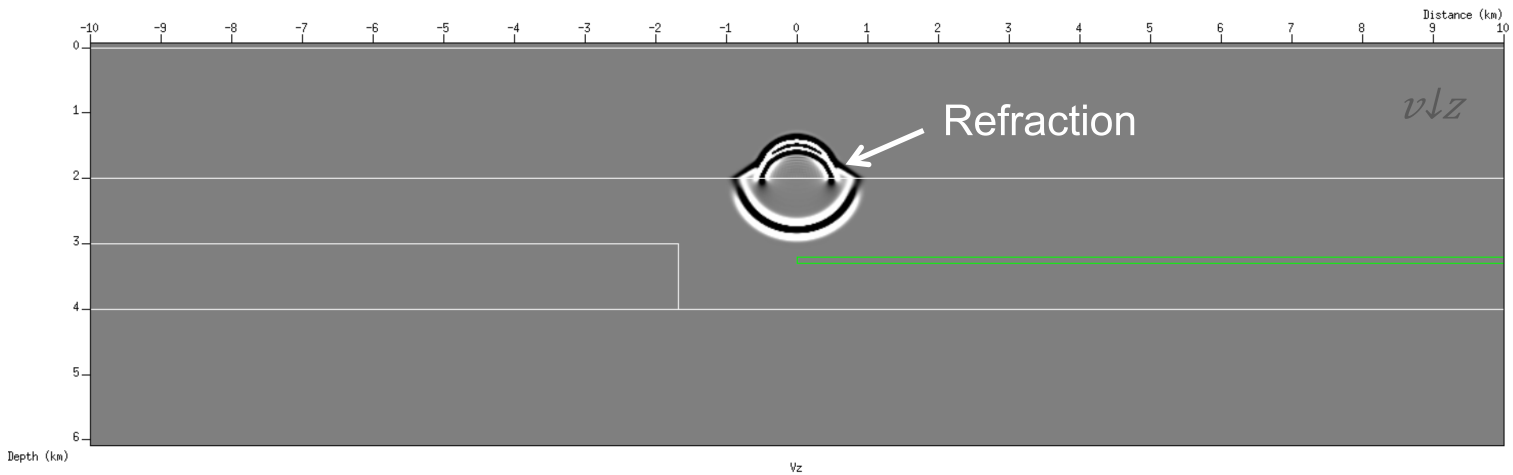
Vertical component of particle velocity



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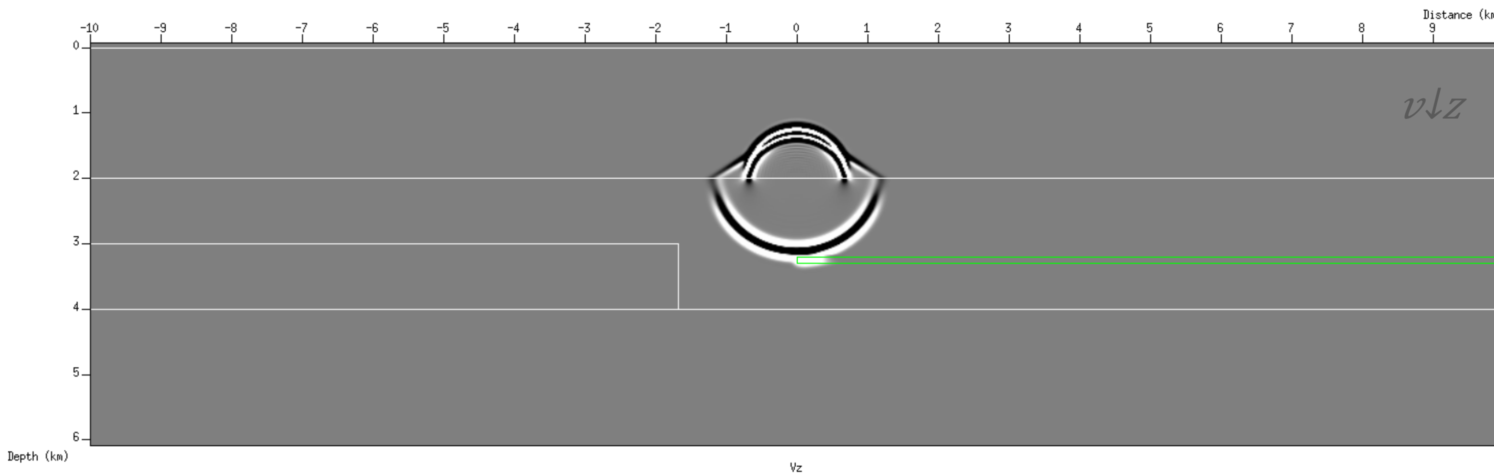
Distance (km)
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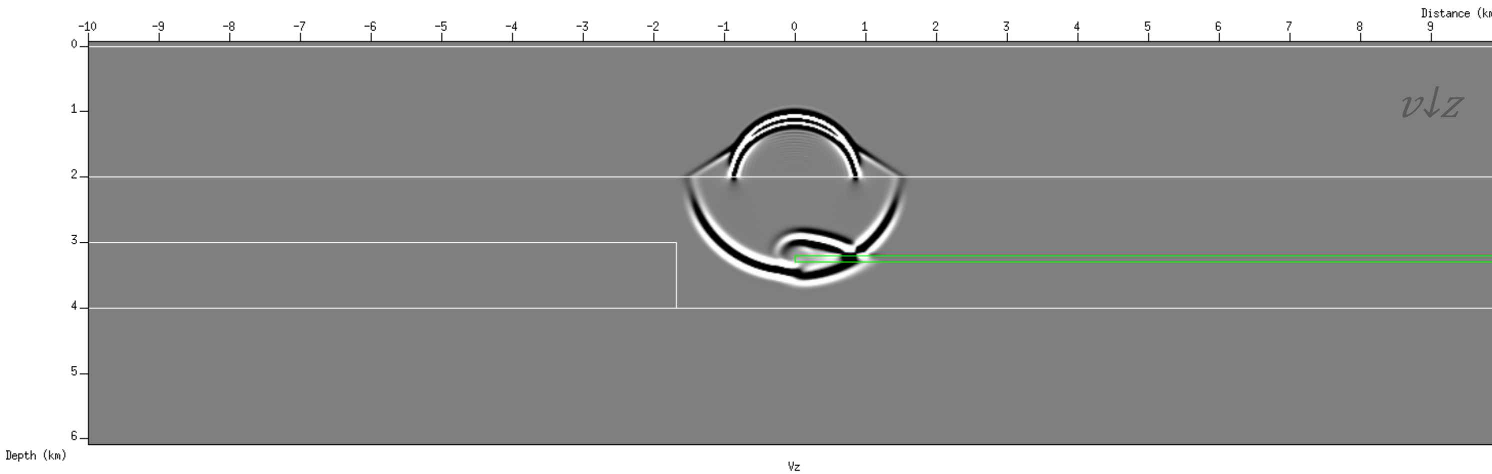
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
0
1
2
3
4
5
6
Depth (km)
Vz
$v\downarrow z$


Distance (km)
-10
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-8
-7
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-3
-2
-1
0
1
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8
9
10
0
1
2
3
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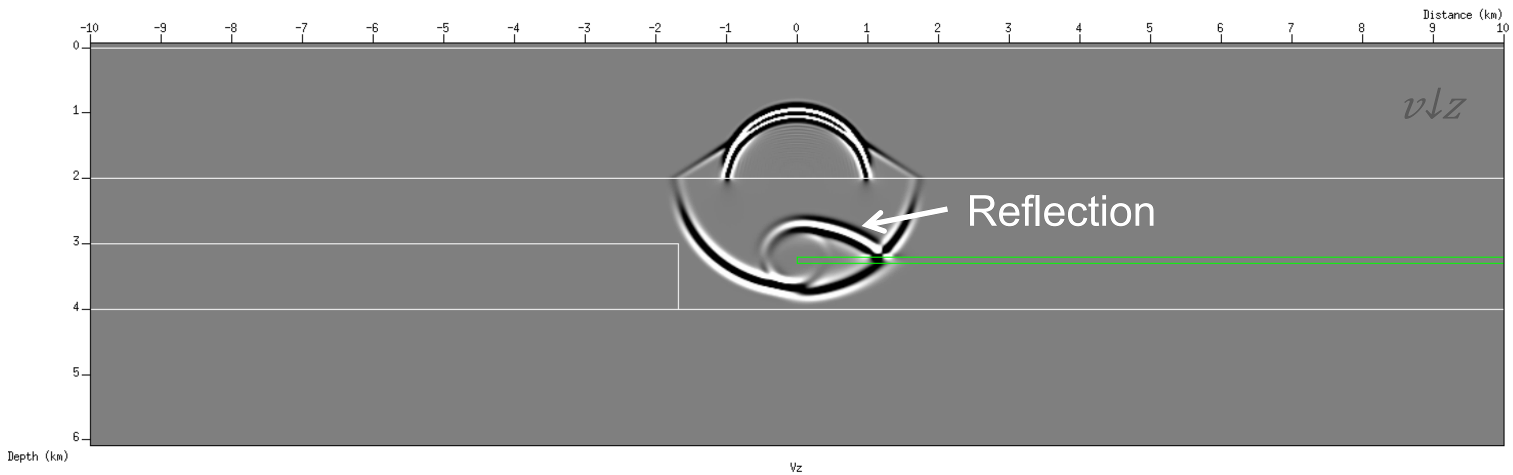
5
6
Depth (km)
v↓z
Vz


Distance (km)
-10
-9
-8
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-5
-4
-3
-2
-1
0
1
2
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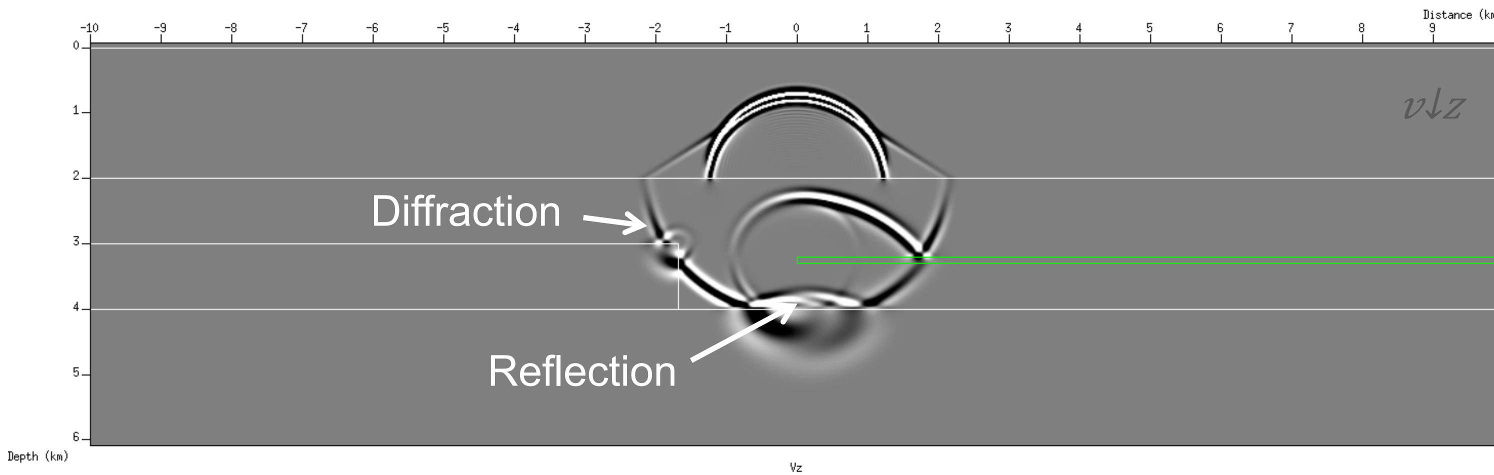
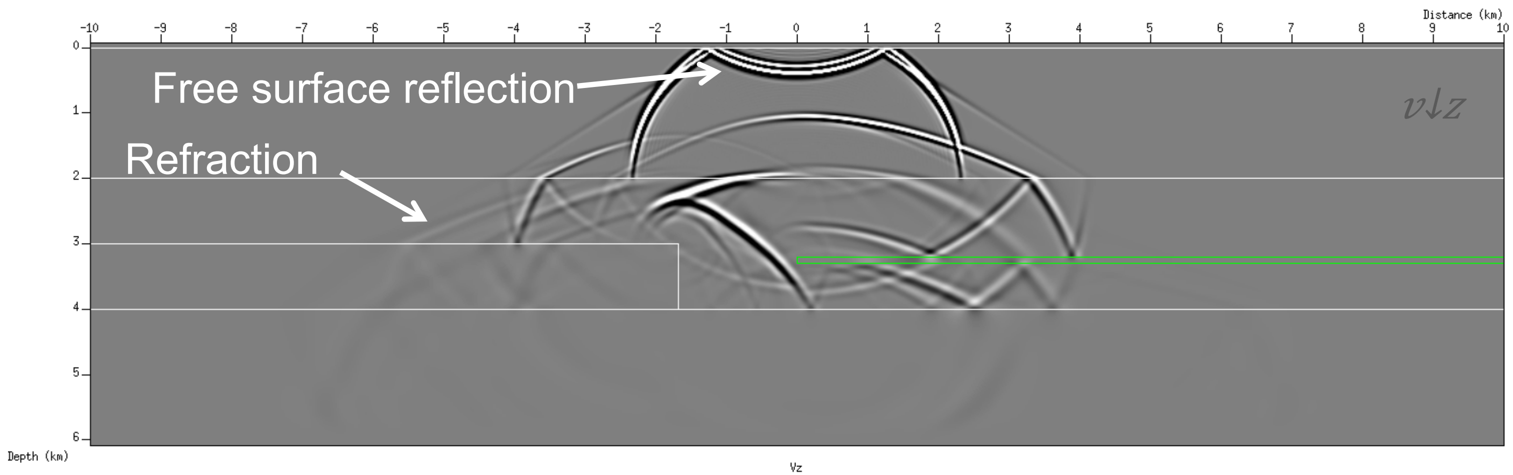
4
5
6
7
8
9
10
0
1
2
3
4
5
6
Depth (km)
Reflection
Vz

