

# Elastic dispersion derived from a combination of static and dynamic measurements

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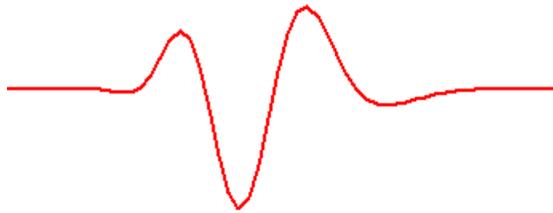
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## Long term challenge:

- measuring acoustic velocities at seismic frequencies in the laboratory



Wavelength  $\gg$  sample size  
 $\Rightarrow$  no travelling wave

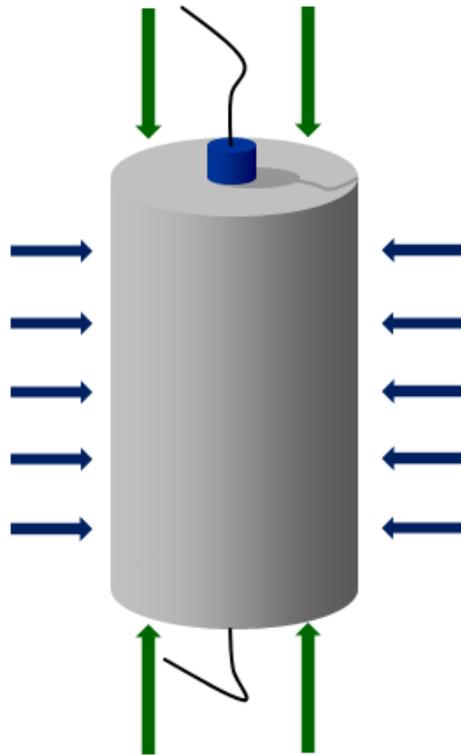
Extremely small deformations ( $\epsilon \sim 10^{-7}$ )  
 $\Rightarrow$  resolution is a problem

Stress path vs wave mode  
 $\Rightarrow$  anisotropy can be a problem

## We claim that:

- velocity dispersion between ultrasonic and seismic frequencies can be estimated from a standard rock mechanical test set-up with acoustic velocity measurements

Laboratory tests:



## Standard triaxial set-up + acoustics

Measurements:

Stress

Strain

Acoustic wave velocities

Enables simultaneous measurements of

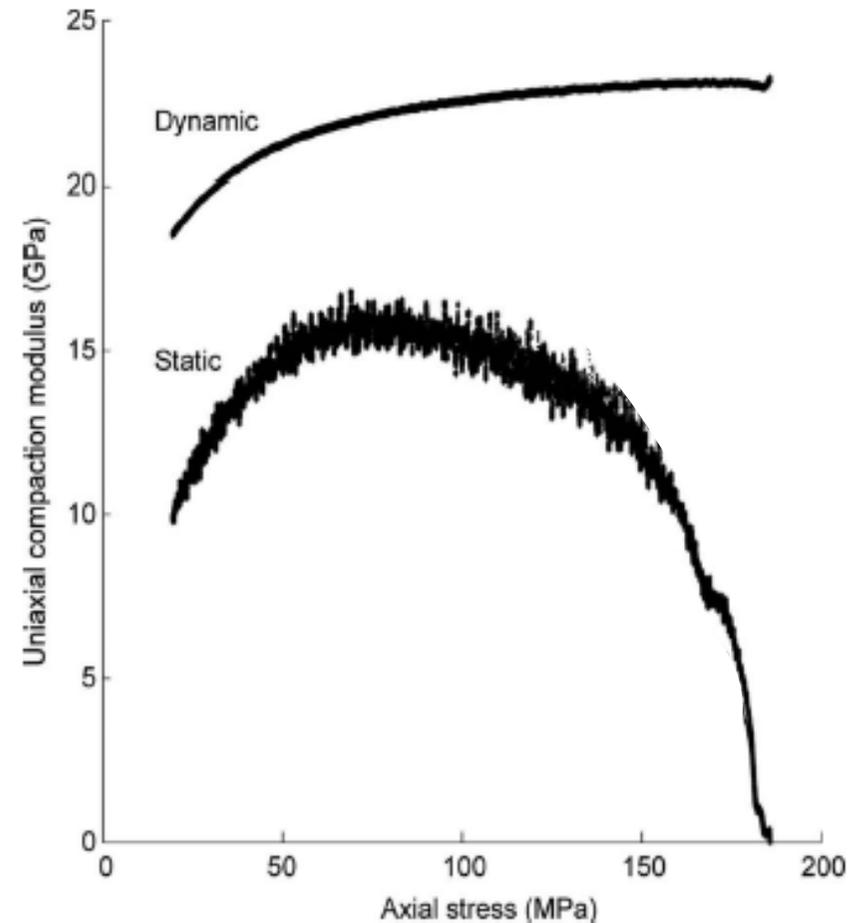
- static moduli (slope of stress-strain curve)
- dynamic moduli (density x velocity<sup>2</sup>)

