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Combining a simplified flow equation and 4D seismic traveltime shifts for pressure and saturation predictions

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- Introduction: main objectives
- Methodology for pressure-saturation discrimination modeling
- Results
 - single well: synthetic
 - single well: Snøhvit CO₂ injection site
 - multi well: synthetic
- Conclusions and Outlook

Objectives

- linking the fields of reservoir engineering and 4D seismic
- separation of fluid saturation and pressure effects on 4D seismic data
- a method faster than reservoir simulation to understand simple, first order effects of reservoir behaviour using a superpositioning principle



Pressure – saturation discrimination

Combining an engineering pseudo-steady state flow equation and 4D seismic traveltime shifts:

(pseudo-steady state: all reservoir boundaries have been felt and the reservoir as a whole is contributing to the flow.)

$$\Delta T = dz \frac{v_1 - v_2}{v_1 v_2} + D \frac{1 - \left(1 - \frac{dp}{p_0}\right)^\gamma}{v_1 \left(1 - \frac{dp}{p_0}\right)^\gamma}$$

saturation (Gassmann) pressure (Hertz-Mindlin)

dz = thickness of gas column
 v_1 = V_p prior to injection
 v_2 = V_p after injection
 D = reservoir thickness
 g = Mindlin exponent
 p_0 = initial reservoir pressure
 dp = differential pressure $p - p_0$

• radial pressure distribution (injection case)

$$p(r) = p_w - \frac{qB\mu}{2\pi hk} \left(\ln \frac{r}{r_w} - \frac{1}{2} \frac{r^2}{r_e^2} \right)$$

$$p(r) = p_w - a * \ln\left(\frac{r}{r_w}\right)$$

assuming r_e is large

using p_w and a as fitting parameters

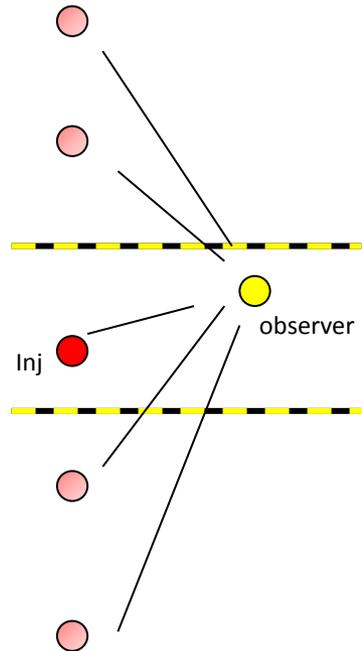
p = pressure
 p_w = well radius
 q = flow rate
 B = volume factor
 m = viscosity
 h = thickness
 k = permeability
 r_w = well radius
 $r_e \gg r_w$
 $r^2 \gg r_w^2$

(Eq 13.33 Zolotukhin & Ursin, 2000)

Superpositioning principle – one well

The theorem states: any linear combination of individual solutions to the diffusivity equation is also a solution to that equation.

Case 1: two no flow boundaries

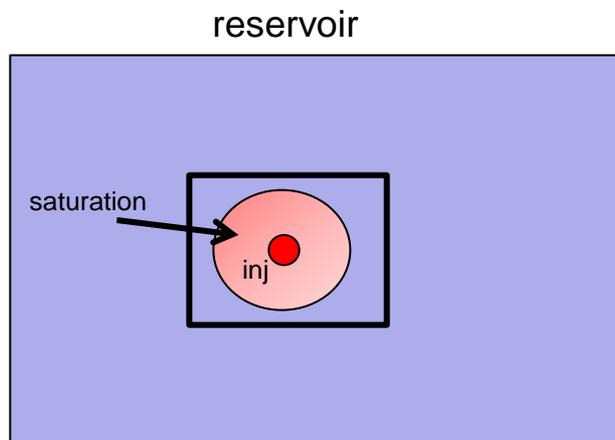


- Removal of physical boundaries and replacing by mirror images of well location
- The mirroring develops into an infinite series, the total pressure at any point is given by the well and mirror image contribution:

$$P_{total}(r) = \sum_{i=1}^{nmir} P_i(r)$$

Saturation

- velocity change pre- and post injection by Gassmann's fluid substitution
- modeling of gas column thickness



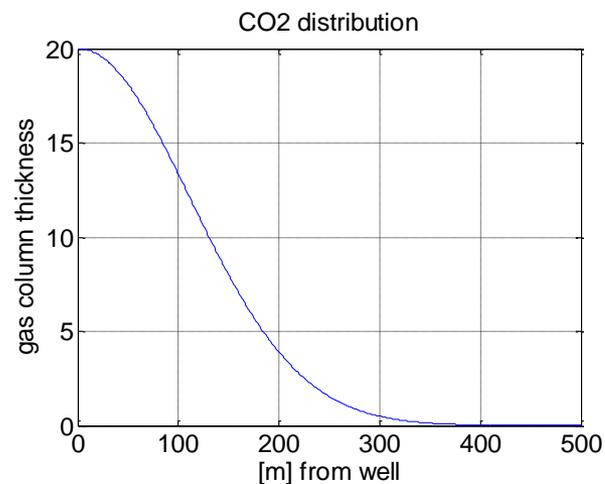
- define an area inside the reservoir
- dz is a function of location (x,y)

=> parameterization of dz

- circular

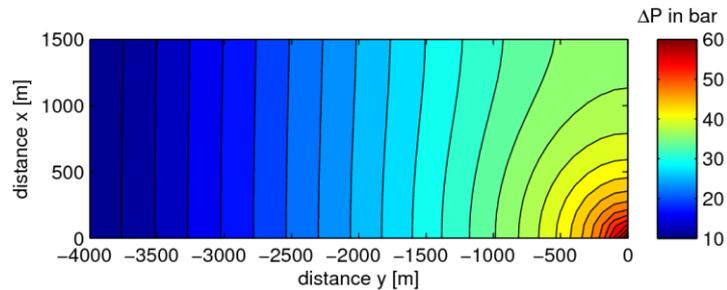
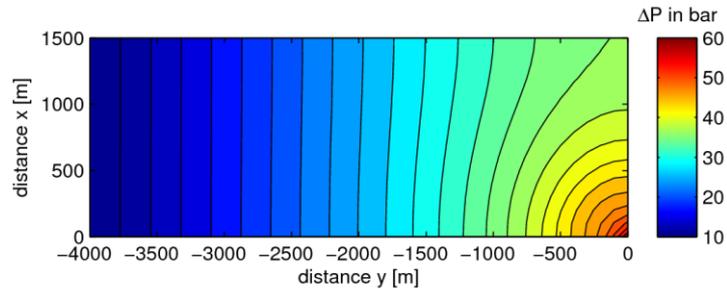
- with decreasing characteristics of a Gaussian distribution:

$$dz(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2} dz_0(r)$$

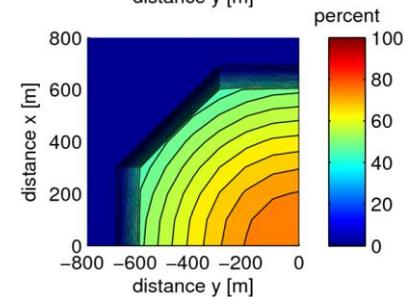
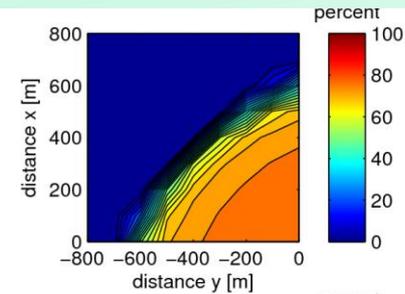