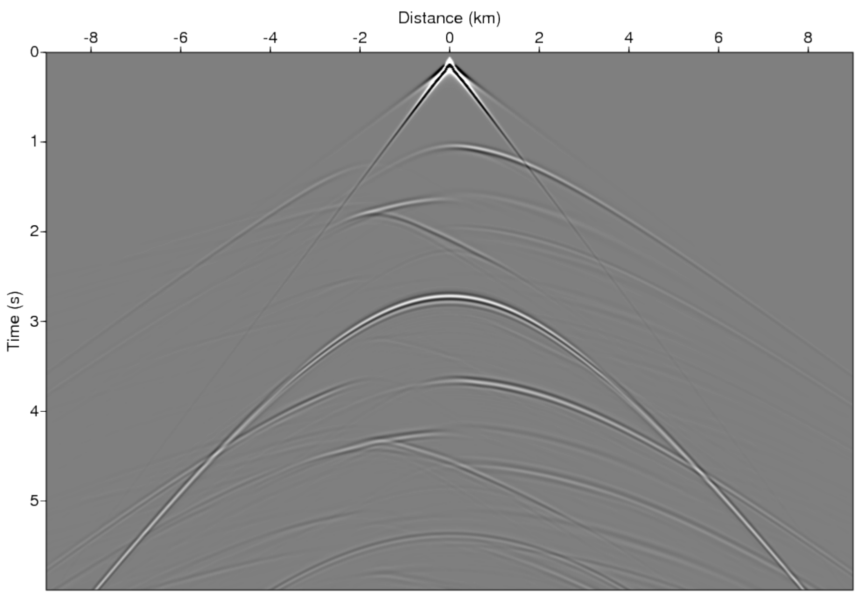


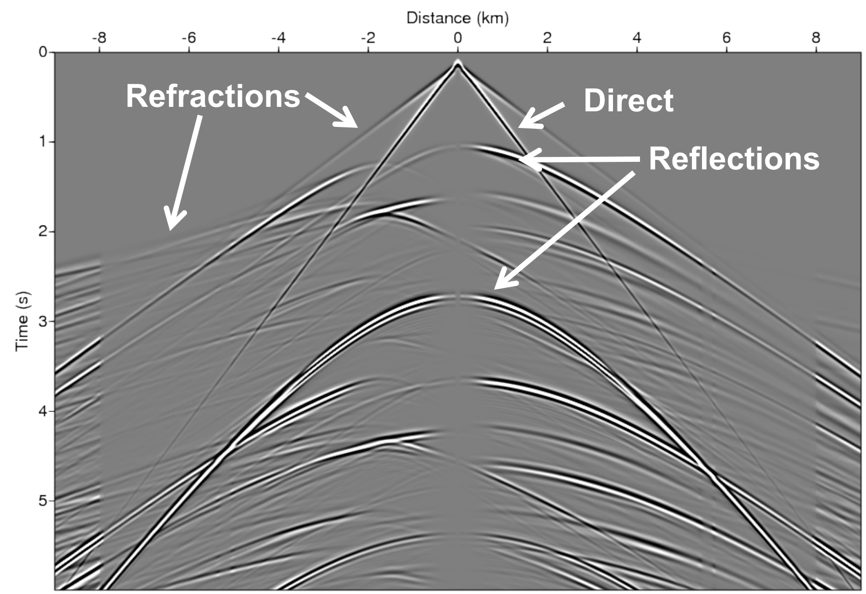
Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
v_z
Diffraction
Reflection
Vz

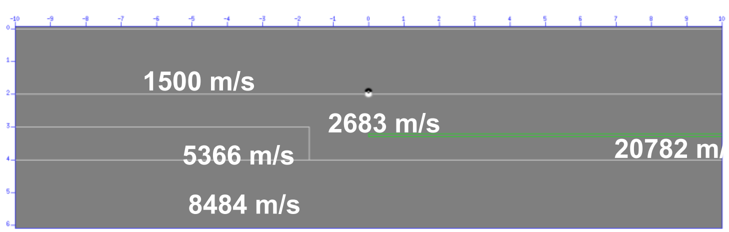


Air-water reflection at seabed after ~ 2.7 s (4000 m / 1500 m/s)

Traces normalized to unity

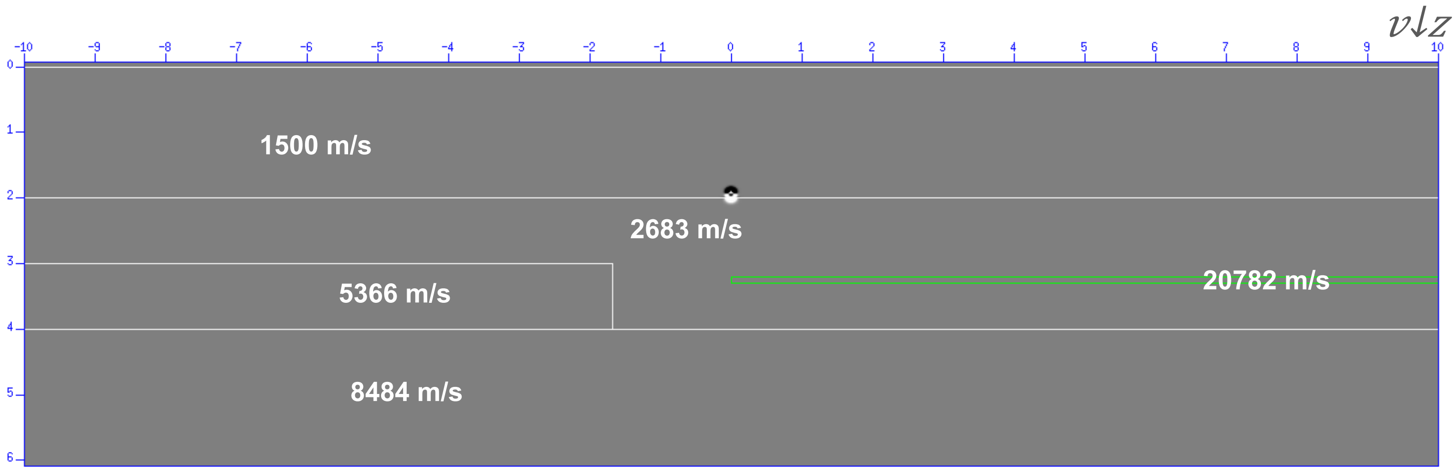


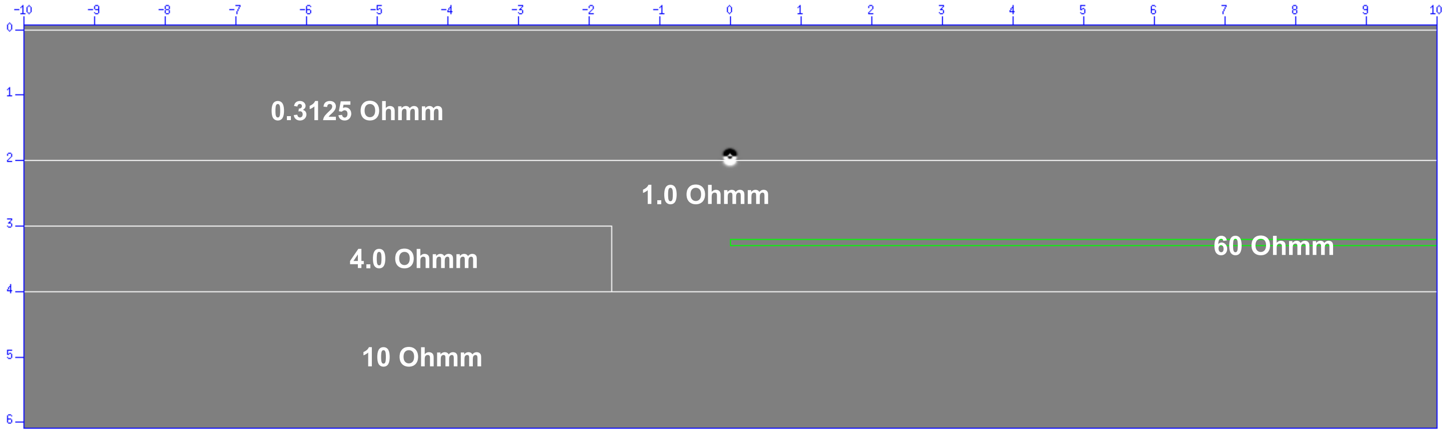




Also diffractions and multiples

3D acoustic simulation and 3D EM simulation





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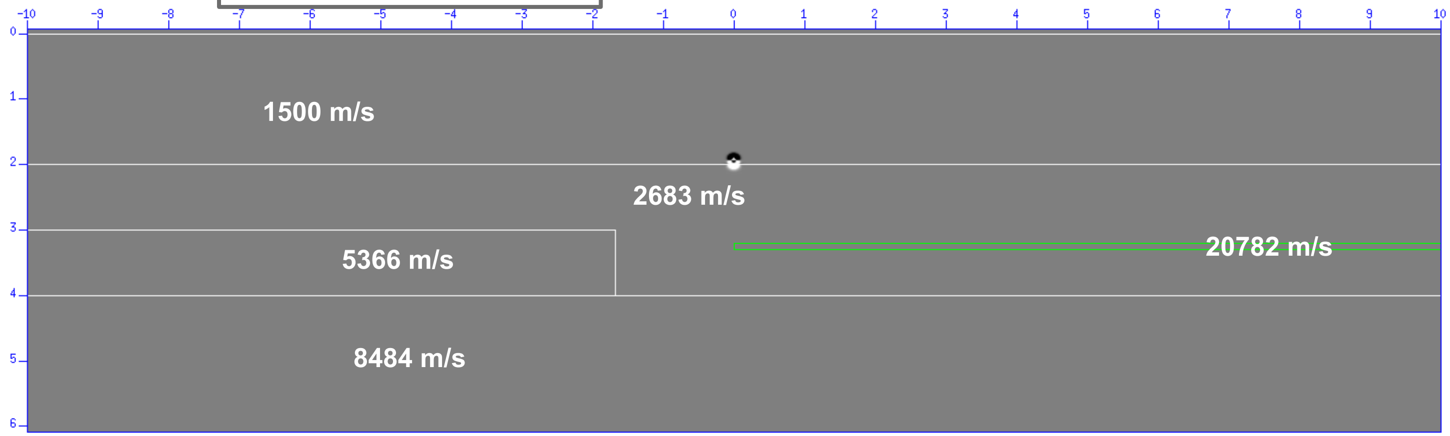
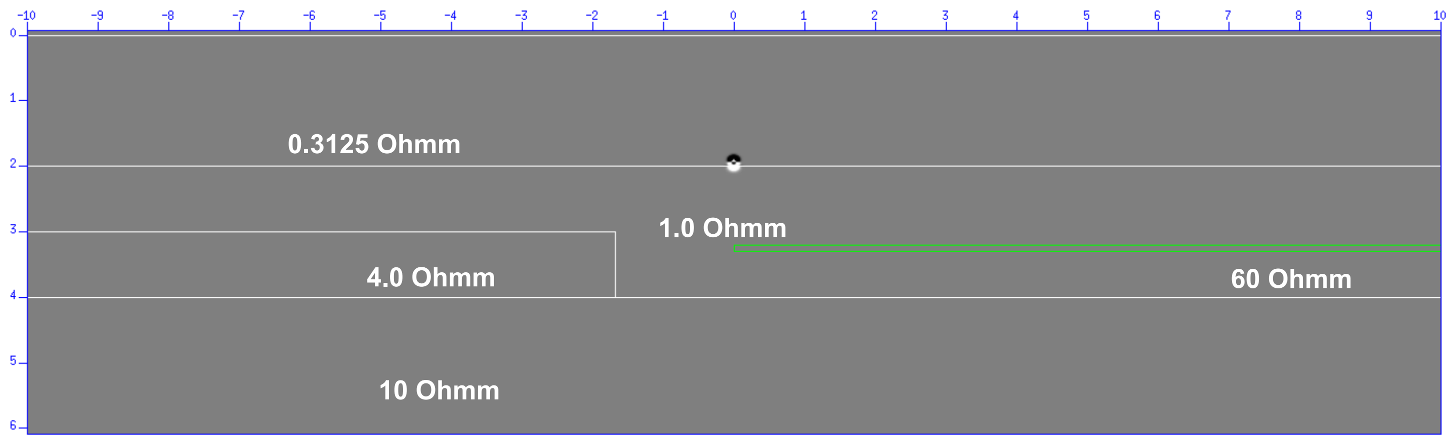
3D acoustic simulation and 3D EM simulation

