ROSE consortium meeting 2013

The offset-midpoint traveltime pyramid in TTI media

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Background

- Analytical traveltime equation is very important for pre-stack Kirchhoff migration and velocity inversion.
- It is difficult to obtain the analytical traveltime equation for anisotropic media, since the exact explicit relation between group velocity and ray angle does not exist.
- Alkhalifah (2000) derived the offset-midpoint traveltime equation, the Cheop's pyramid equation for VTI media using the stationary point method.

Objectives

- Derive the analytical expressions for traveltime pyramid in TTI media.
- Find the shape of traveltime pyramid in TTI media.

$$T = T_1 + T_2 = ?$$

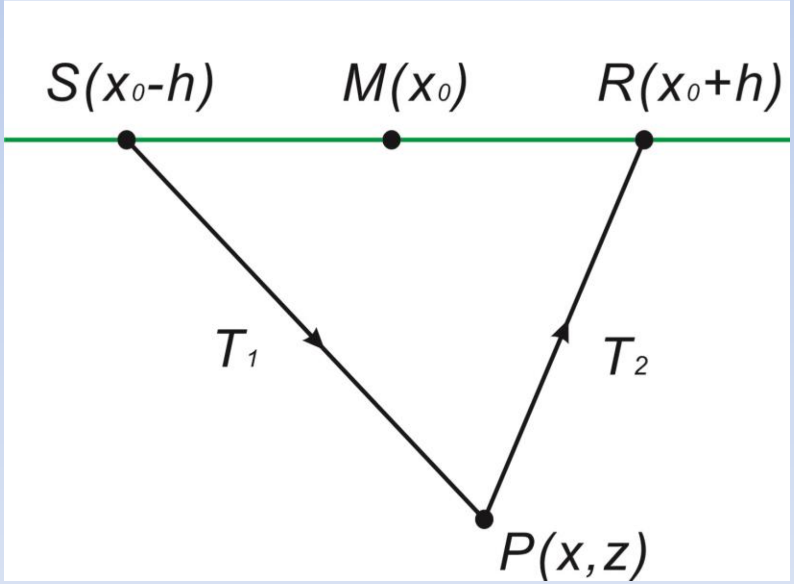


Figure 1 *Schematic plot of scattering ray propagating in homogeneous TTI media*

Outline

- **Pre-stack phase-shift migration in offset-midpoint domain for TTI media**
- **The slowness at a stationary point**
- **Slowness surface in a TTI medium**
- **Traveltime pyramid in TTI media**
- **Numerical Example**
- **Conclusions**

Pre-stack phase-shift migration in offset-midpoint domain for TTI media

Phase-shift migration operator in offset-midpoint domain

$$P(x,h=0,z,t=0)=\int d\omega \tilde{P}(x_0,h_0,z=0,\omega)\int dk_h \int dk_x \exp(-i\omega T)$$

where traveltime shift T is ,

$$T=(q_s+q_g)z+\boxed{2p_x(x-x_0)-2p_h h_0}$$

$$\boxed{p_s y_s + p_g y_g}$$

where,

- x_0, h_0 are midpoint and offset, respectively
- x, z are the lateral and vertical position of image point, respectively.
- P is seismic data after migration
- $\tilde{P}$ is seismic data before migration
- q_s, q_g are vertical projections of the slowness vector defined at source and receiver positions, respectively
- p_x, p_h are horizontal projections of the slowness vector defined in midpoint-offset space

The slowness at stationary point

- **Acoustic VTI slowness surface equation**

$$F_{VTI} = -2\eta\upsilon_0^2\upsilon_{nmo}^2 p_v^2 q_v^2 + \upsilon_0^2 q_v^2 + (1+2\eta)\upsilon_{nmo}^2 p_v^2 - 1 = 0$$

- **Stationary point equation for acoustic VTI media**

$$\frac{\partial T}{\partial p_v} = 0$$

p_v, q_v are horizontal and vertical slowness, respectively.