One well – synthetic

- 31x141x51 cells in x-y-z direction (central part 3000x8000x110m)
- permeability = 50mD in x, y, z
- porosity = 14%
- PVT properties for 90 degree celsius and 14% salinity (Span & Wagner, 1996)
- BHP controlled CO₂ injection over 2.5years



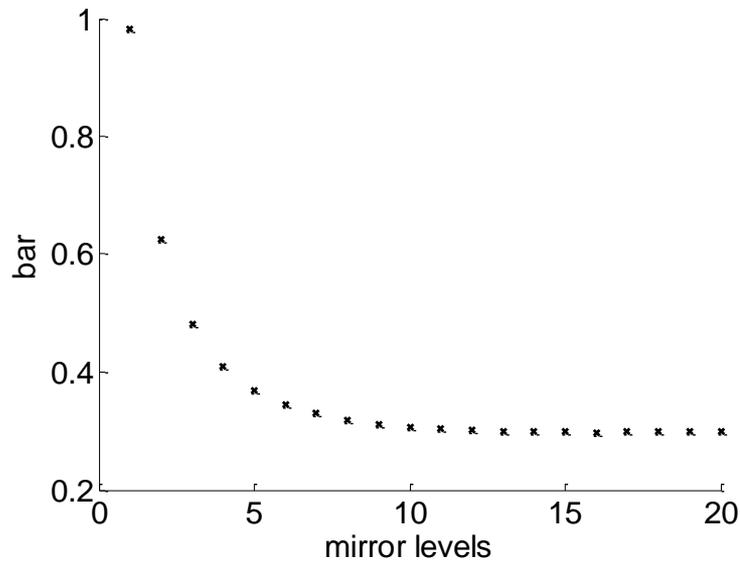
ΔP



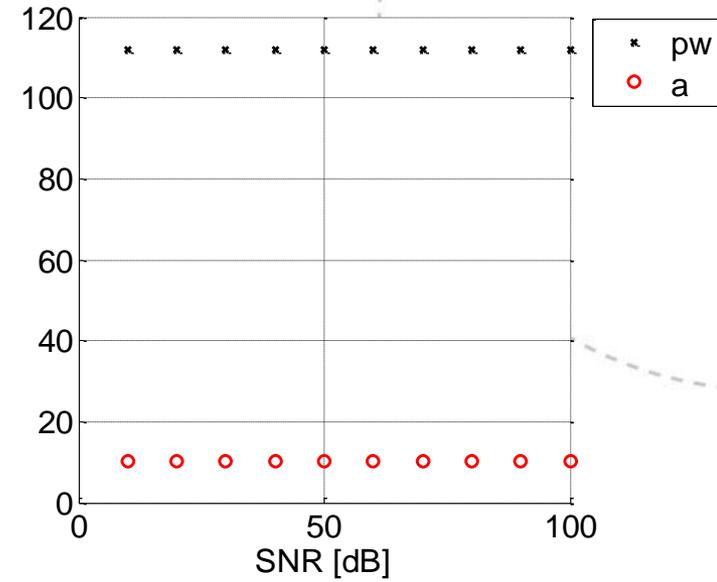
ΔS

top: Eclipse model, bottom: inverted model 20 mirror levels

One well – synthetic



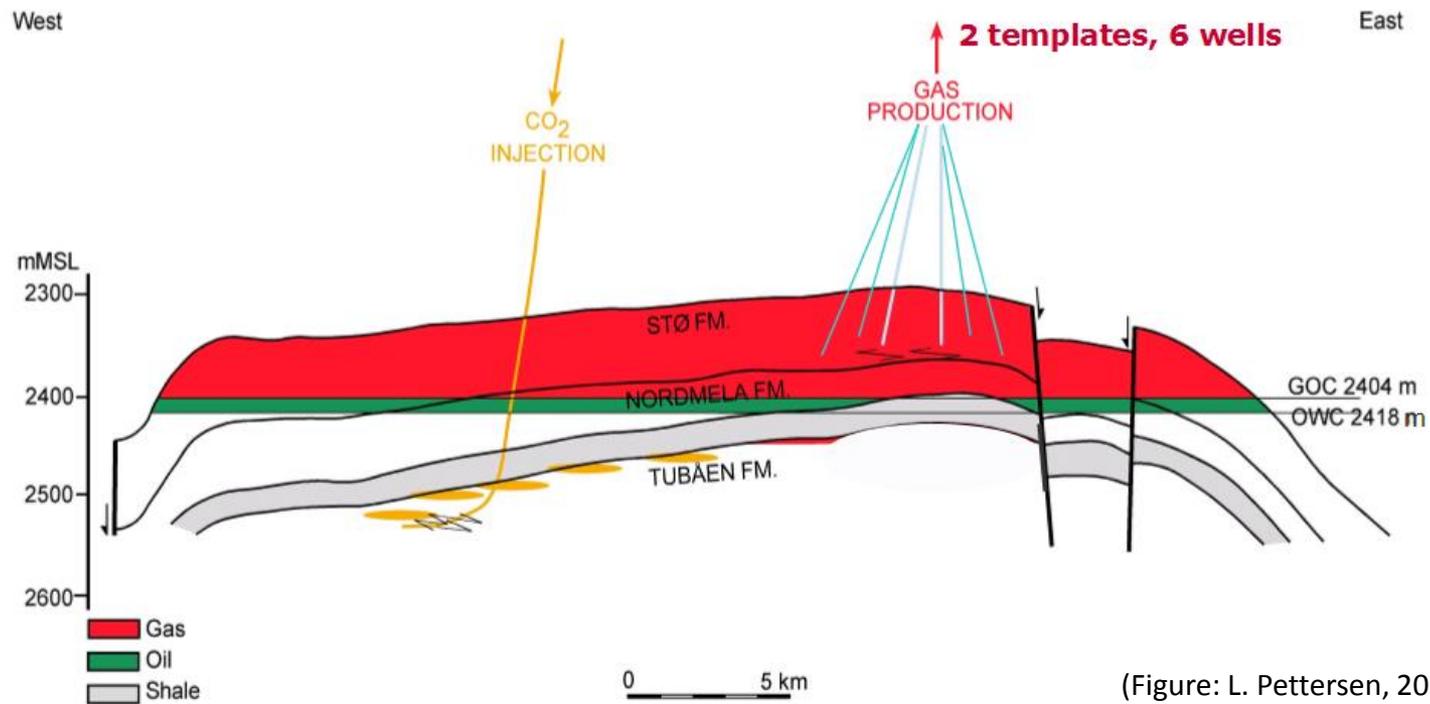
- mean error
- the error reduces strongly during the first 10 mirror levels
- due to difference close to the well the average error does not reach zero