$$c(\boldsymbol{x}) = \sqrt{2\omega_0 \rho(\boldsymbol{x})/\mu_0}$$

$$\omega_0 = 2\pi f_0 \qquad f_0 = 0.7198\ Hz$$

v_z





Resistivity/conductivity model maps into velocity model

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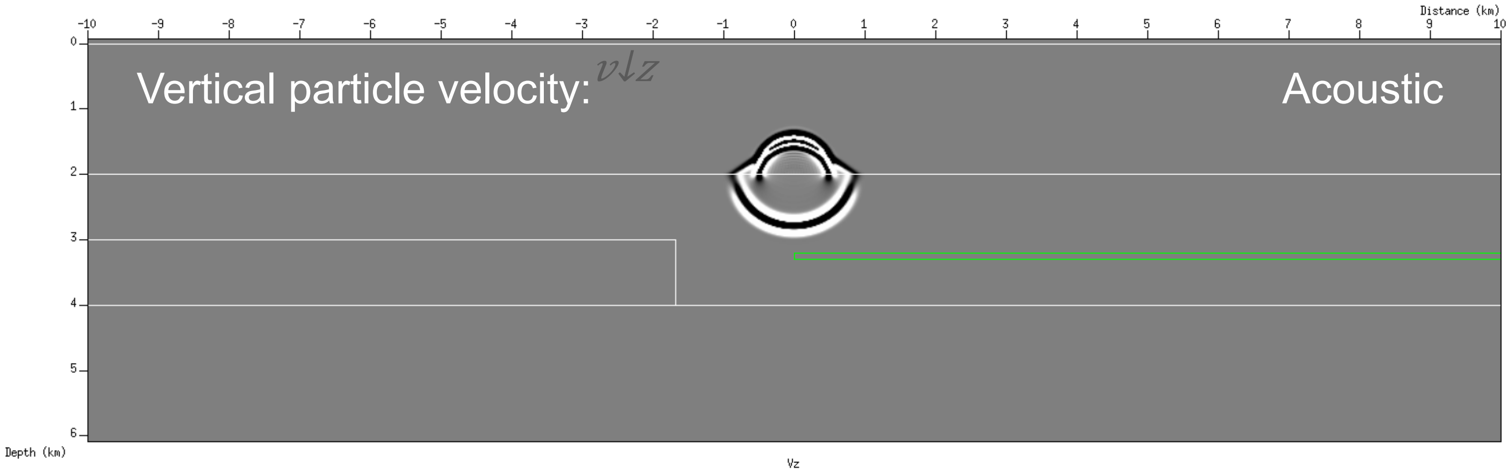
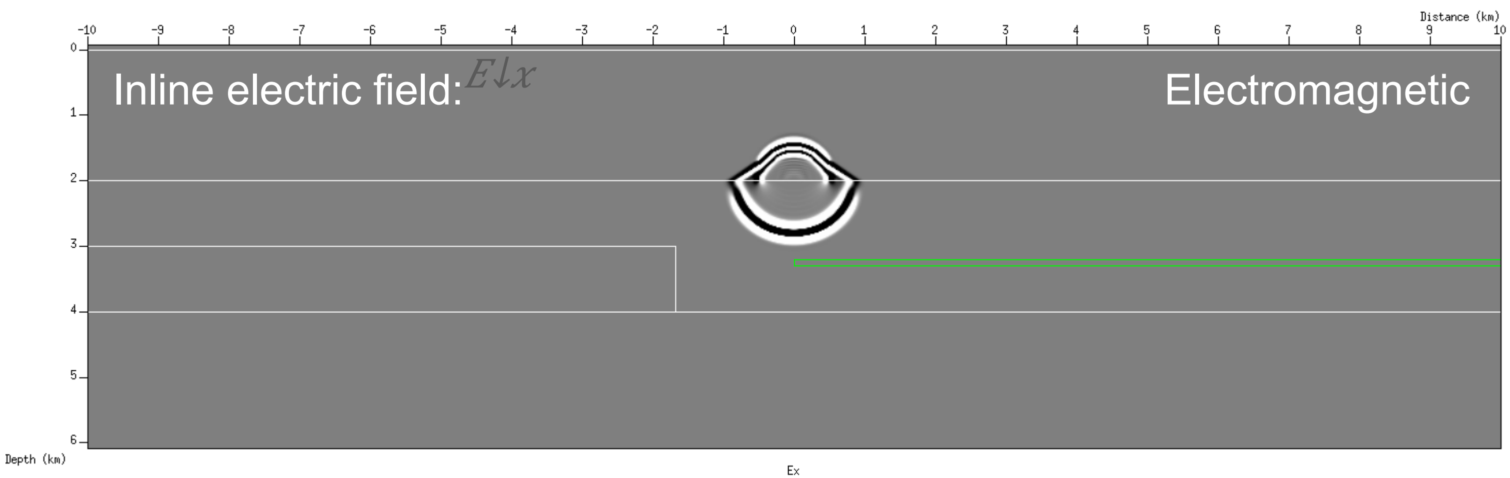
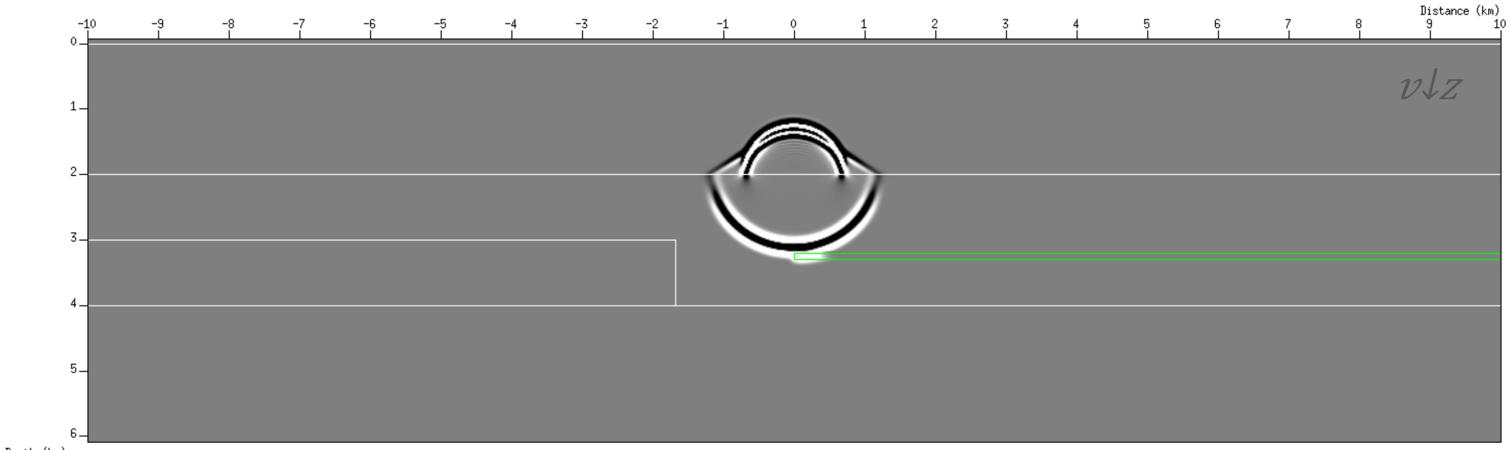
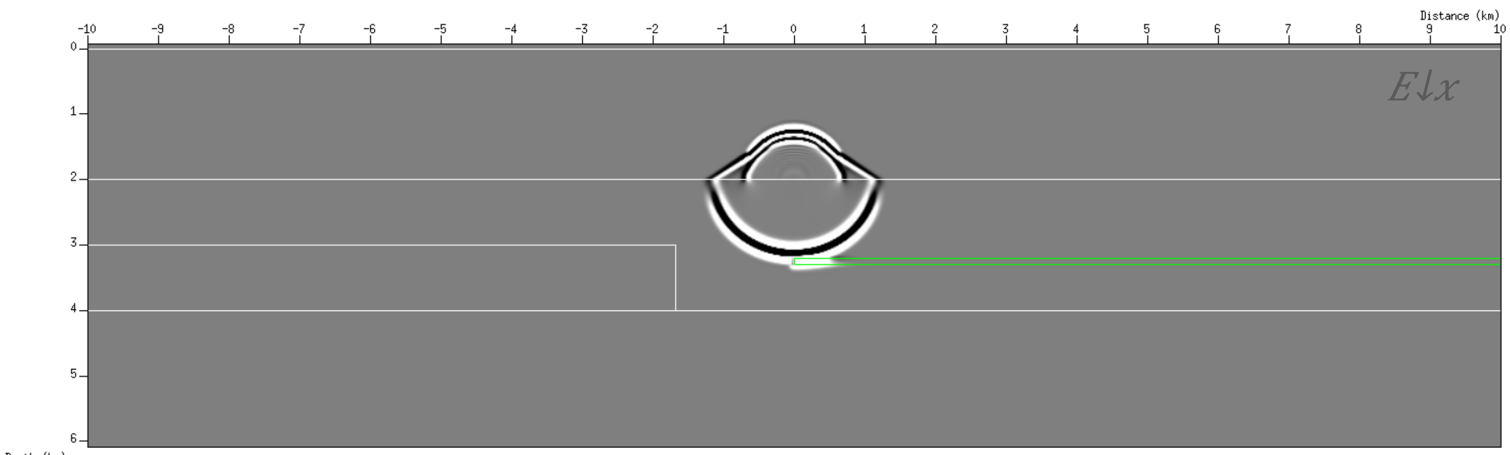