υ_0 is the vertical velocity

υ_{nmo} is the NMO velocity $\upsilon_{nmo} = \upsilon_0\sqrt{1+2\delta}$

δ Thomsen anisotropic parameter,

η anellipticity parameter $\eta = \dfrac{\varepsilon - \delta}{1+2\delta}$

- **Taylor expansion method to compute horizontal slowness**

$$p_v^2 = p_{v0}^2 + p_{v1}^2(2\eta) + p_{v2}^2(2\eta)^2 + \dots$$

The accuracy of p_v^2 can be improved by Shanks transformation

$$y = y_0 + \frac{2y_1^2\eta}{y_1 - 2y_2\eta}$$

- **Traveltime calculation at stationary point**

$$T(x, x_0, h, \tau) = 0.5\left(\sqrt{1-\frac{2\upsilon_{nmo}^2 p_s^2}{1-\upsilon_{nmo}^2\eta p_s^2}} + \sqrt{1-\frac{2\upsilon_{nmo}^2 p_g^2}{1-\upsilon_{nmo}^2\eta p_g^2}}\right)\tau + p_s y_s + p_g y_g$$

where

$$y_s = (x - x_0 + h_0) \qquad y_g = (x - x_0 - h_0)$$

Slowness surface in a TTI medium

The slowness surface in TTI and VTI

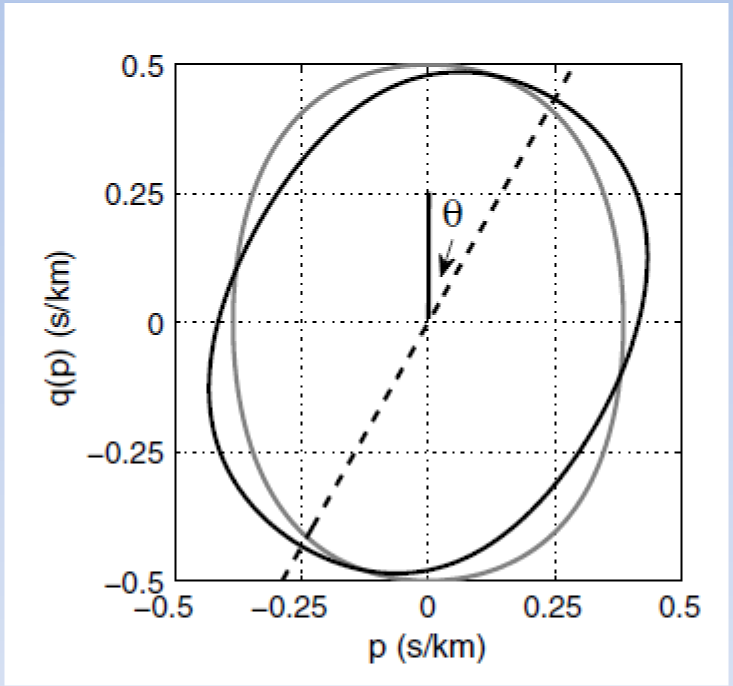


Figure 2 Slowness surfaces for VTI and TTI, respectively (from Golikov and Stovas, 2012).

$$p_v = p\cos\theta - q\sin\theta$$

$$q_v = p\sin\theta + q\cos\theta$$

$(p_v,\ q_v)$ Slowness in VTI media

$(p,\ q)$ Slowness in TTI media

The stationary point equation

$$\frac{\partial T}{\partial p} = 0 \quad \Rightarrow \quad \frac{dq}{dp} = -\frac{y}{z}$$

results in

$$\frac{dq_v}{dp_v} = -\frac{y\cos\theta - z\sin\theta}{z\cos\theta + y\sin\theta} = -a$$

Considering the VTI slowness surface, we obtain

$$p_v^2 \upsilon_{nmo}^4 - a^2 \upsilon_0^2 (-1 + 2 p_v^2 \upsilon_{nmo}^2 \eta)^3 (-1 + p_v^2 \upsilon_{nmo}^2 (1 + 2\eta)) = 0$$

$$a^2 q_v^2 \upsilon_0^4 - (1 - q_v^2 \upsilon_0^2) \upsilon_{nmo}^2 (1 + 2\eta - 2 q_v^2 \upsilon_0^2 \eta)^3 = 0$$

- **Trial Solutions -> expansions for slownesses**

$$p_v = p_{v0} + p_{v1}(2\eta) + p_{v2}(2\eta)^2 + \ldots$$

$$q_v = q_{v0} + q_{v1}(2\eta) + q_{v2}(2\eta)^2 + \ldots$$

- **Rotation operator**

$$p = p_0 + p_1(2\eta) + p_2(2\eta)^2 + \ldots$$

where

$$p_i = p_{vi} cos\theta + q_{vi} sin\theta \quad i = 0,1,2,$$

- **Vertical slowness approximation** (Stovas and Alkhalifah, 2012)

$$q = q_0 + \frac{2q_1^2\eta}{q_1 - 2q_2\eta}$$

where the coefficients q_i , *i=0*, 1 2, are the first- and second-order perturbation coefficients. This equation could be used to calculate vertical slowness for source and receiver. For a given horizontal slowness , we can evaluate two vertical slownesses corresponding to down- and up-ward going waves.