- fitting parameters with respect to different SNR
- strong robustness against noise – the inversion parameters remain almost unchanged with different noise levels

One-well – Snøhvit field

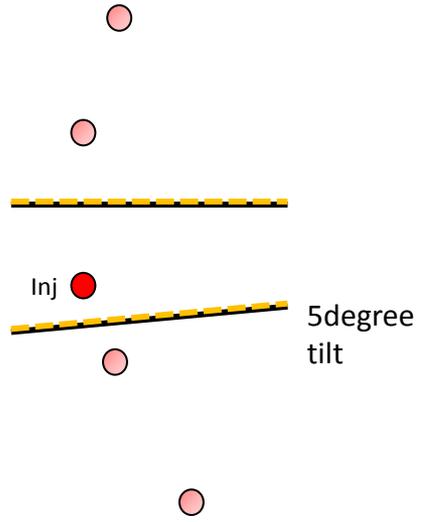
- 2 sealing faults East-West
- reservoir thickness ~110m
- CO₂ Injection from April 2008 into Tubåen formation
- interpreted as part of a delta plain environment
- distributary channel systems observed in core analysis



(Figure: L. Pettersen, 2011)

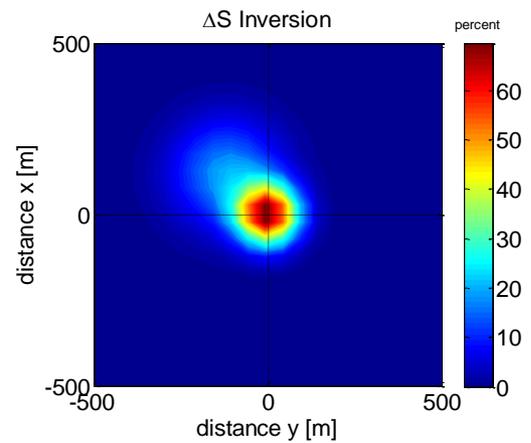
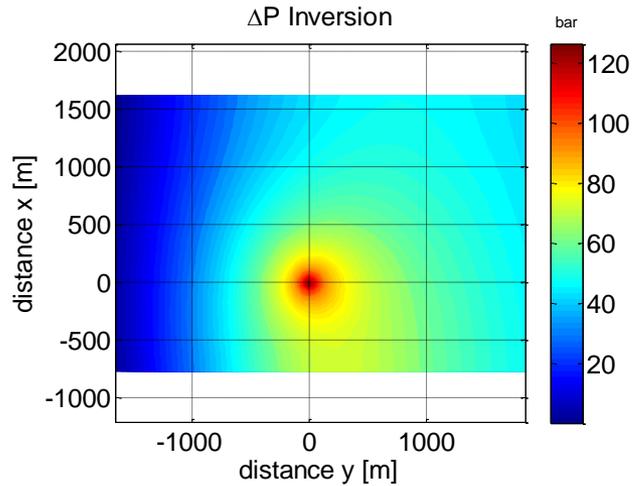
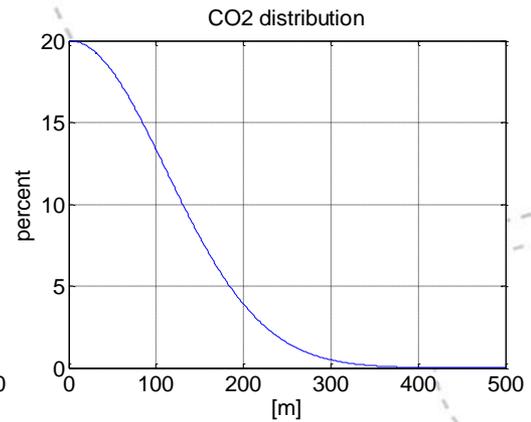
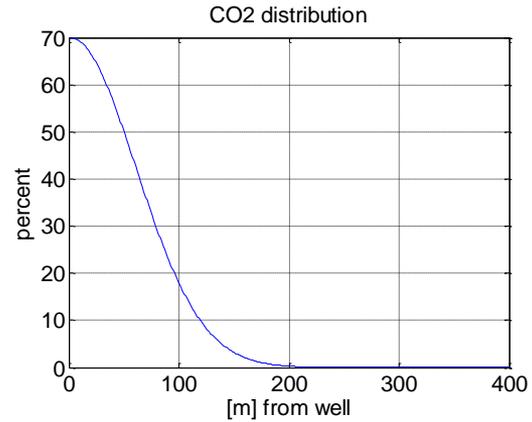
One-well – Snøhvit field