# Static and dynamic moduli of soft rocks are different.

The difference changes along the stress path.

Potential causes for the difference between static and dynamic moduli:

- Strain rate
- Rock volume involved
- Drainage conditons
- Anisotropy
- Non-elastic processes



# Measures to isolate strain rate effects

## 1. Homogeneous rock

No wavelength effect

Same rock involved for both static and dynamic measurements

Potential causes for the difference between static and dynamic moduli:

- Strain rate
- ~~Rock volume~~ involved
- Drainage conditons
- Anisotropy
- Non-elastic processes



# Measures to isolate strain rate effects

## 2. Consistent drainage conditions

Static moduli: usually drained

Dynamic moduli: undrained

⇒ dry, partially saturated, or undrained

Potential causes for the difference between static and dynamic moduli:

- Strain rate
- ~~Rock volume involved~~
- ~~Drainage conditons~~
- Anisotropy
- Non-elastic processes



# Measures to isolate strain rate effects

## 3. Stress path: $K_0$

Static modulus = slope of stress-strain curve:

$$H = \frac{\Delta\sigma_z}{\Delta\varepsilon_z}$$

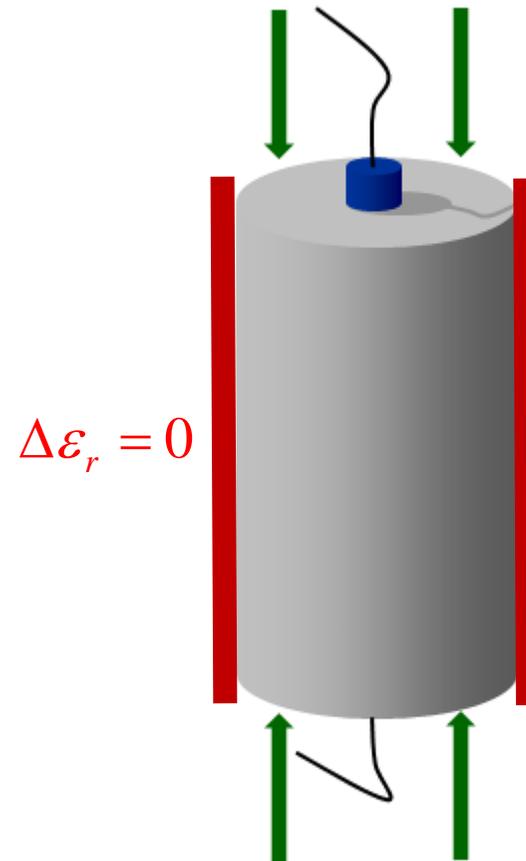
Dynamic modulus given by axial P-wave velocity:

$$H_e = \rho V_P^2$$

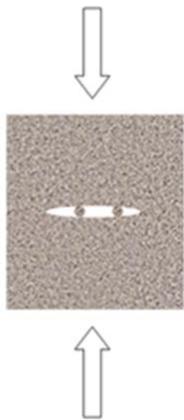
⇒ Same modulus, static and dynamic

Potential causes for the difference between static and dynamic moduli:

- Strain rate
- ~~Rock volume involved~~
- ~~Drainage conditons~~
- ~~Anisotropy~~
- Non-elastic processes