Traveltime Pyramid in TTI media

Traveltime shift:

$$T=(q_s+q_g)z+2p_x(x-x_0)-2p_h h_0$$

slowness relation between offset-midpoint and source-receiver domains

$$T=(q_s+q_g)z+p_s y_s+p_g y_g$$

Vertical slowness approximation from Stovas and Alkhalifah (2012)

traveltime pyramid in depth-domain:

$$T(x,x_0,h,z)=\left(q_{s0}+\frac{2q_{s1}^2\eta}{q_{s1}-2q_{s2}\eta}+q_{g0}+\frac{2q_{g1}^2\eta}{q_{g1}-2q_{g2}\eta}\right)z+p_s y_s+p_g y_g$$

This equation describes the traveltime pyramid in depth domain for TTI media, which is also called the Cheop's pyramid.

Setting $h=0$ and $x=x_0$, we obtain

$$\tau = T(x, x_0 = x, h=0, z) = 2q_{z0}z$$

$$q_{z0} = \frac{1}{2}\left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1}-2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1}-2q_{g2}\eta} \right)\Bigg|_{x=x_0,\ h=0}$$

traveltime pyramid in time-domain:

$$T(x, x_0, h, \tau) = \frac{1}{2q_{z0}}\left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1}-2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1}-2q_{g2}\eta} \right)\tau + p_s y_s + p_g y_g$$

This equation describes the Cheop's pyramid in time domain for TTI media.