Case 2: two no flow boundaries: one tilted

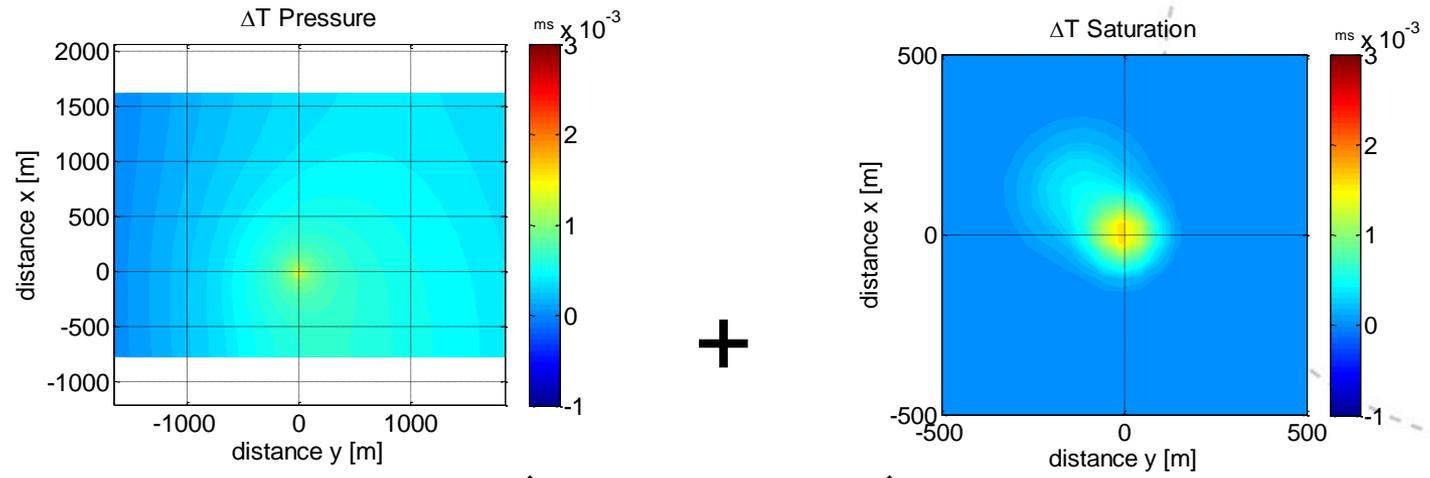


Saturation model

- a positive dipole => fct. 2 center shifted by (120m/-120m)

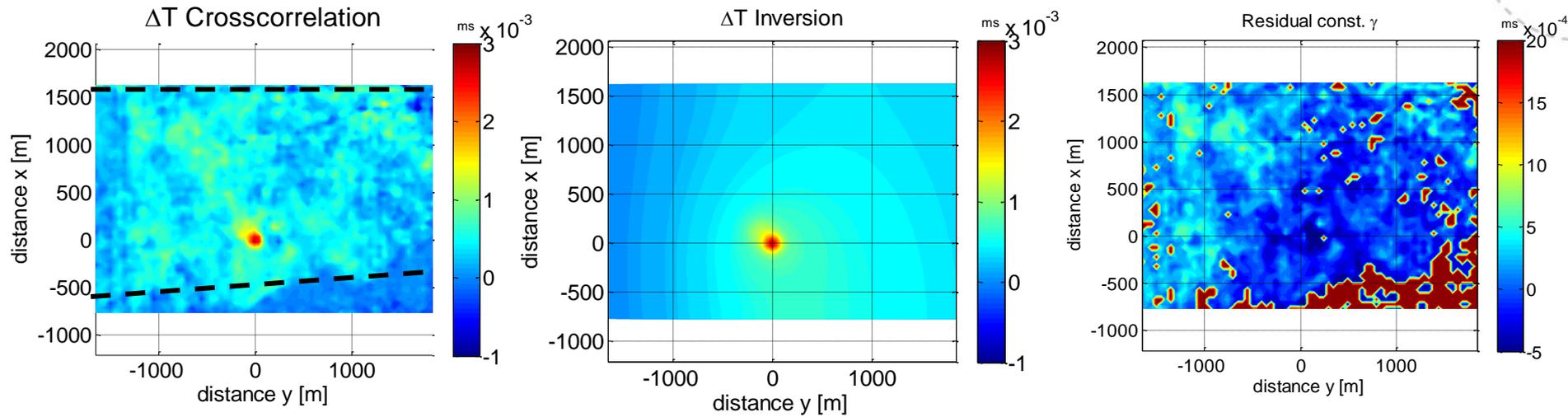


One-well – Snøhvit field



$$D \frac{1 - \left(1 - \frac{dp}{p_0}\right)^\gamma}{v_1 \left(1 - \frac{dp}{p_0}\right)^\gamma}$$

$$dz \frac{v_1 - v_2}{v_1 v_2}$$



- lateral heterogeneities are not captured by our method

One-well – Snøhvit field

1. The measured timeshifts are heterogeneous

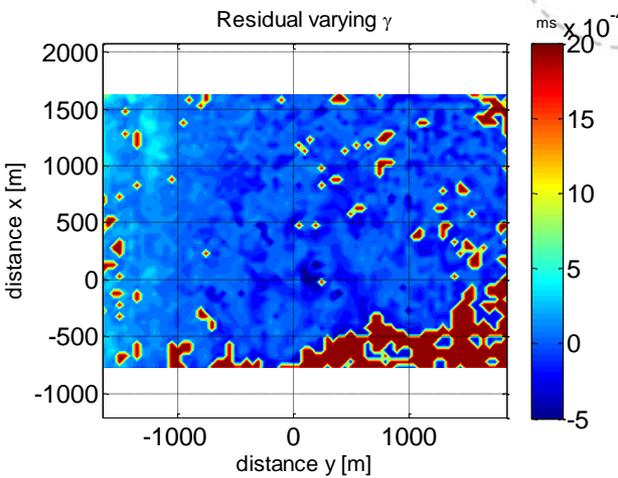
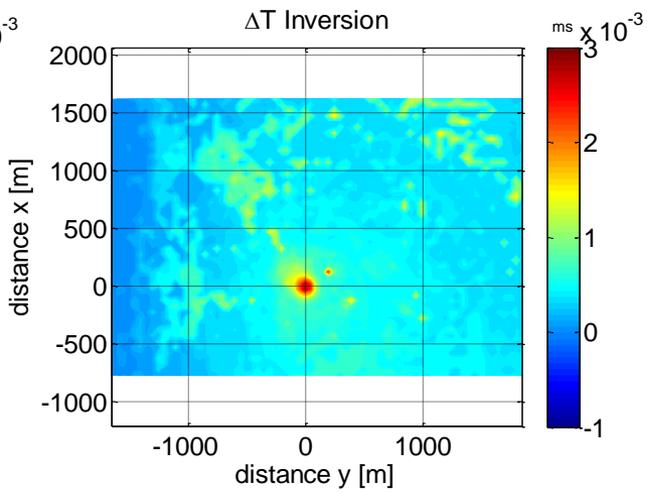
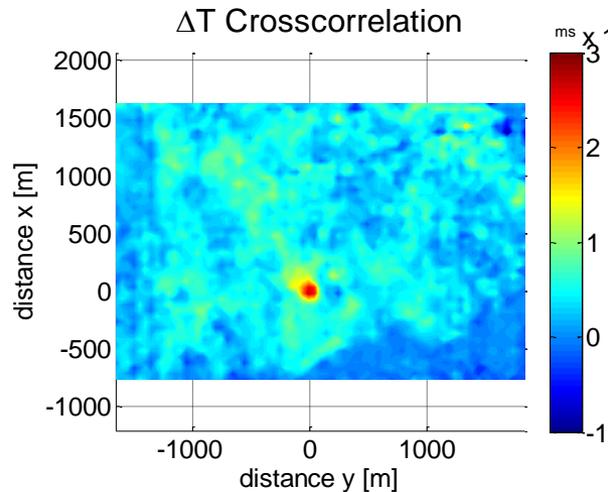
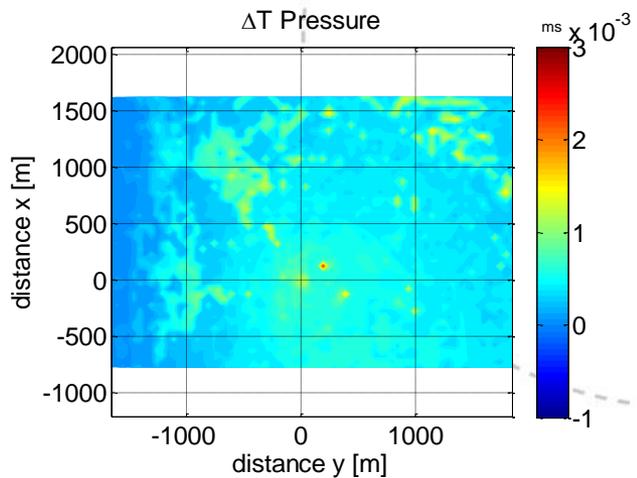
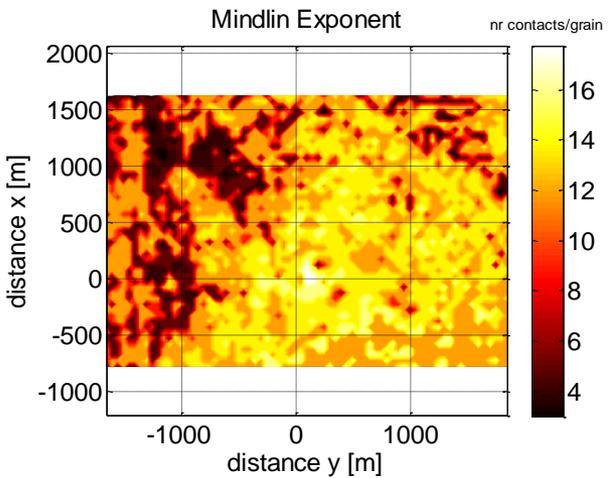
2. It is most likely that these are caused by