Vertical particle velocity: v_z
Acoustic
Distance (km)
Depth (km)
Vz


Inline electric field: E_x
Electromagnetic
Distance (km)
Depth (km)
Ex

emgs

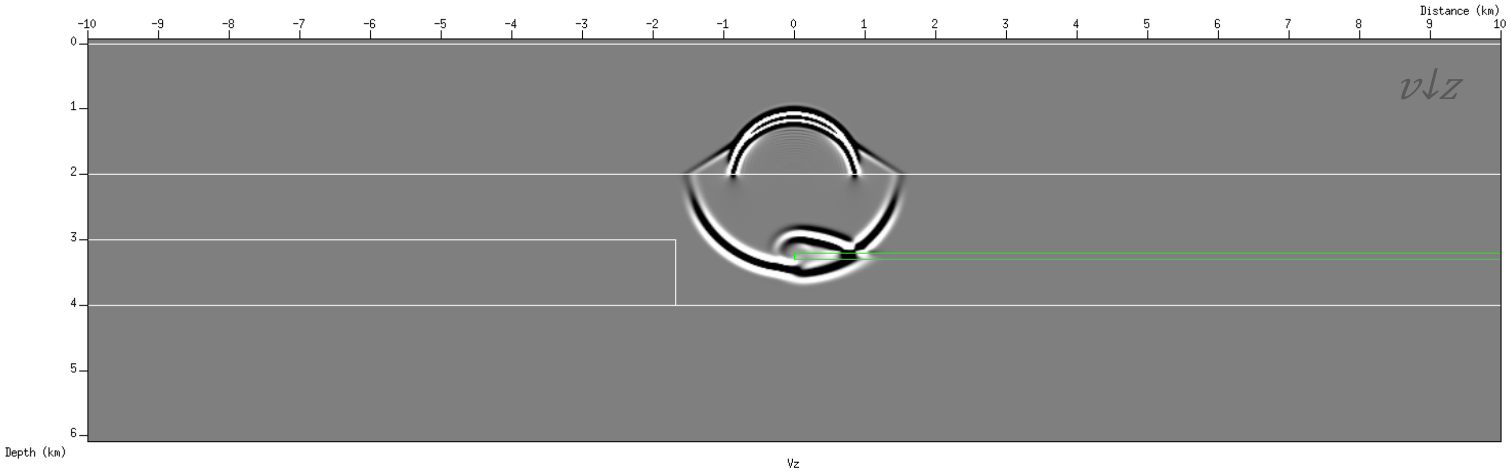


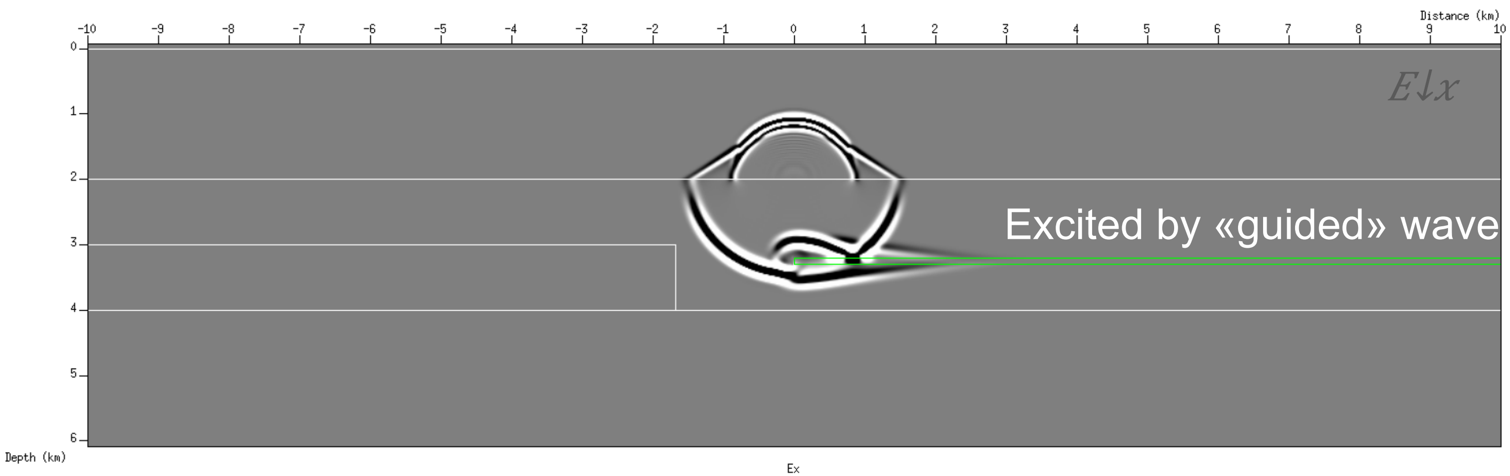
Vz



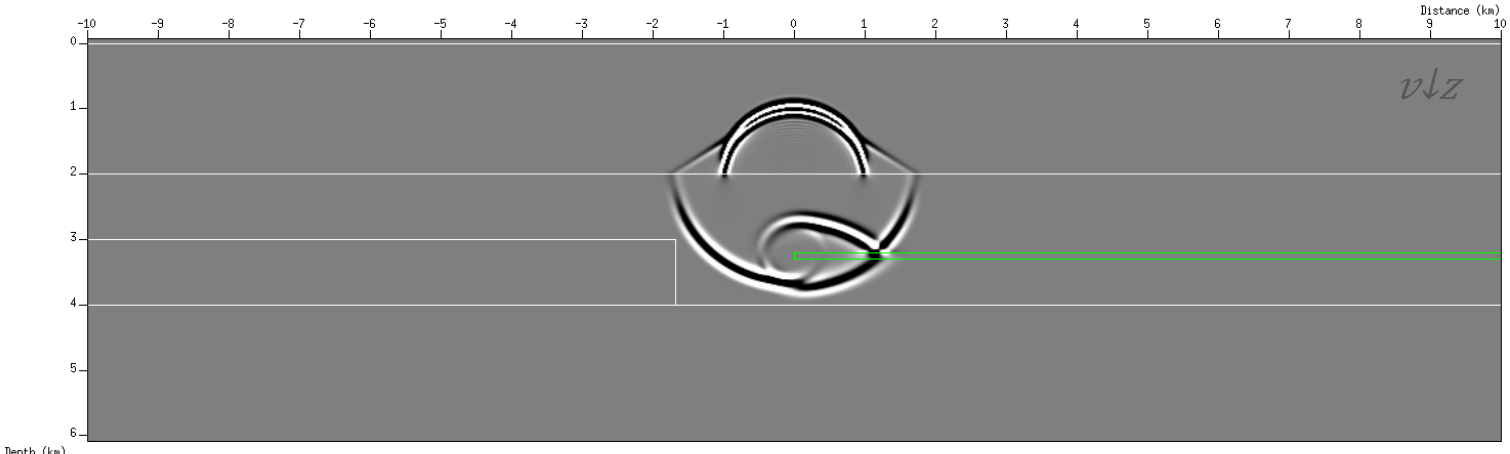
Ex

emgs


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
v_z
Vz


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
E_x
Excited by «guided» wave
Ex

emgs



Distance (km)
-10
-9
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-7
-6
-5
-4
-3
-2
-1
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1
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7
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9
10
0
1
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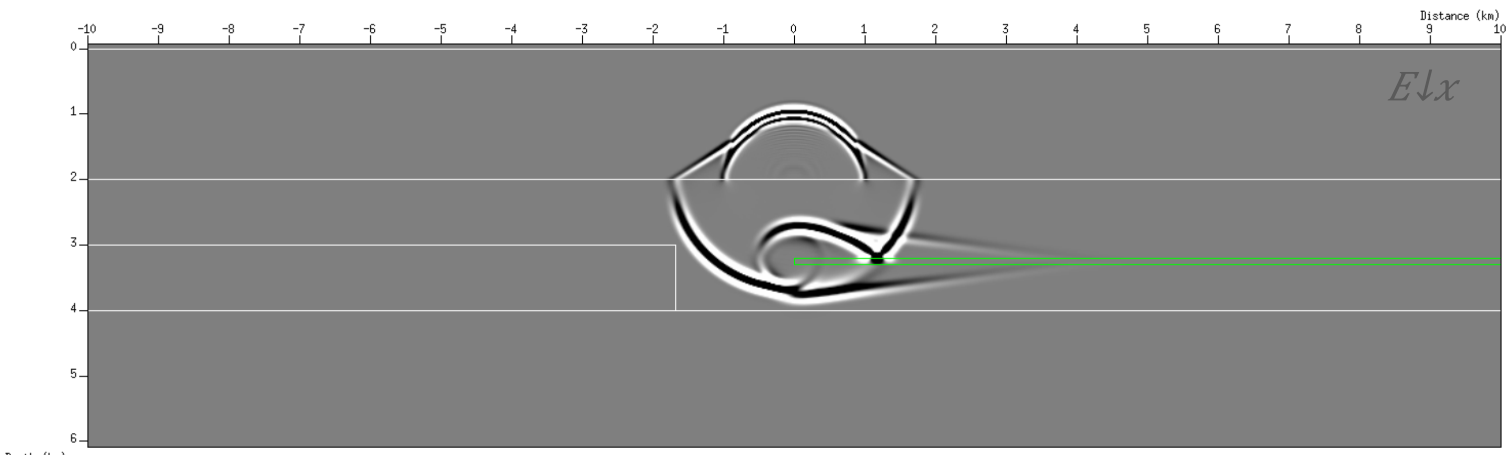
3
4
5
6
Depth (km)
v_z

Vz



Distance (km)
-10
-9
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-5
-4
-3
-2
-1
0
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9
10
0
1
2
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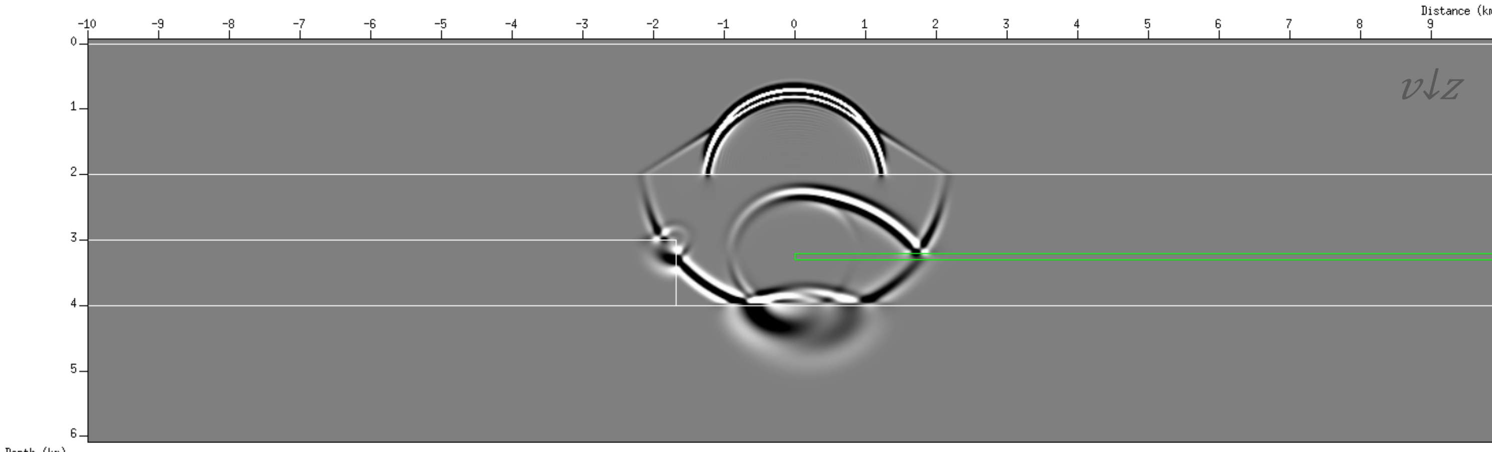
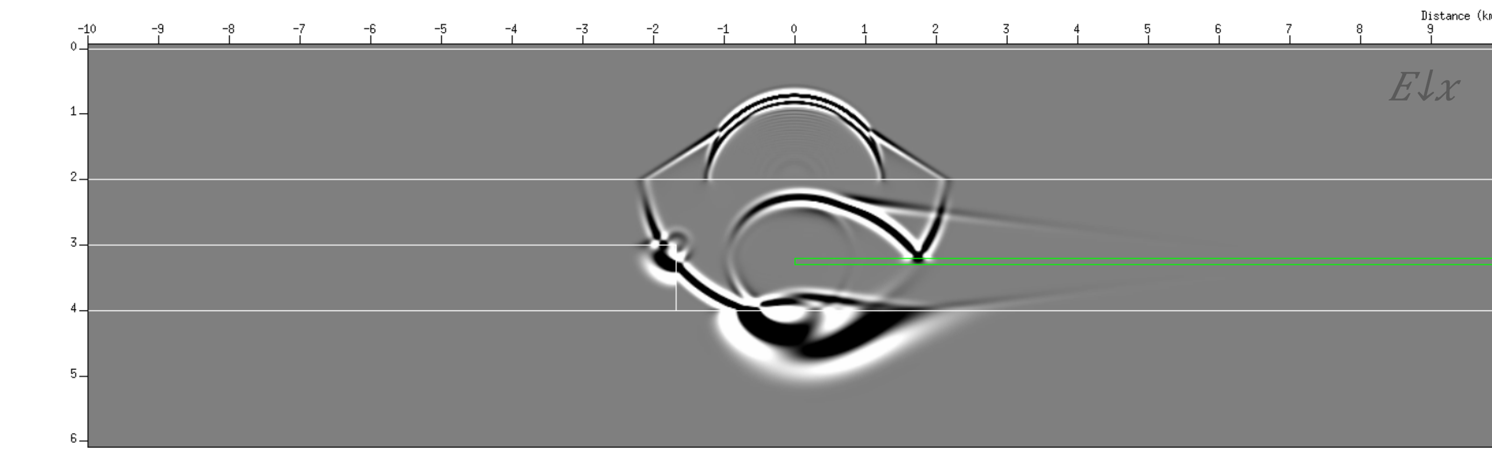
5
6
Depth (km)
E_x