Numerical example

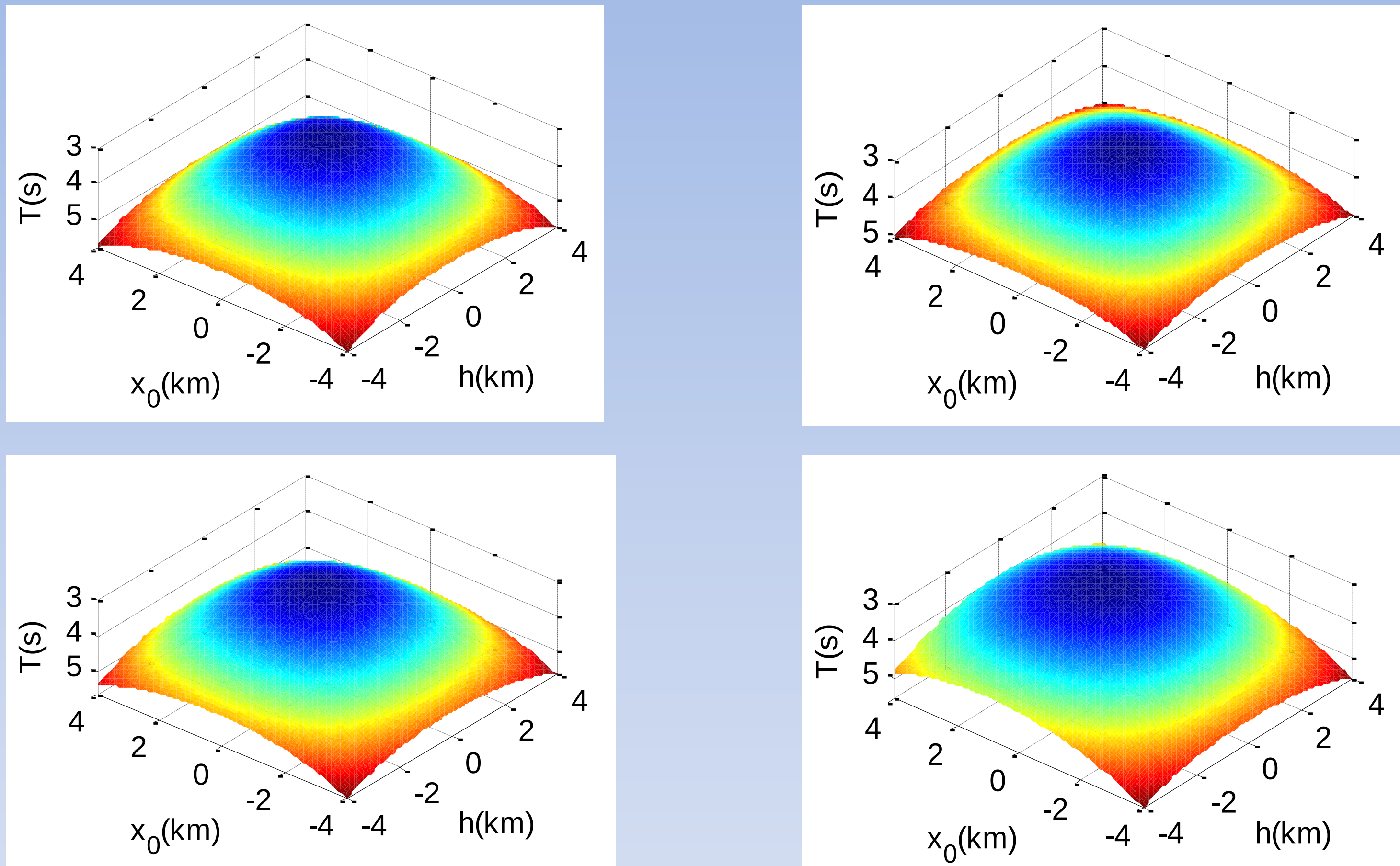
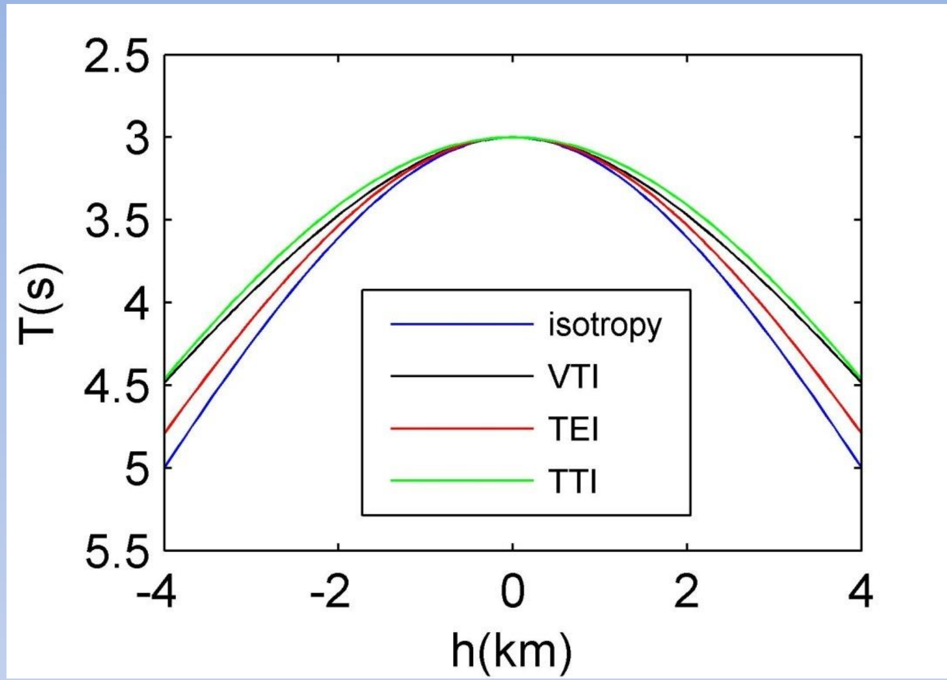
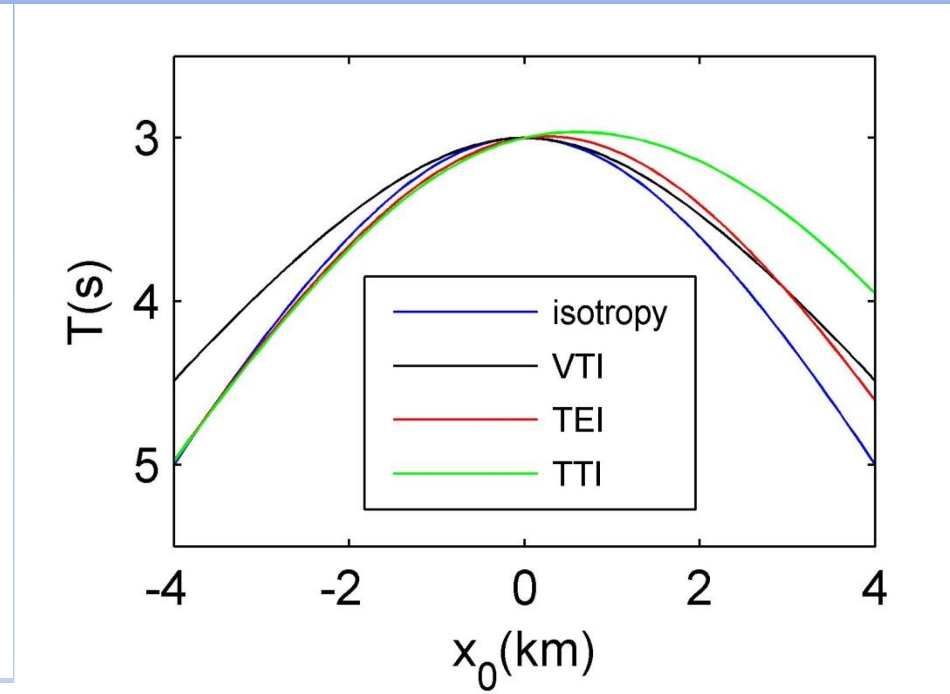