=> noise

=> local: variation in pressure (since we assume that saturation changes are confined to the near well area)

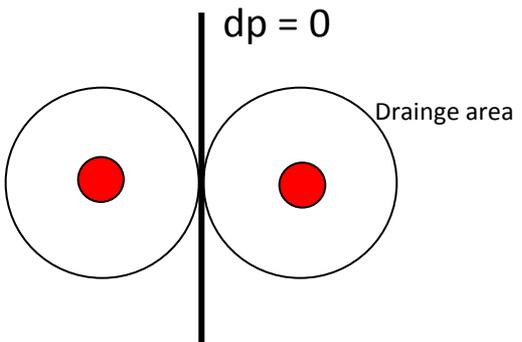
3. We suggest to let the Mindlin-coefficient vary and interpret these variations with respect to rock stiffness

One-well – Snøhvit field



- with variation of Mindlin exponent (between 1/3 and 1/18) main trends can be captured

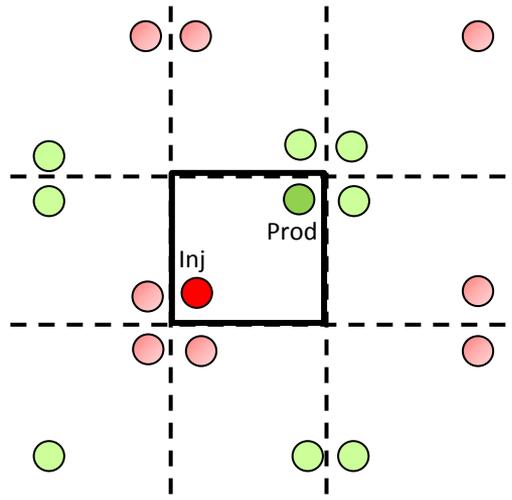
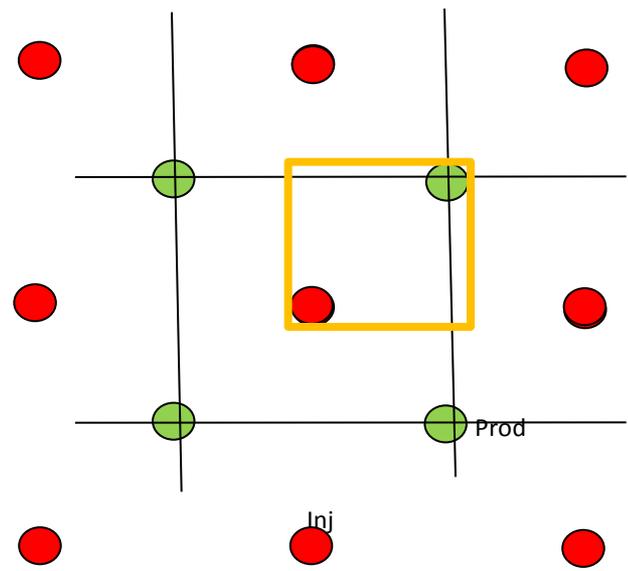
5 spot – synthetic



In simulation:

- modeling a quarter of one five spot pattern (Green, D.W., et.al., 1998)