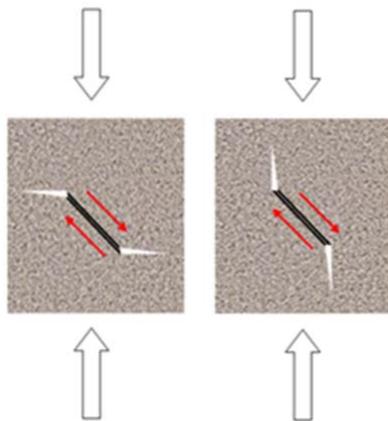


## Non-elastic processes causing differences between static and dynamic moduli in dry rocks:



Crushing of asperities at grain contacts or crack faces

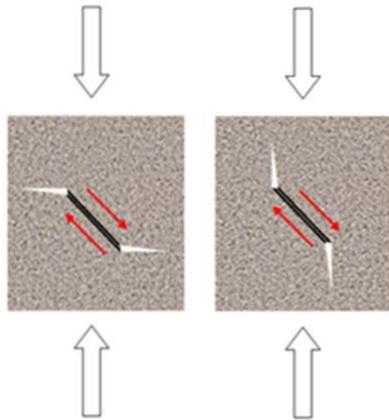
Occurs only during loading



Friction controlled shear sliding of closed cracks  
(accompanied by opening or closing of "wing cracks")

Occurs both during loading and unloading

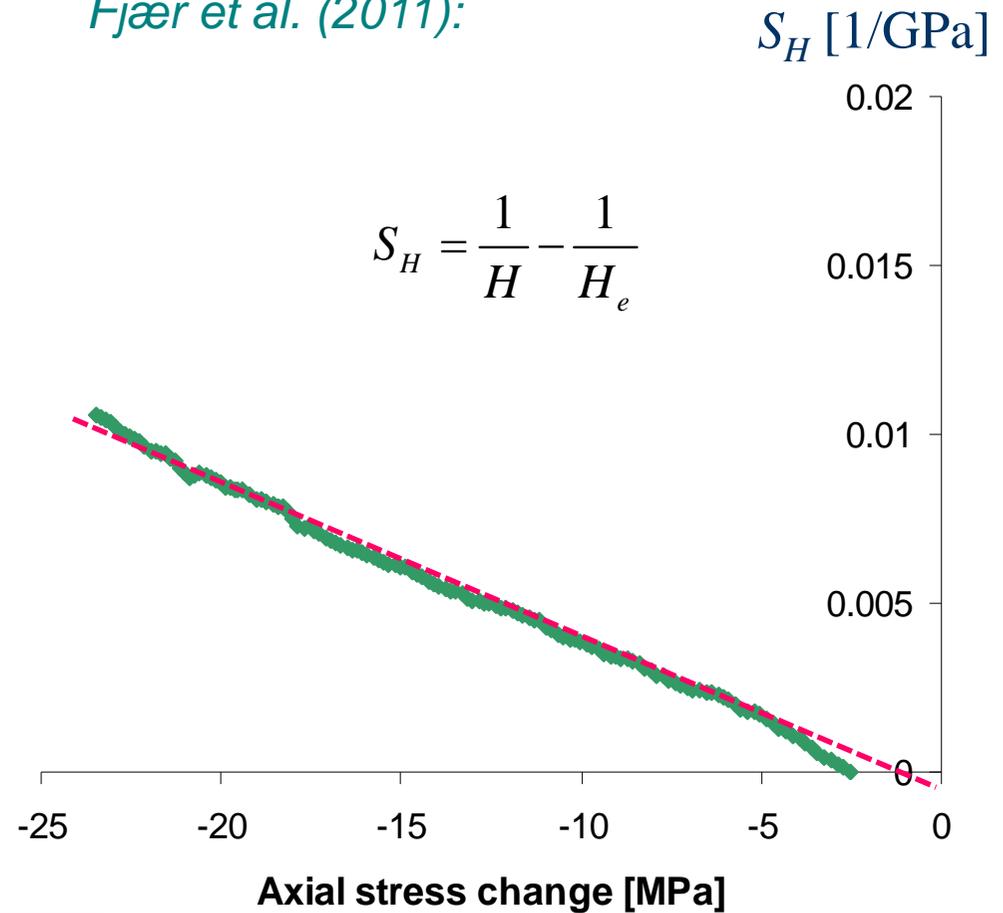
Unloading:



Non-elastic processes

*Fjær et al. (2011):*

$$S_H = \frac{1}{H} - \frac{1}{H_e}$$



Observation:

Non-elastic compliance increases linearly with decreasing stress

$$S_H = \frac{1}{H} - \frac{1}{H_e}$$

# Measures to isolate strain rate effects

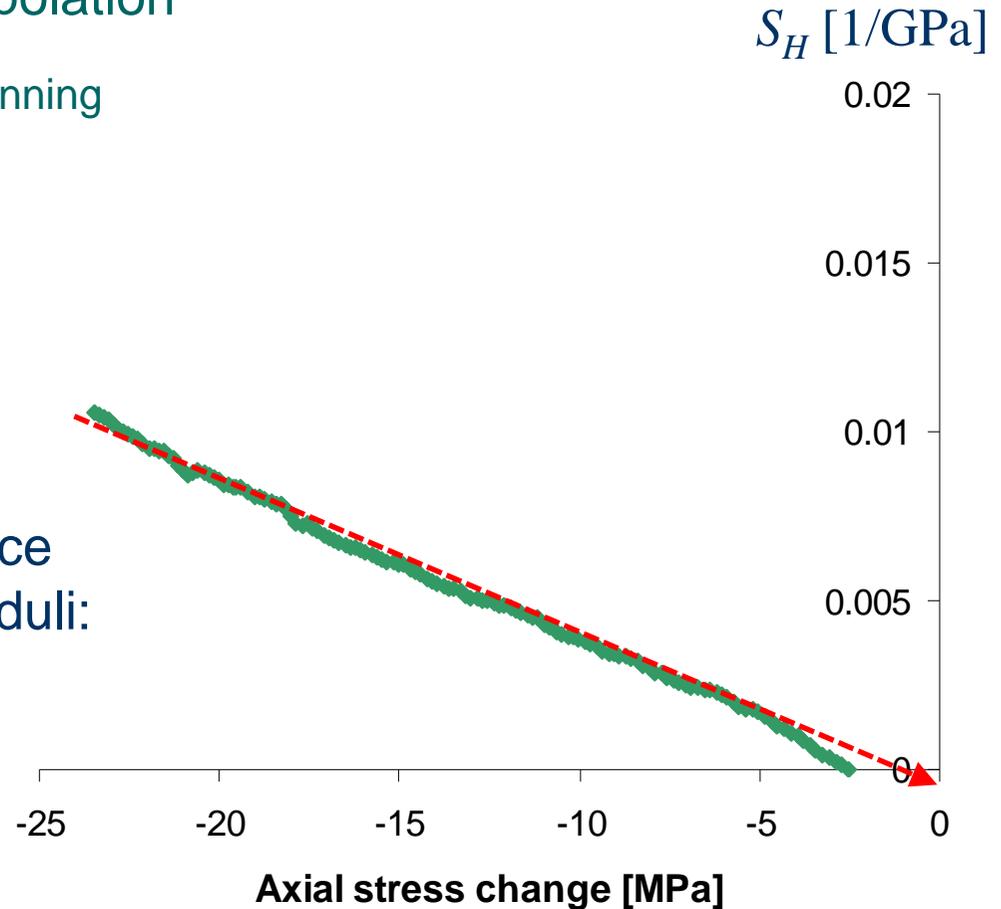
## 4. Unloading & backwards extrapolation

Linear extrapolation towards the beginning of the unloading path eliminates non-elastic contributions

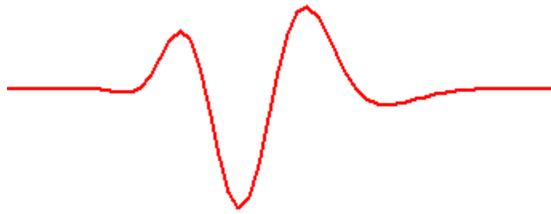
$$\rightarrow H^{tp}$$

Potential causes for the difference between static and dynamic moduli:

- Strain rate
- ~~Rock volume involved~~
- ~~Drainage conditons~~
- ~~Anisotropy~~
- ~~Non-elastic processes~~



## Strain rate



Average strain rate for dynamic measurements:

$$\langle \dot{\varepsilon} \rangle = 4 f \Delta \varepsilon$$

$$\Delta \varepsilon \sim 10^{-7} \quad = \text{strain amplitude}$$

$$f = 5 \cdot 10^5 \text{ Hz} \quad = \text{frequency}$$

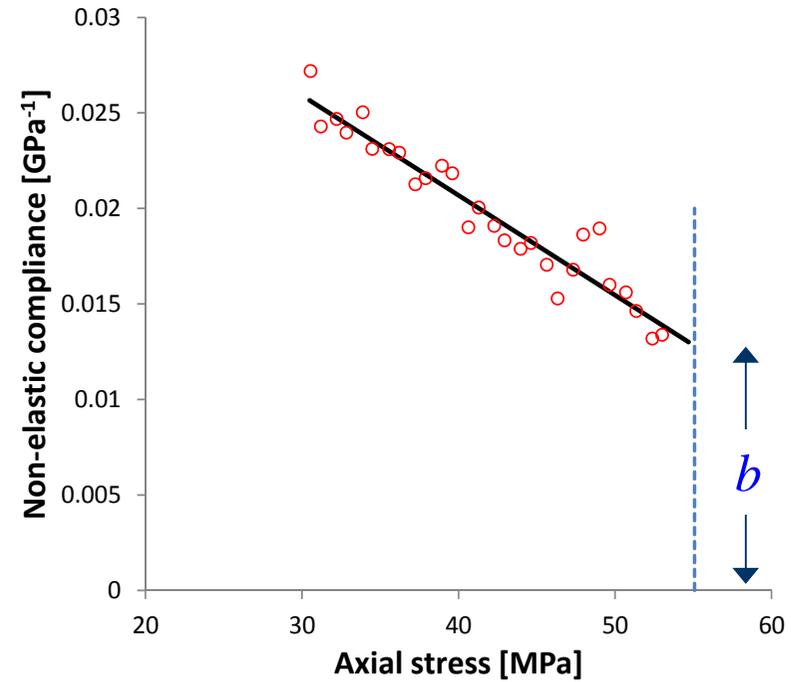
$$\Rightarrow \langle \dot{\varepsilon} \rangle \approx 10^{-1} \text{ s}^{-1}$$

# Mancos shale

Partly saturated,  
with 13% illite/smectite,  
5% kaolinite, etc.

– probably significant dispersion

$$V_{P,Low} = \frac{V_{P,High}}{\sqrt{1 + b\rho V_{P,High}^2}}$$



## Main sources of uncertainty:

$\Delta C_{33}$  (static stiffness)

– related to stress and strain measurements

$\Delta V_{P,High}$  (ultrasonic velocity)

– related to ~~calibration~~ and travelttime measurements

$\Delta b$  (linear trendline parameter)

– related to fluctuations in the non-elastic compliance

$\Delta \rho$  (density)

– related to weight and volume measurements

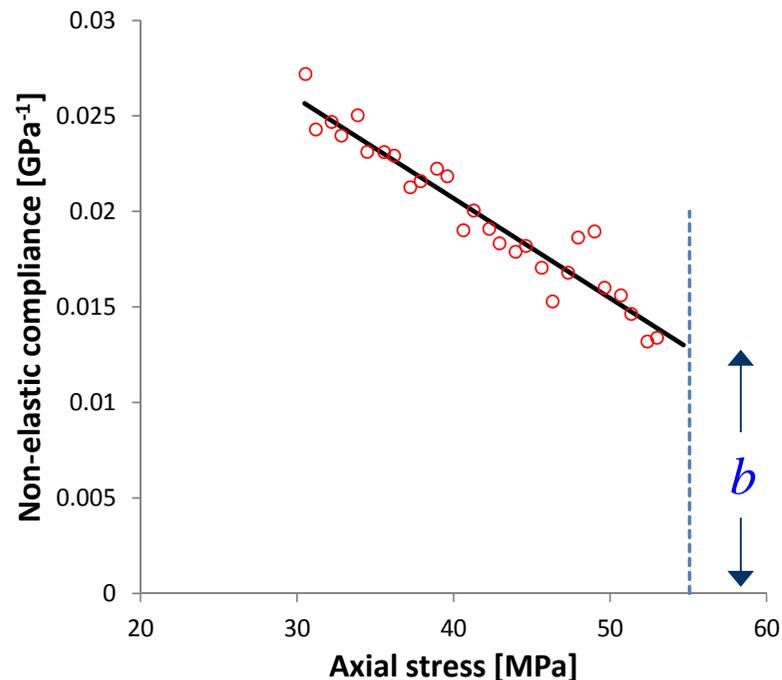
Comparing  
repeated measurements

# Mancos shale

Partly saturated,  
with 13% illite/smectite,  
5% kaolinite, etc.