Figure 3 *Traveltime as a function of half offset* h *and midpoint* x_0 *for isotropic case (top left), VTI case with* η *=0.2,* δ *=0.1 (top right), tilted elliptical isotropic(TEI) medium with* η *=0,* δ *=0.1 and* θ *=*30^0 *(bottom left) and TTI case with* η *=0.2,* δ *=0.1 and* θ*=*30^0 *(bottom right). On-axis velocity is 2km/s. The two-way zero-offset traveltime is 3s.*





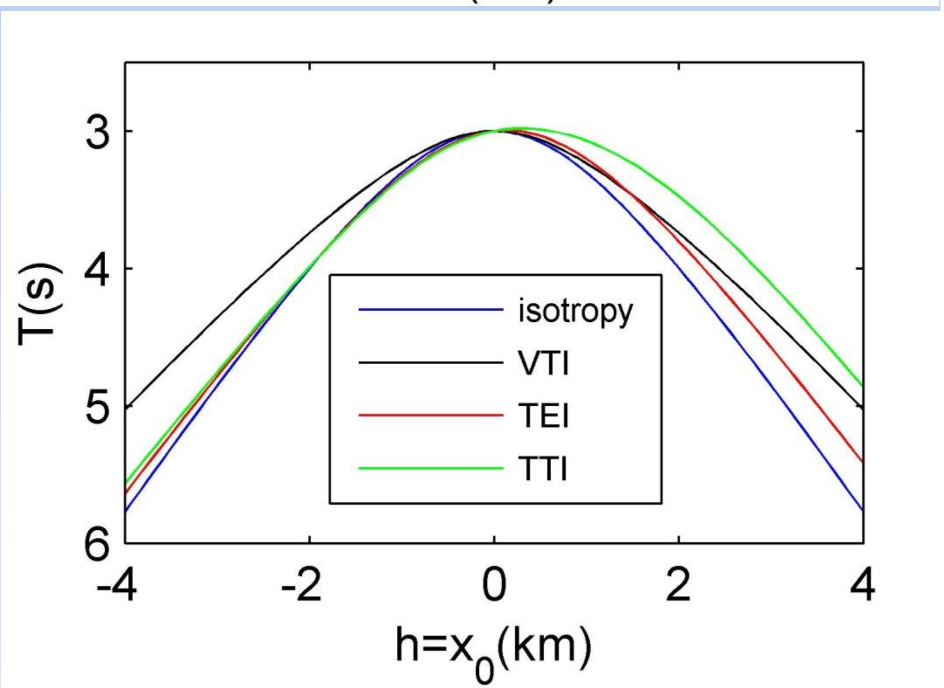


Figure 4 *Comparison of slices extracted from traveltime pyramids for x_0=0 (top left), h=0 (top right) and x_0=h (bottom right) in figure 3.*

Conclusions

- The offset-midpoint traveltime equation for TTI media is derived using the stationary phase method.
- Perturbation in anisotropic parameter η and the following Shanks transformation are involved to derive a relatively simple analytical form for the offset-midpoint traveltime pyramid in depth and time domain.

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