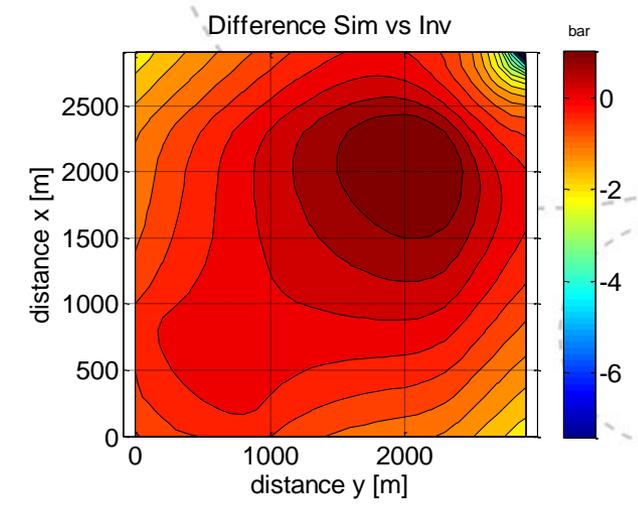
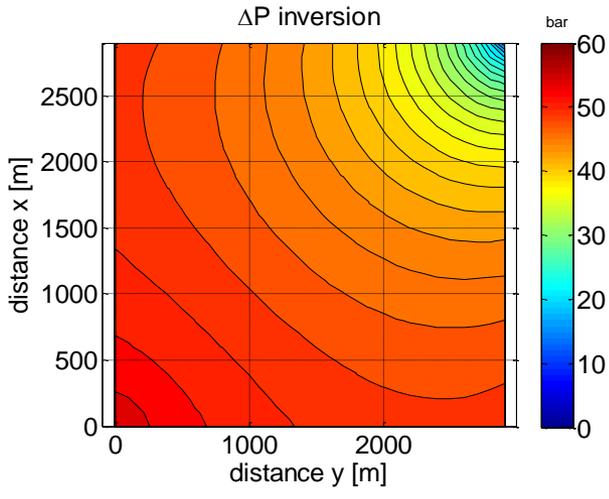
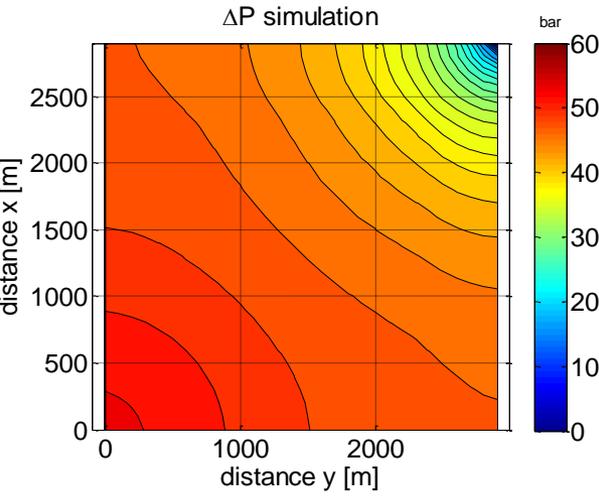
Series of 5 spot pattern in an ideal field



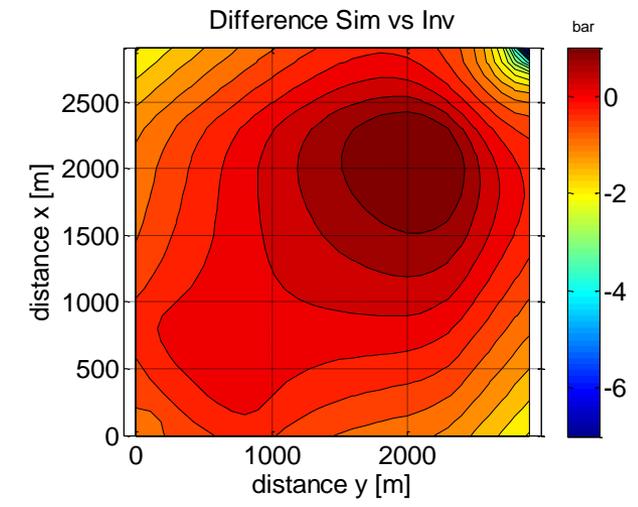
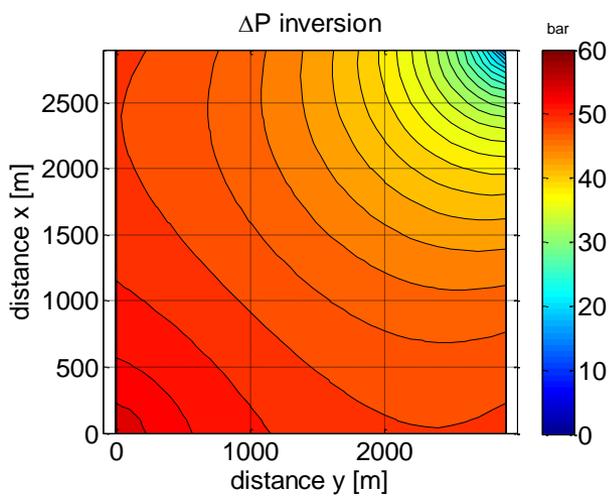
8 mirror points per mirror level and for each well
 => mirror level 10 has 440 mirror points/well

5 spot – synthetic: pressure modeling

- 30x30x3 cells in x-y-z direction (3000x3000x30m)
- permeability = 500mD in x, y, z
- porosity = 30%
- rate controlled gas injection and oil production after 3 years



- pressure modeling for 5 (top) and 10 (bottom) mirror levels using a non-linear least square fitting method
- sum over differences reduces from 487.7 to 478.4)



=> local minimum instead of global minimum?

5 spot – synthetic: pressure modeling

analytical solution

$$P_{inj} = \sum_{n \in inj} [pw_i - a_i * \log(\frac{r_n}{r_w})] \quad \text{pressure from injector + mirror images}$$

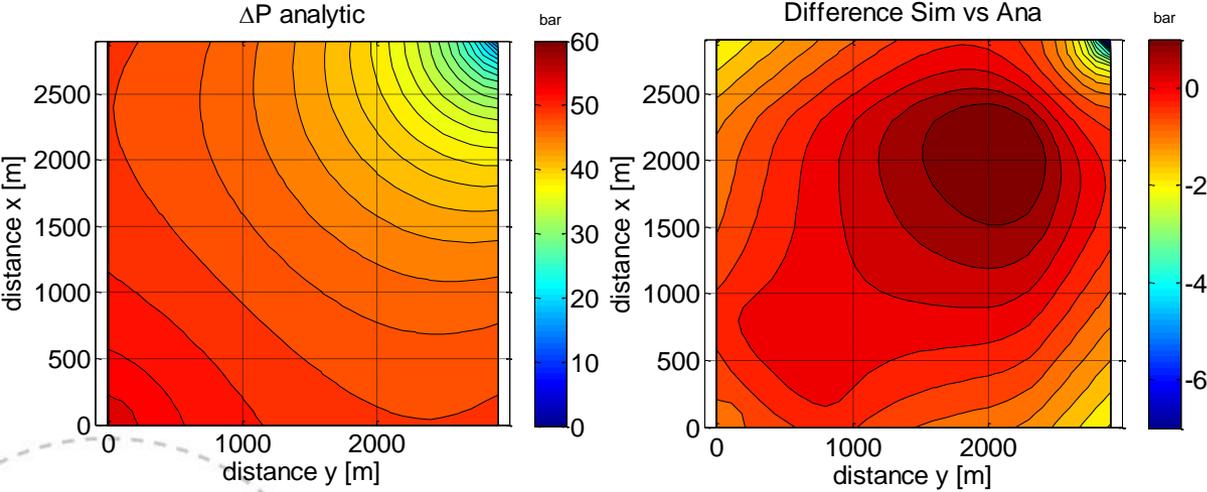
$$P_{pro} = \sum_{n \in pro} [pw_p - a_p * \log(\frac{r_n}{r_w})] \quad \text{pressure from producer + mirror images}$$

$$P_{tot} = \underbrace{pw_i - pw_p}_{\text{dependent}} - [a_i \sum_{n \in inj} \log(\frac{r_n}{r_w}) + a_p \sum_{n \in pro} \log(\frac{r_n}{r_w})] \quad \text{total pressure from injector and producer}$$