– probably significant dispersion

$$V_{P,Low} = \frac{V_{P,High}}{\sqrt{1 + b\rho V_{P,High}^2}}$$



$\sigma_z$ MPa	$\sigma_r$ MPa	$b$ $10^{-4} \text{ GPa}^{-1}$	$V_{P,High}$ m/s	$V_{P,High} - V_{P,Low}$ m/s
55	18	$129 \pm 12$	$4\,163 \pm 42 (\pm 8)$	$846 \pm 148 (\pm 114)$

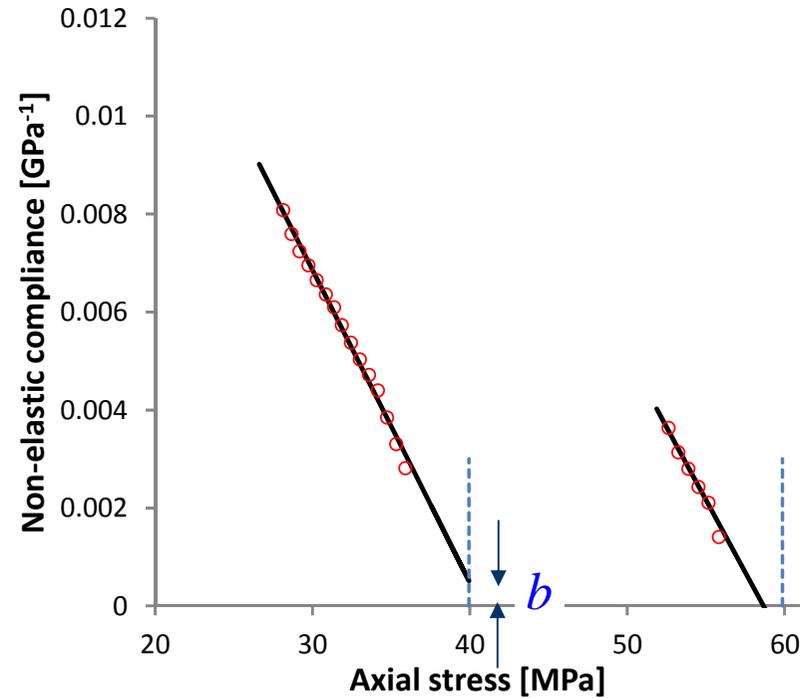
Significant dispersion, far beyond the resolution limit for the method