Ex

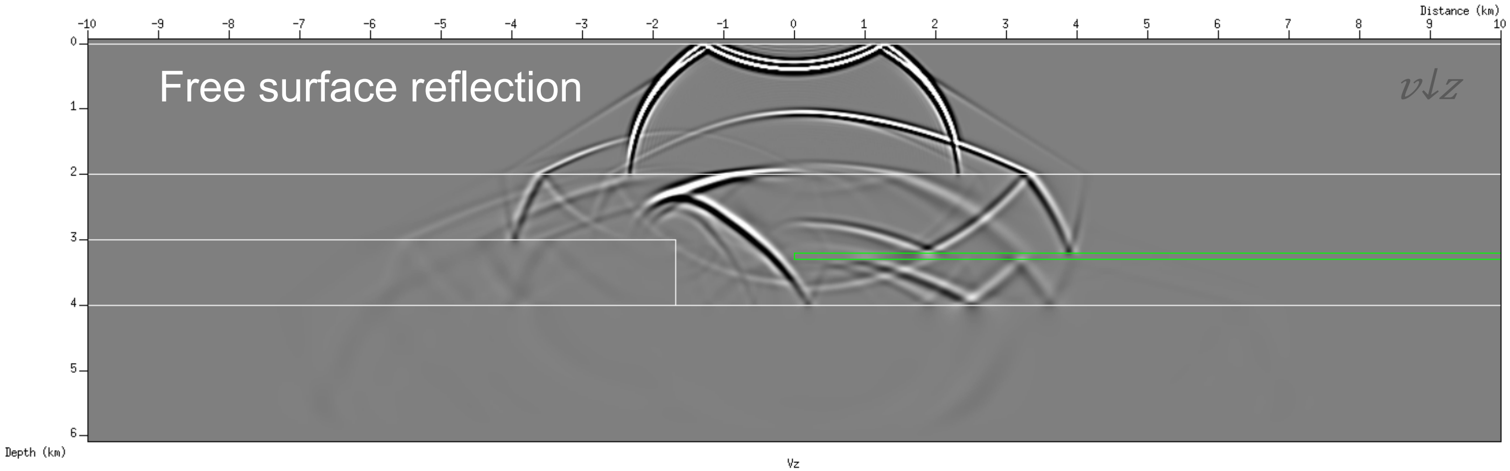
emgs


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
v↓z

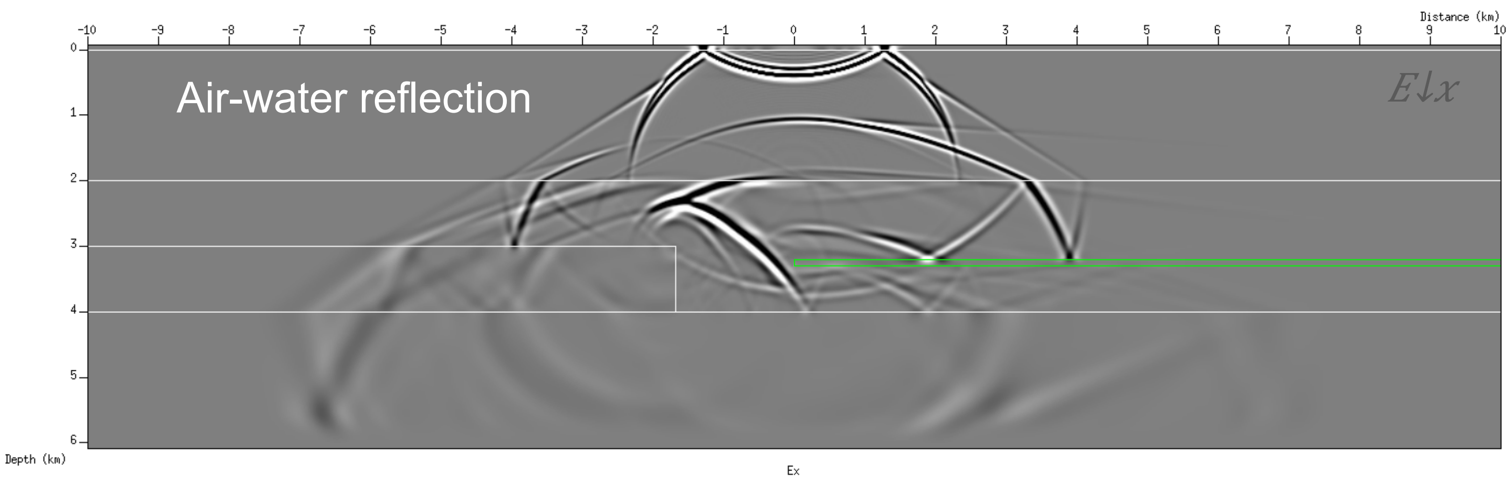
Vz


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
E↓x

Ex

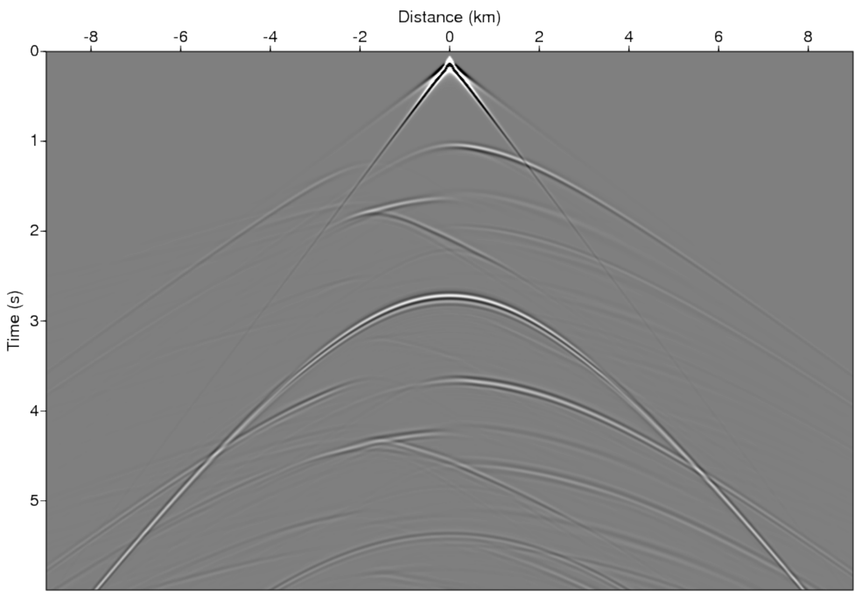


Free surface reflection
v↓z
Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
Vz

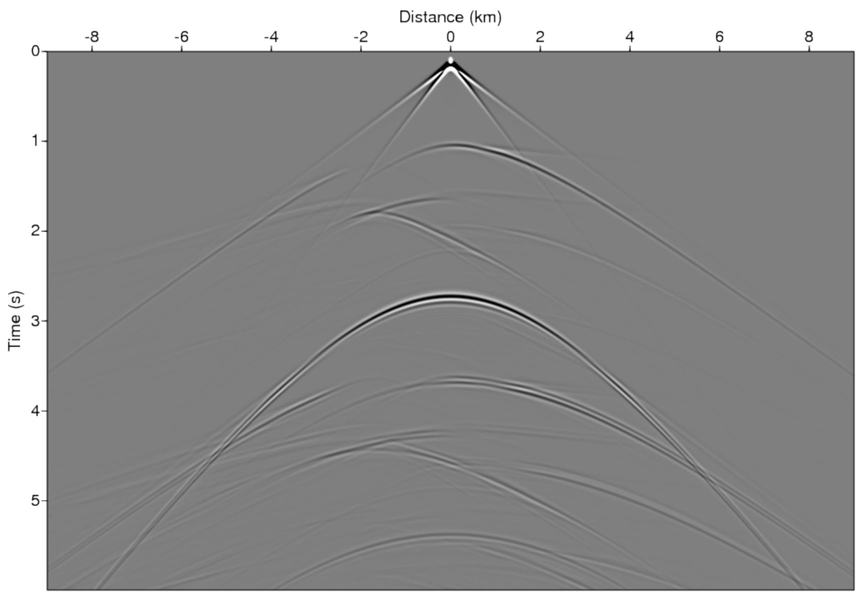


Air-water reflection
E↓x
Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
Ex