 **3 inversion parameters**

workflow

- least square method (min |Psim- Pmod|^2) , taking partial derivatives with respect to inversion parameters
- set up system of equations (A)
- invert (A)⁻¹ to find explicit solutions for the inversion parameters

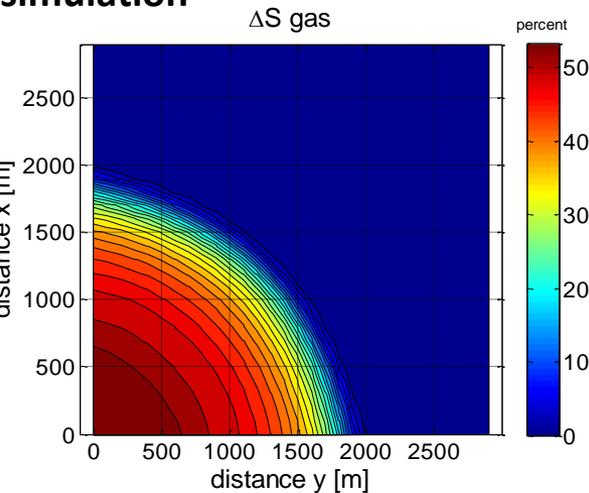


For 10 mirror levels total difference reduces not significantly compared to fitting method

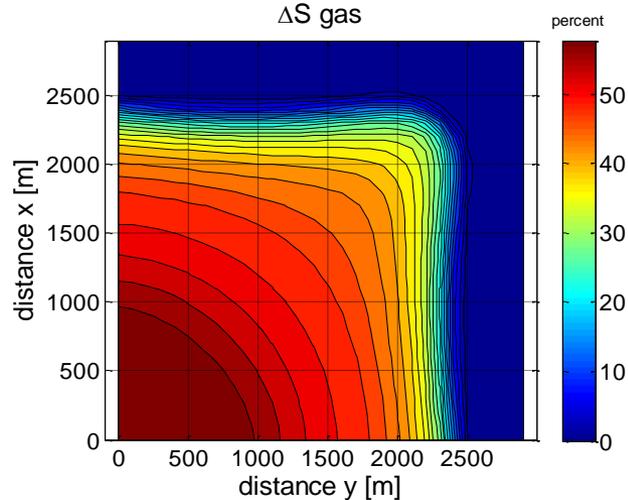
BUT: analytical solution is much faster for high mirror levels

5 spot – synthetic: saturation modeling

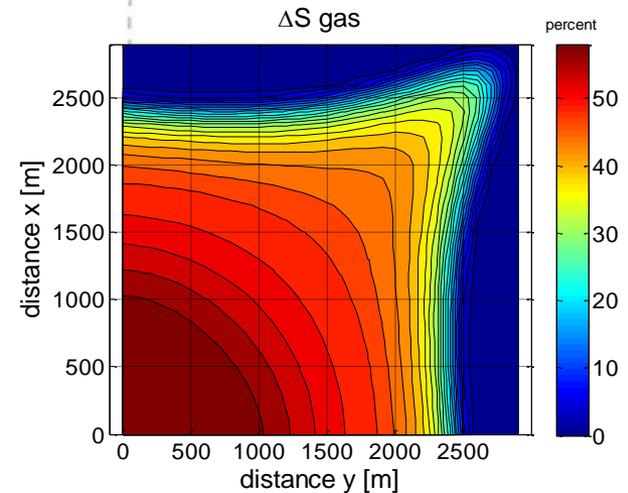
simulation



1 year



3 years

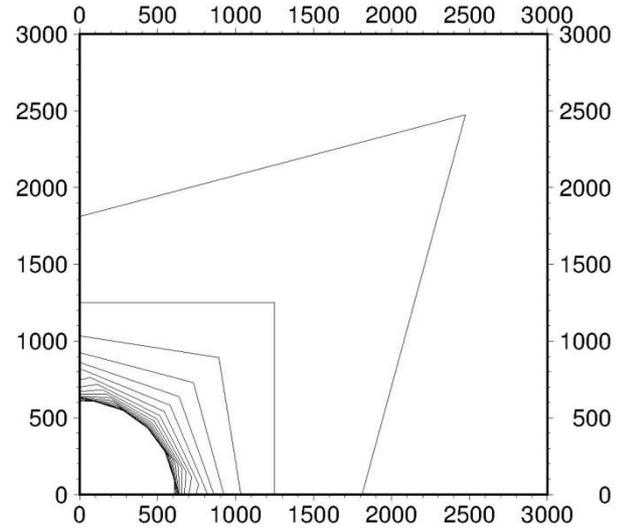
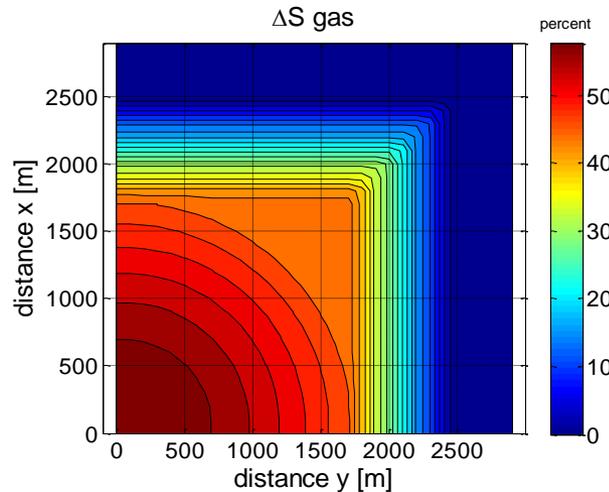