# Castlegate sandstone

Dry, clay free

– presumably no significant dispersion

$$V_{P,Low} = \frac{V_{P,High}}{\sqrt{1 + b\rho V_{P,High}^2}}$$



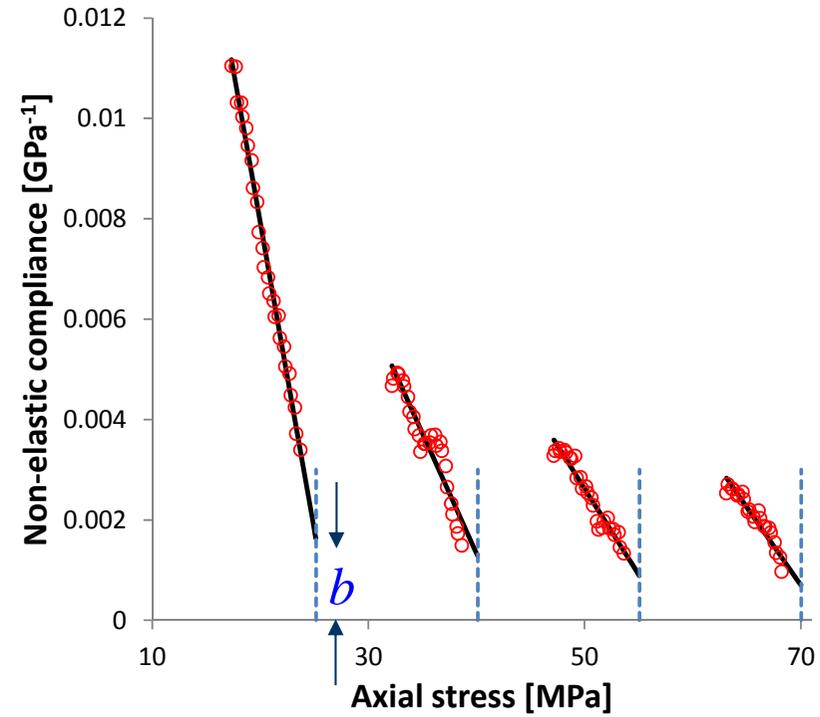
No measurable dispersion

# Berea sandstone

Dry, 8% clay

– possibly some dispersion

$$V_{P,Low} = \frac{V_{P,High}}{\sqrt{1 + b\rho V_{P,High}^2}}$$

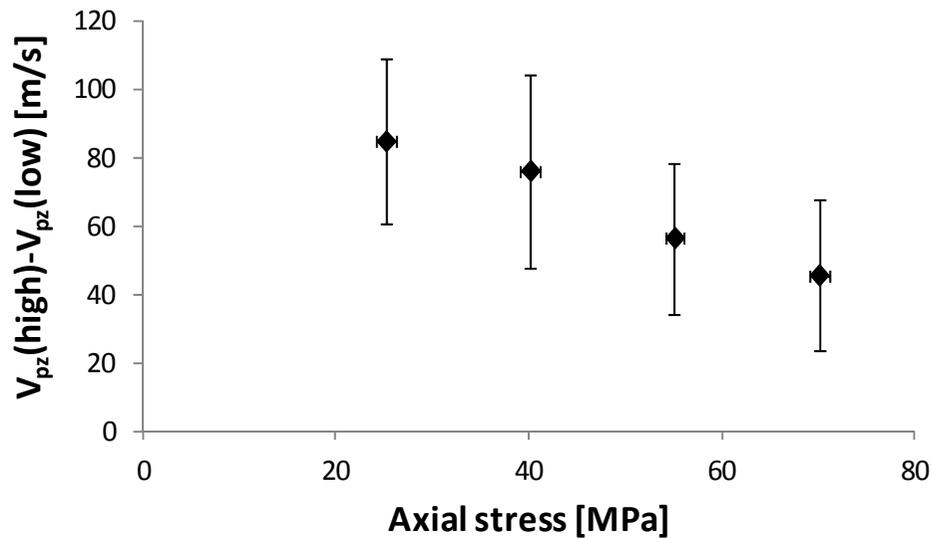
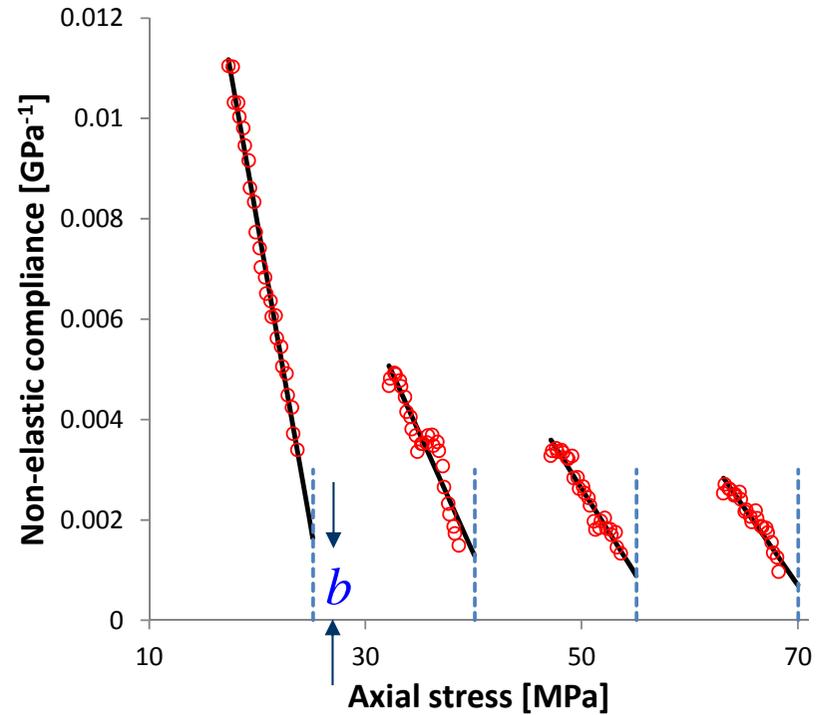


# Berea sandstone

Dry, 8% clay

– possibly some dispersion

$$V_{P,Low} = \frac{V_{P,High}}{\sqrt{1 + b\rho V_{P,High}^2}}$$



Significant,  
measurable dispersion,  
decreasing with increasing  
stress

## Conclusions:

- We argue that velocity dispersion between ultrasonic and seismic frequencies can be estimated from a standard rock mechanical test with acoustic velocity measurements.
- The demand for accuracy is not extreme. Standard, good quality measurements is sufficient.
- Application of this method on two sandstone and one shale sample indicates that dispersion increases with clay content and decreases with stress. Intrinsic dispersion associated with clay, and patchy saturation, are potential causes.

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