Acoustic



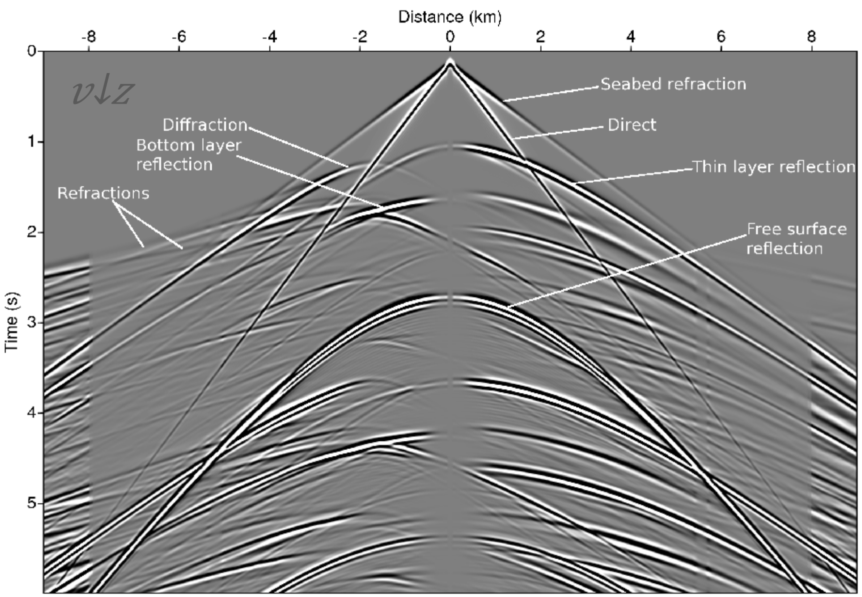
Electromagnetic



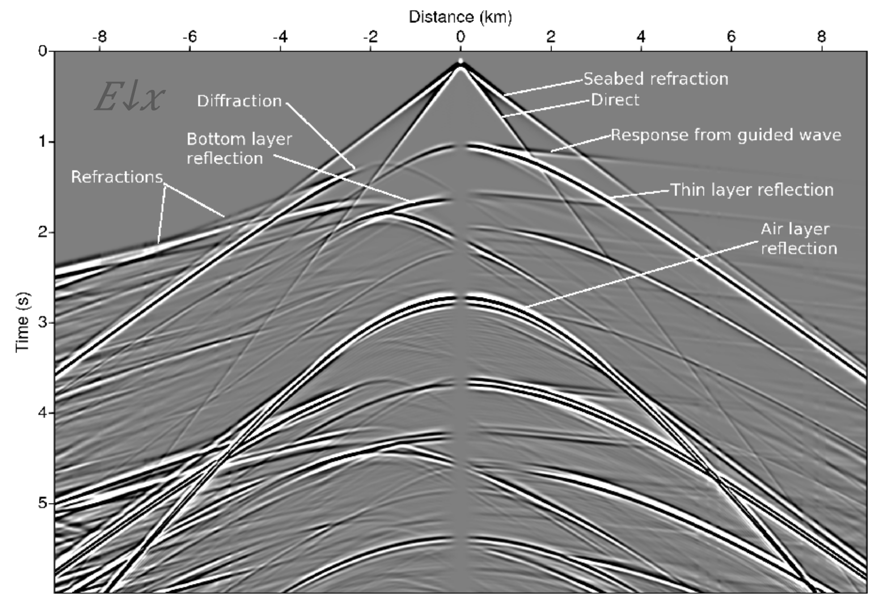
emgs

Traces normalized to unity

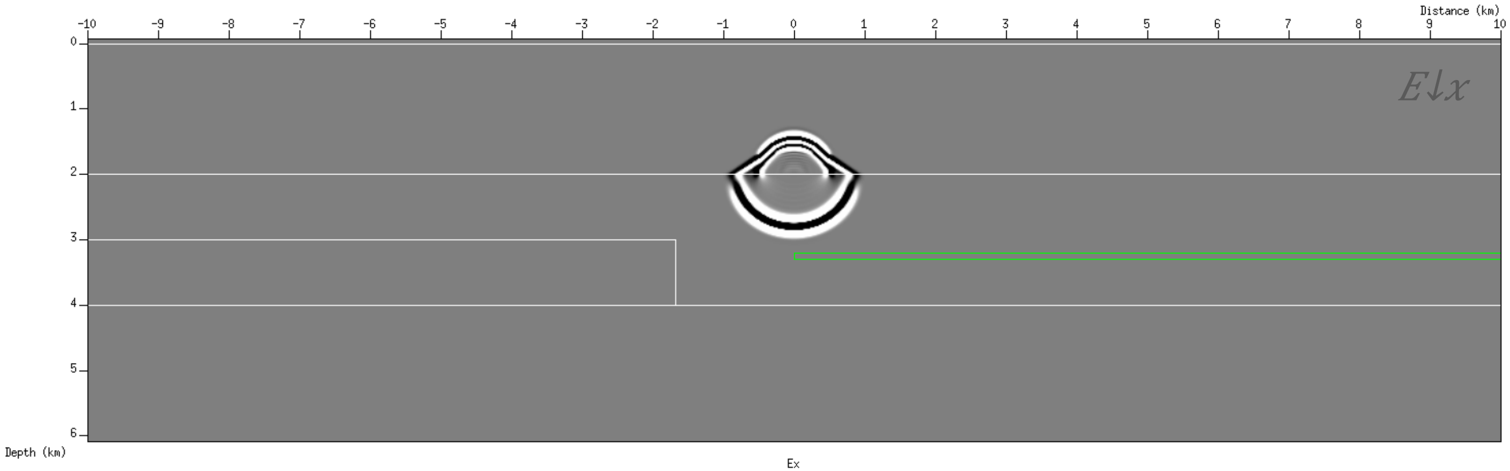
Acoustic

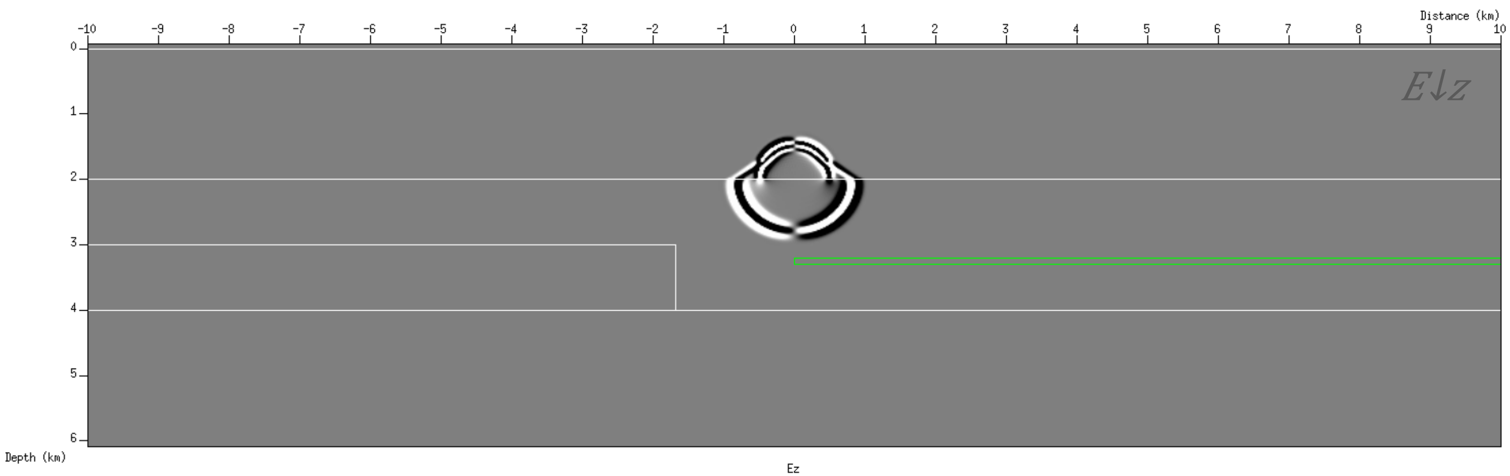


Electromagnetic

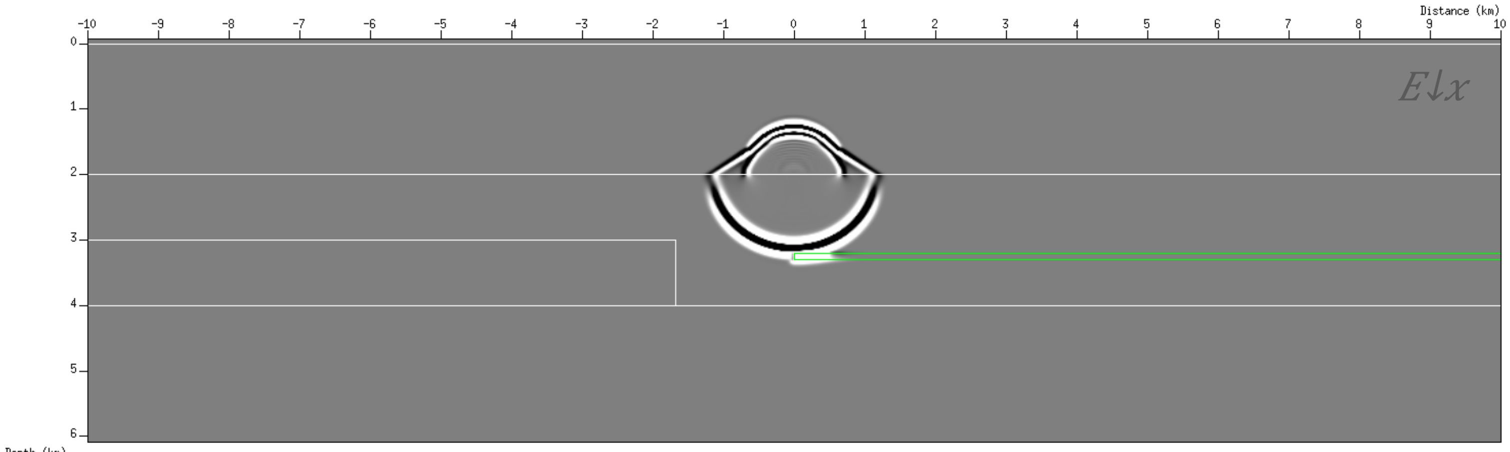


emgs

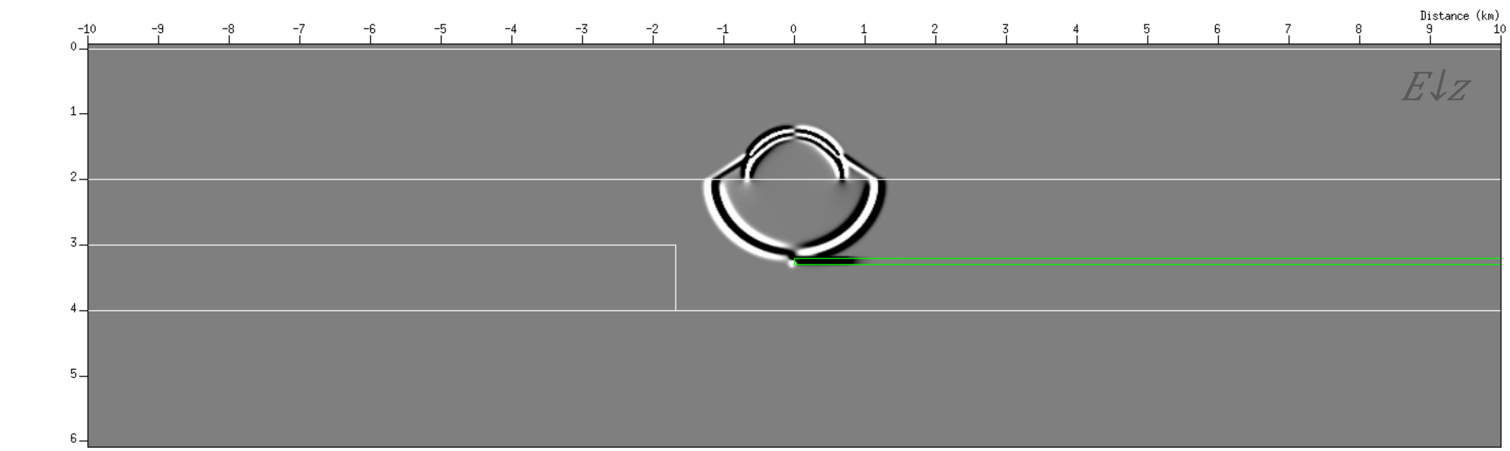

Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
E↓x
Ex


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
E↓z
Ez

emgs


Distance (km)
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6
Depth (km)
E↓x

Ex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-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
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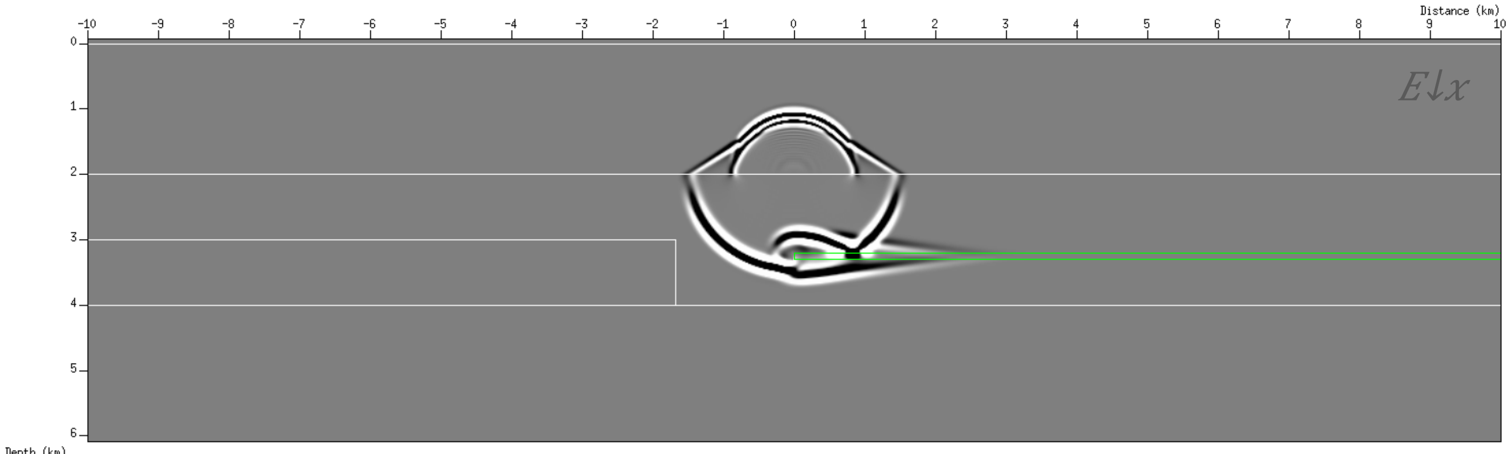
Ez



Distance (km)
-10
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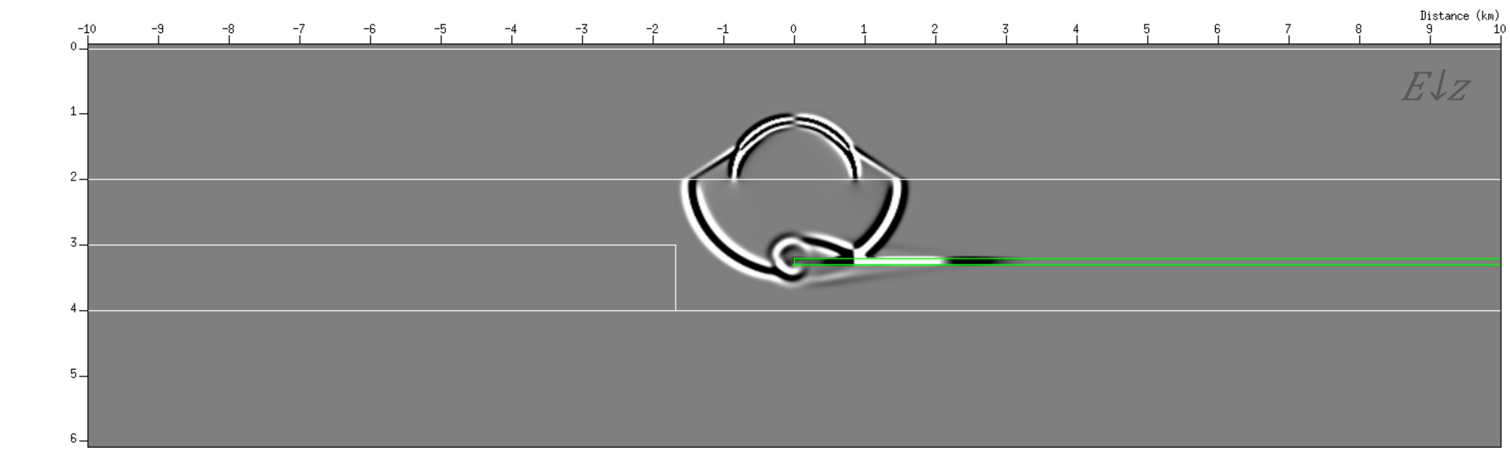
Depth (km)
E↓x

Ex



Distance (km)
-10
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-7
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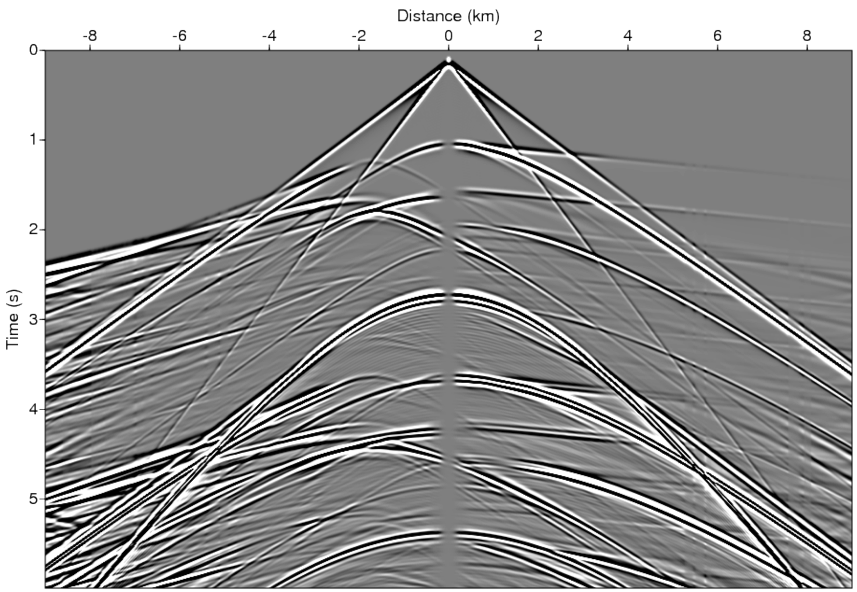
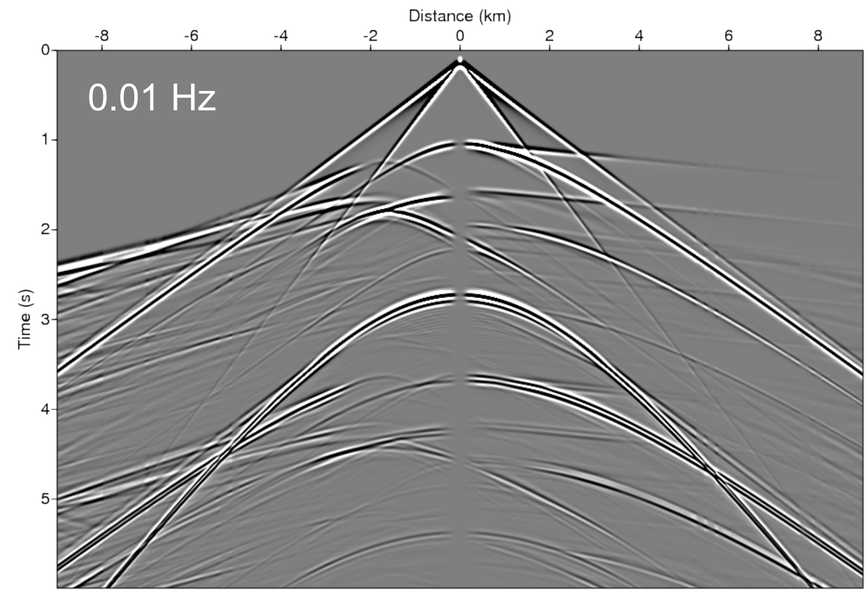
8
9
10
0
1
2
3
4
5
6
Depth (km)
E↓z

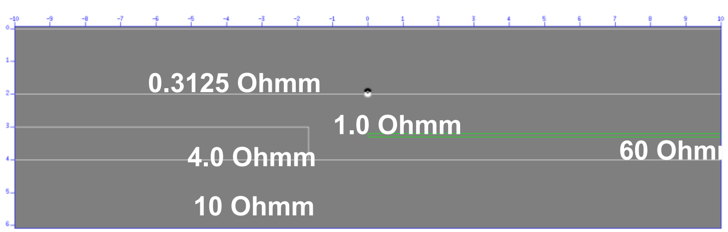
Ez

emgs

$$E_i(\boldsymbol{x},\omega)=\int_0^T dt'\, E'_i(\boldsymbol{x},t')\, e^{-\sqrt{\omega\omega_0}\,t'}\, e^{i\sqrt{\omega\omega_0}\,t'}$$







emgs

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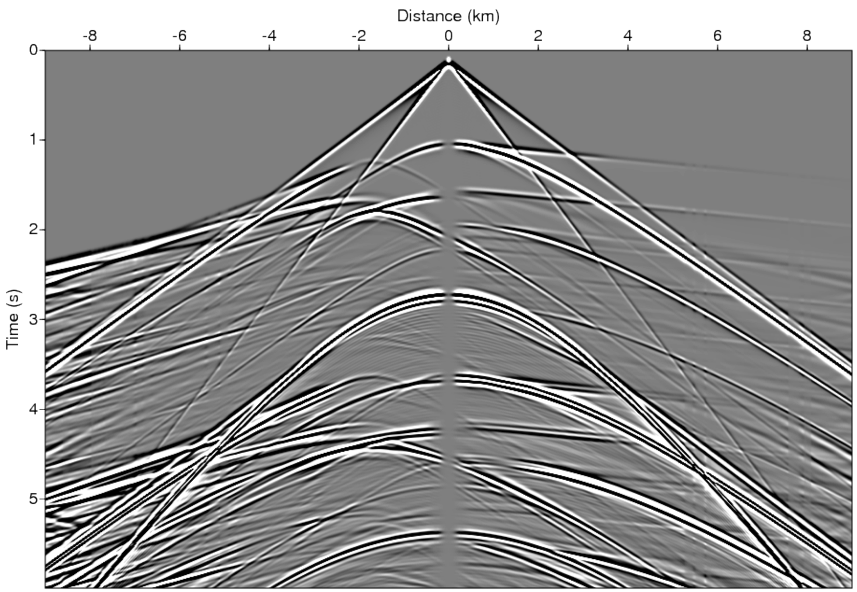

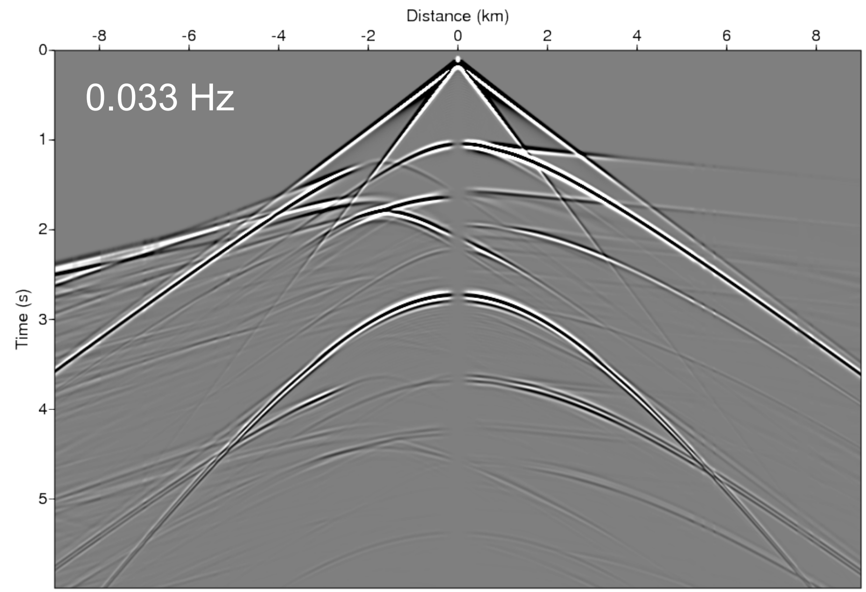

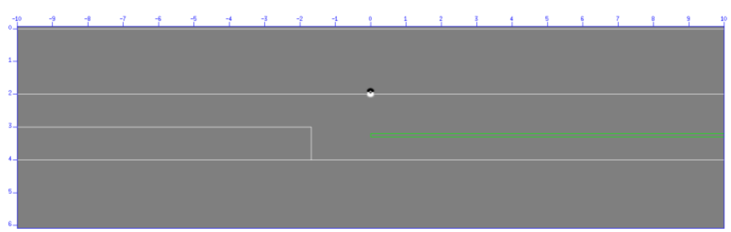

emgs

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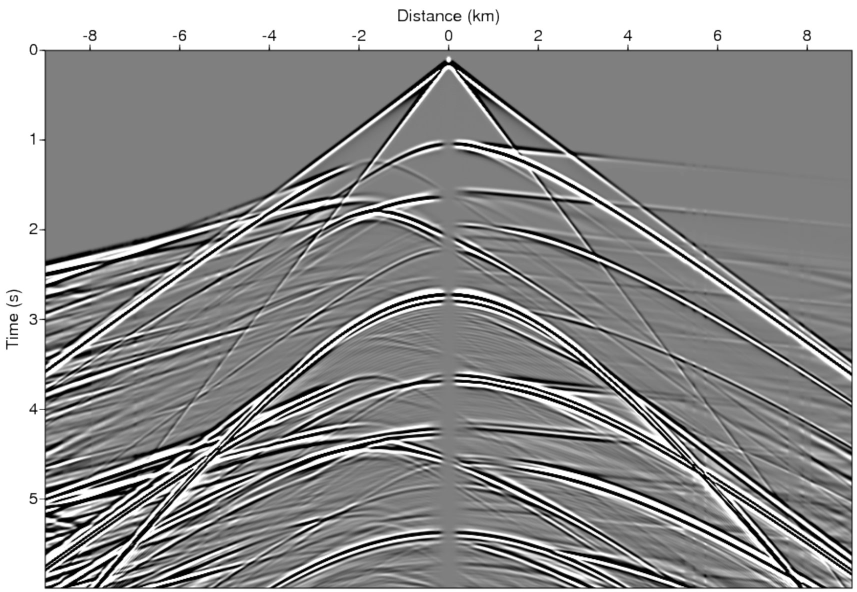

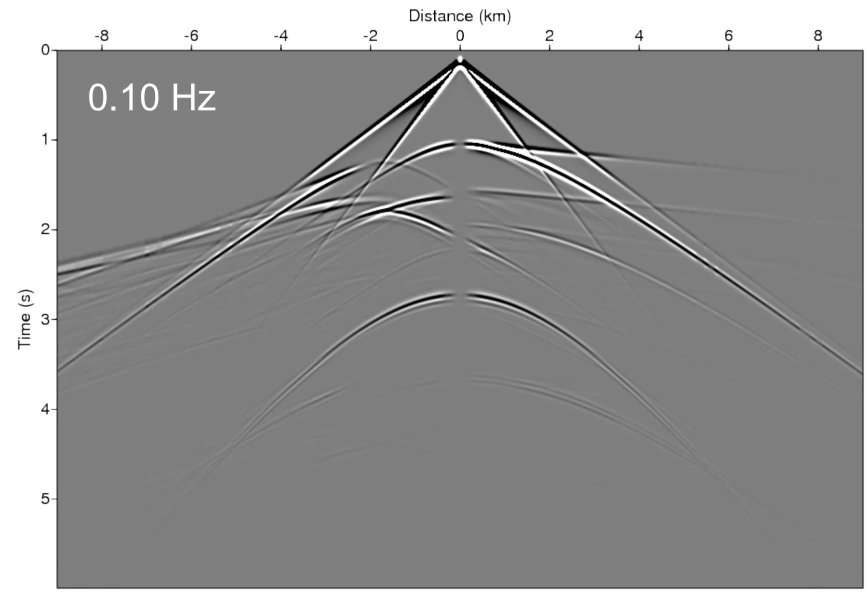

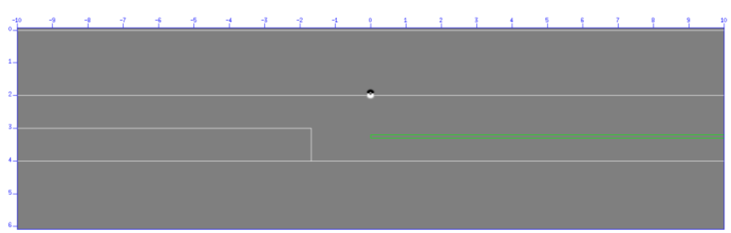

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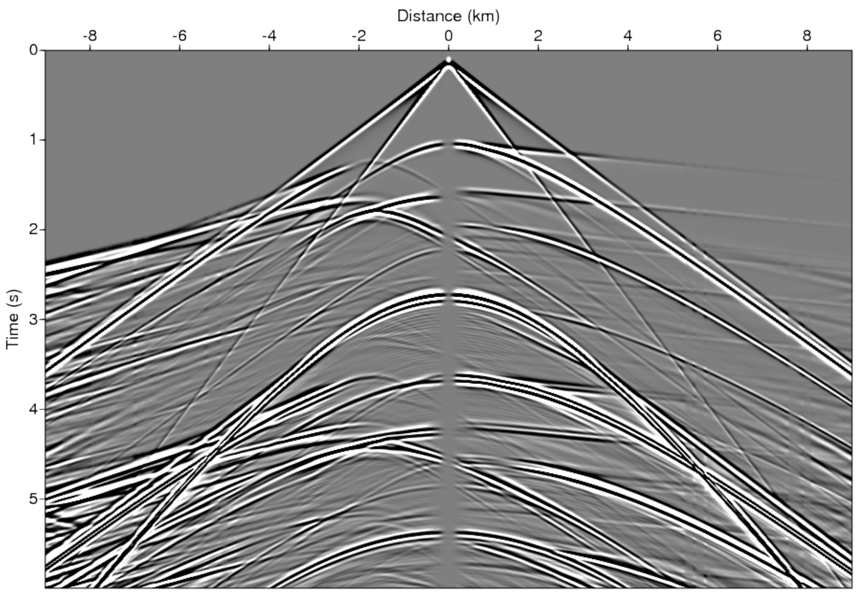

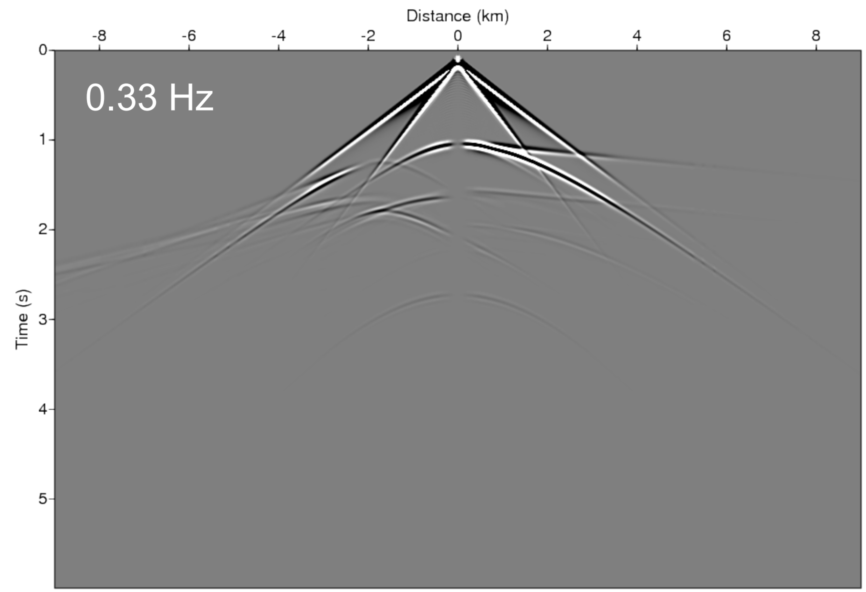

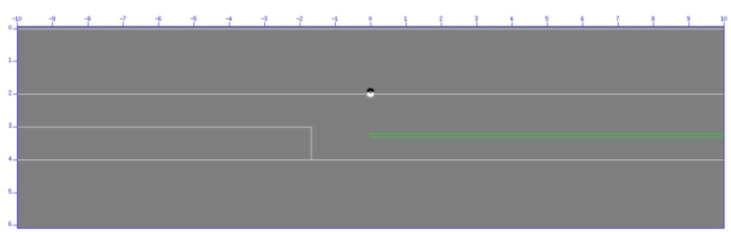

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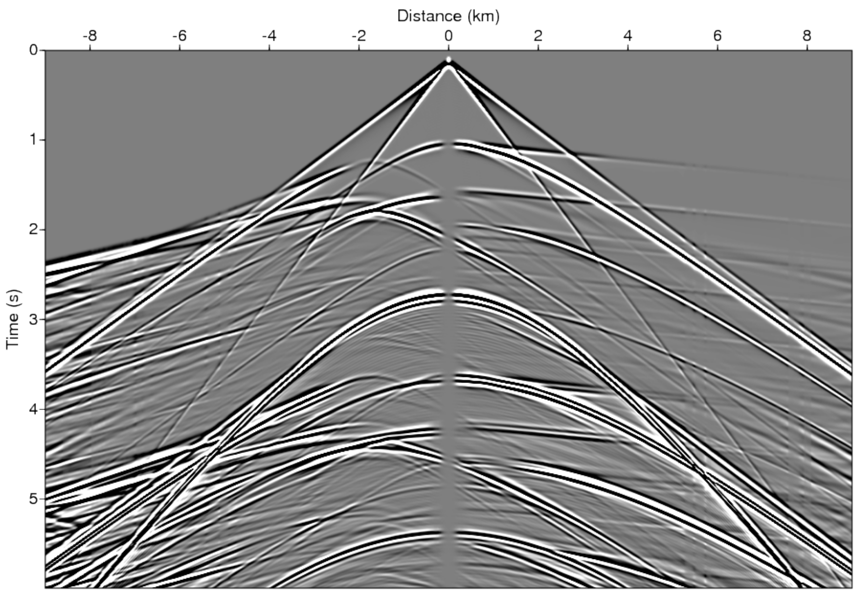

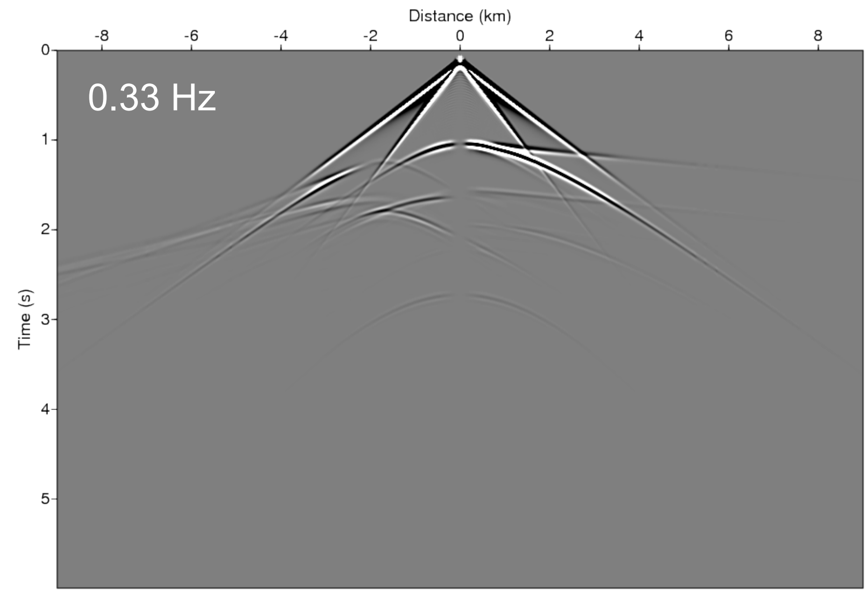

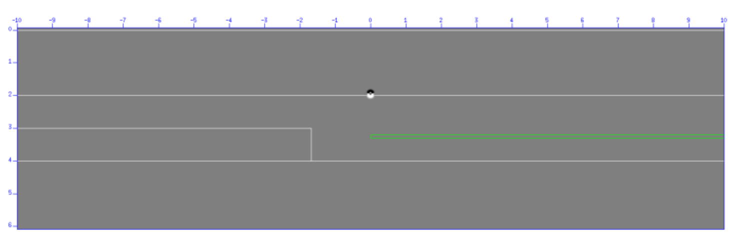

emgs

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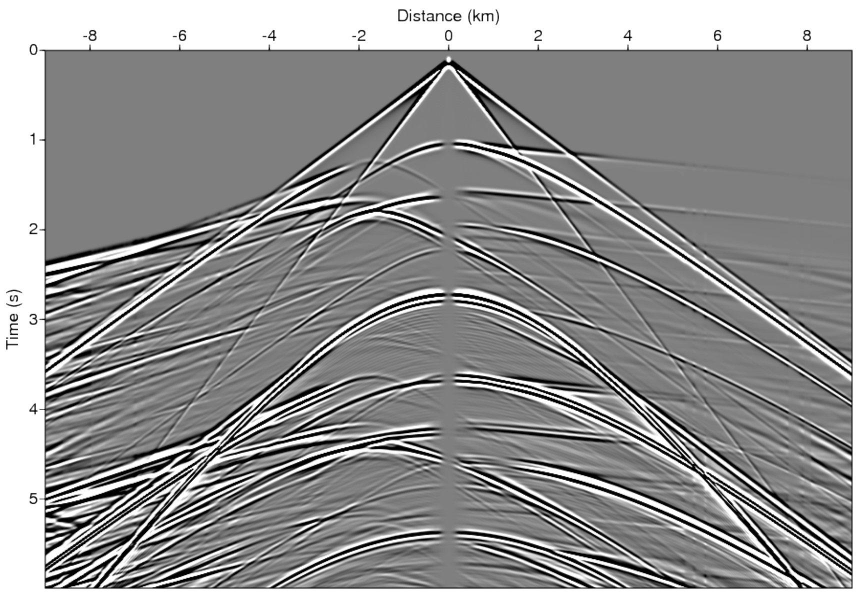

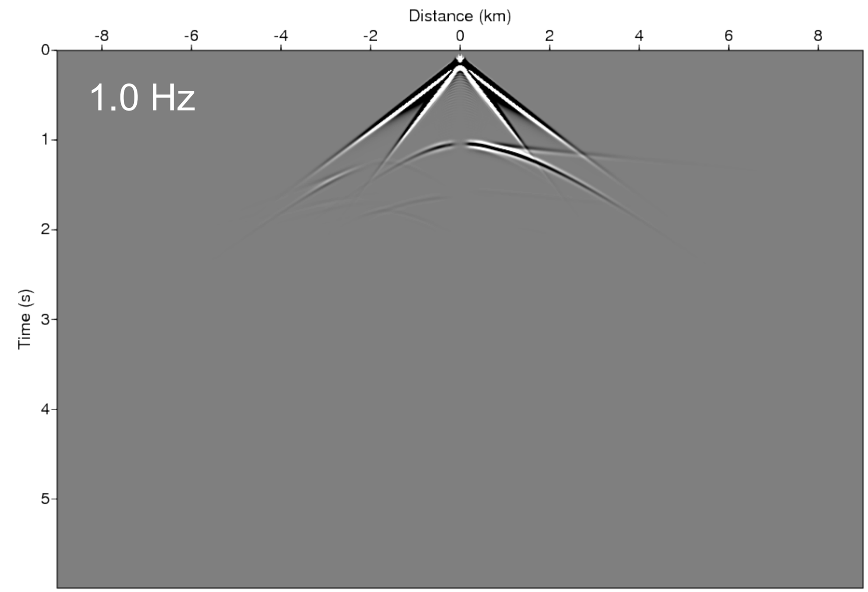

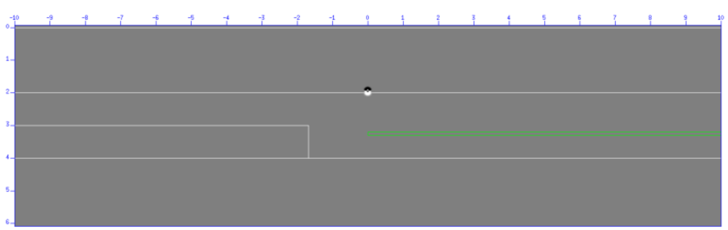

emgs

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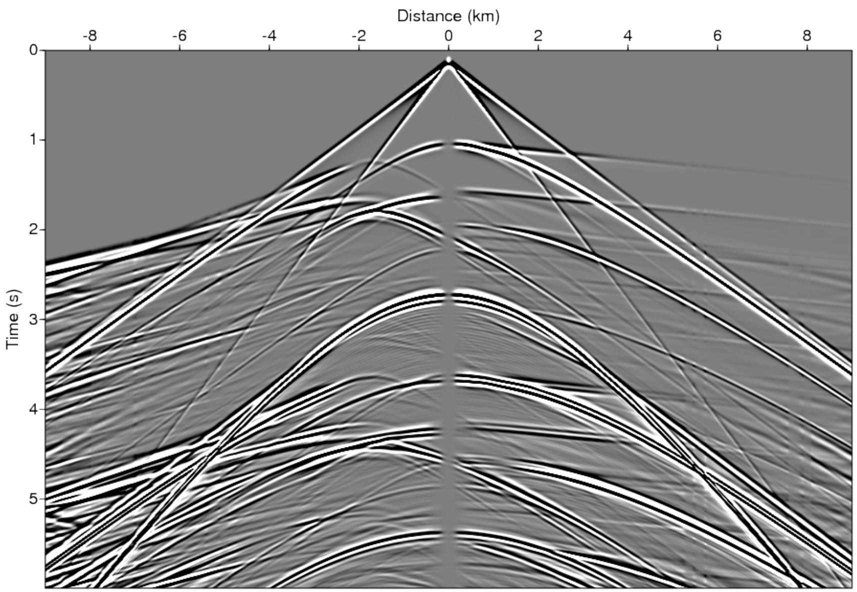

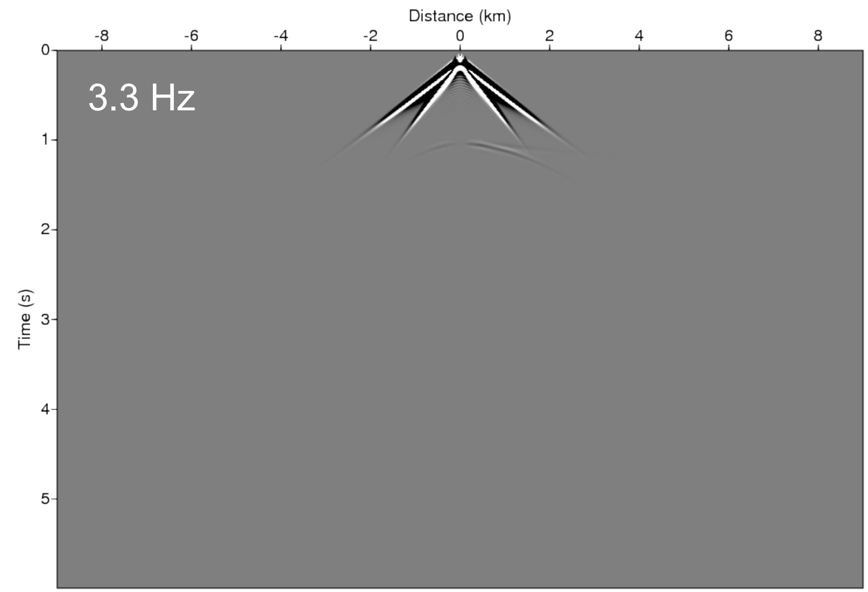

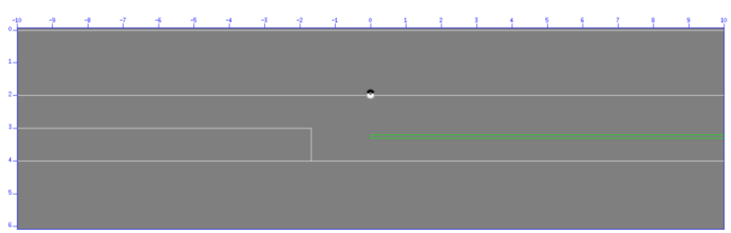

Summary

Transform from fictitious wave-domain time response to «real-world» frequency response by temporal integral

No spatial integration in transform:

What is a reflection or refraction in the EM wave domain stays a reflection or refraction after the transform to the real frequency domain

No problem using concepts like reflections, refractions, diffractions etc for interpretation of marine CSEM responses

Note: «use of ray physics» is in principle possible

Summary

For formation resistivities typical for marine sediments and for typical CSEM frequencies:
First arrivals are important - Refractions and (possibly) reflections at small offsets
- Refractions and guided events at intermediate and large offsets

Guided field in thin resistor gives relatively large electric (and magnetic) fields at large offset. The effect is so strong that it can be used as a hydrocarbon indicator

Marine CSEM responses can be interpreted by inspecting events in the EM wave domain

Marine CSEM data have common properties with refraction seismic data and this may partly explain the relative success of 3D FWI of CSEM data.

emgs

FWI of CSEM data

Typical for 3D CSEM data:

Low frequencies

Large source-receiver offsets

Wide azimuth

Data dominated by refraction (transmission) type events. Reflections «filtered out» by MN.

Source function measured.