3.5 years

model

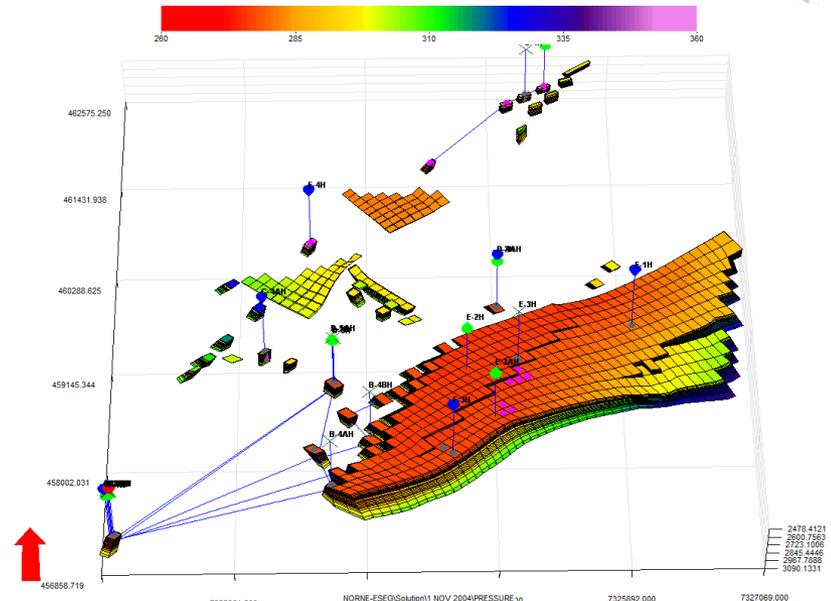
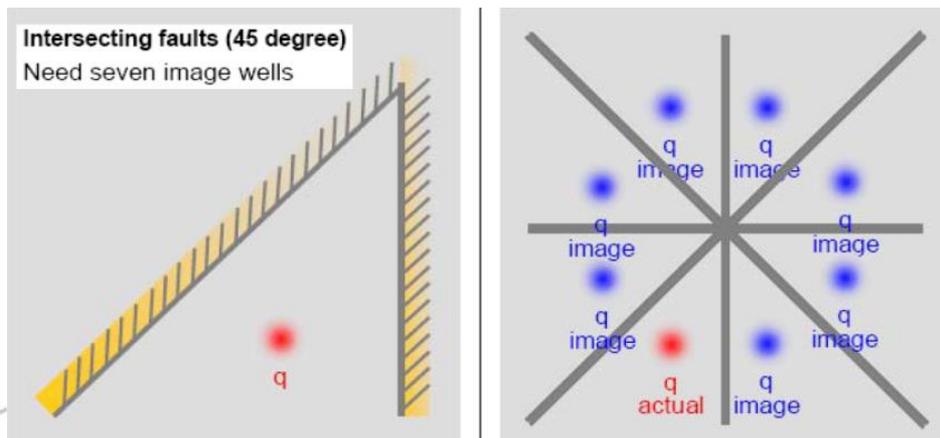
shape of gas distribution:

- circle
- circle and square
- circle, square and triangle
- or as a series of polygons



Conclusions & Outlook

- combining simple pressure modeling and 4D seismic traveltime shifts using the concept of superpositioning in space
- amount of mirror levels depends on well location, length or amount of inj. /prod. rate
- fast and easily applicable to describe first order effects of pressure and saturation behaviour
- limitations: spatially low frequent and valid for homogeneous reservoir conditions
- variation of Mindlin exponent helps to include main heterogeneous trends
- application to hydrocarbon production and injection cases
- many mirror images when a fault is intersecting by 45 degrees
- how will this concept work in a more complex setting like the Norne field?
- how would temperature effects change the result?



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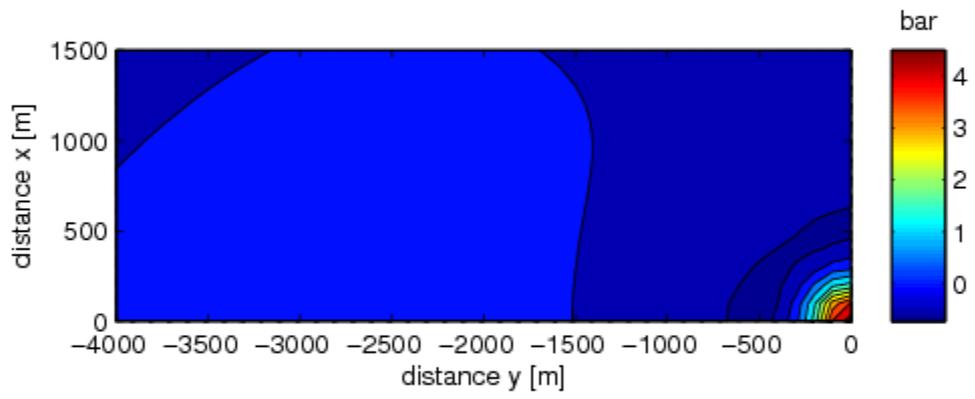
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Considering a homogeneous layer of thickness D , where one embedded portion of it (dz) is filled with CO_2 , and assuming that

$$T_1 = \frac{z_1}{v_0} + \frac{dz}{v_0} \quad \text{and} \quad T_2 = \frac{z_1}{v_0} + \frac{dz}{v_1}, \quad (1)$$

we obtain the following expression for the travelttime change caused by the presence of CO_2 :

$$\Delta T = dz \frac{v_0 - v_1}{v_0 v_1}. \quad (2)$$

here, v_0 defines the background velocity and v_1 the velocity for the CO_2 filled rock respectively. Velocity changes are modeled using Gassmann fluid substitution (Gassmann, 1951).

For simplicity, we neglect reservoir compaction and imply that the induced pressure effect is causing a velocity change over the entire reservoir thickness D , so that the travelttime shifts resulting from pressure changes can be defined as:

$$\Delta T = D \left(\frac{1}{v_0} - \frac{1}{v_1} \right), \quad (3)$$

where v_1' denotes the new P-wave velocity resulting from pressure changes. Based on the Hertz-Mindlin theory (Mindlin, 1949) a relation between P-wave velocity and effective pressure is stated as:

$$\frac{v_1'}{v_0} = \left(\frac{P}{P_0} \right)^\gamma, \quad \text{where} \quad P = P_0 + dP. \quad (4)$$

P_0 denotes the in situ effective reservoir pressure, P the new effective pressure, dP the change in effective pressure and γ the Mindlin exponent. Finally, combining equation 2, 3 and 4 leads to an expression quantifying timeshifts caused by pressure and saturation changes:

$$\Delta T = dz \frac{v_0 - v_1}{v_0 v_1} + D \frac{1 - \left(1 - \frac{dP}{P_0}\right)^\gamma}{v_0 \left(1 - \frac{dP}{P_0}\right)^\gamma}. \quad (5)$$

When modelling the trailing term of the travelttime shift equation, we are setting the constrained that:

$$1 - \left(1 - \frac{dP}{P_0}\right) > 0 \quad (6)$$

to ensure a non complex return of fitting results. The thickness of the gas column dz is modeled with decreasing characteristics of a normal distribution: