



Statoil

Module 2: Exorcizing seismic ghosts

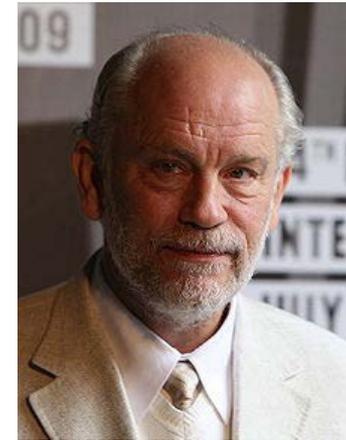
Lasse Amundsen and Martin Landrø

April 24, 2013

Geophysicists do not lack self confidence, but will we ever solve the deghosting problem?

“I wouldn’t describe myself as lacking in confidence, but I would just say that – the ghosts you chase you never catch“

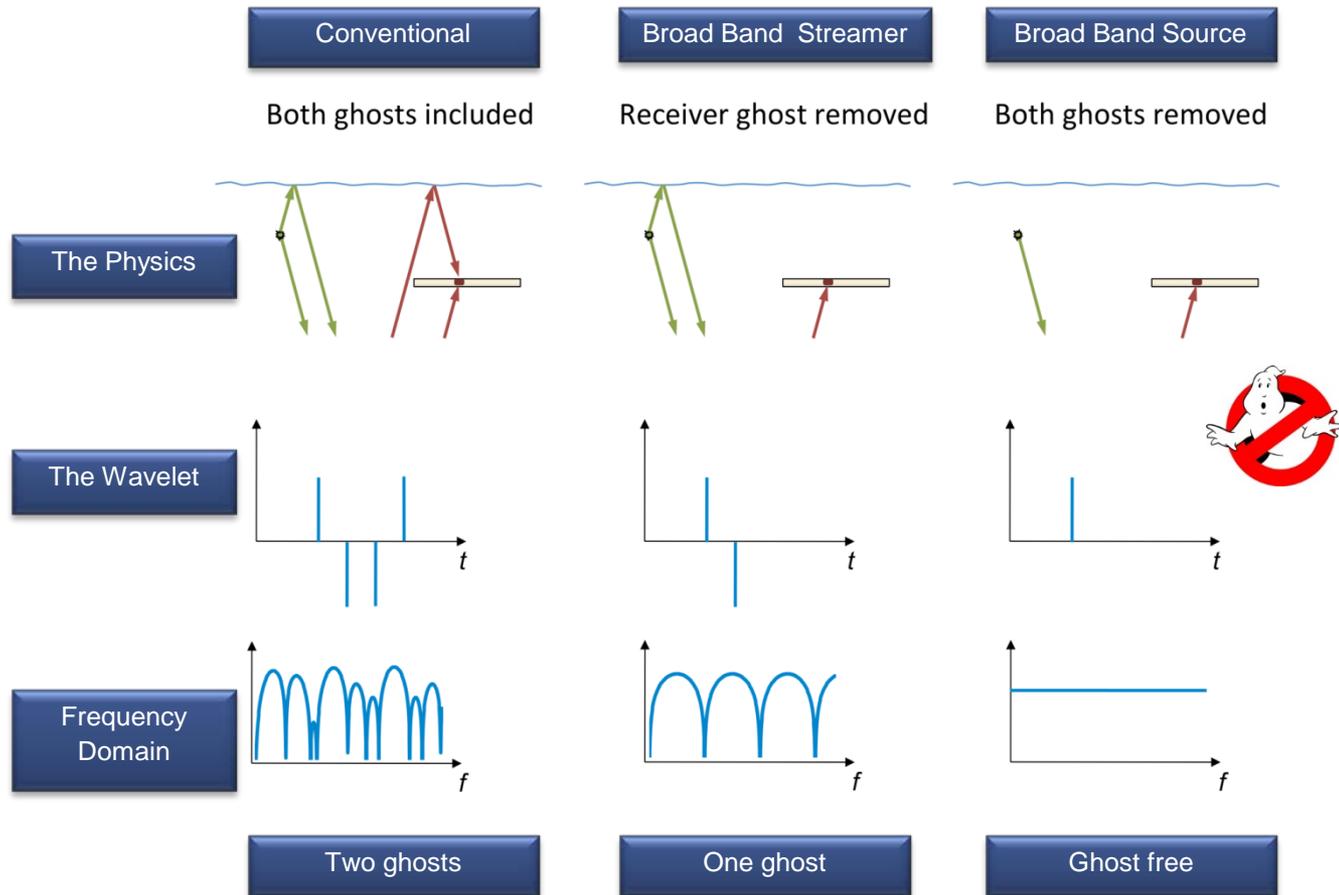
John Malkovich (1953–)



To understand how geophysical priests exorcize seismic ghosts, either through developing sensor technologies or new data processing methods, you need to understand what ghosts are and how they bandlimit seismic data

Long-standing, and today “hot” research area.

Ghosts and de-ghosting



A source ghost is an event starting its propagation upward from the source, and a receiver ghost ends its propagation moving downward at the receiver. They both have a reflection at the sea surface, which leads to a reduction of the useful frequency bandwidth and therefore damages seismic resolution.

Some history

- The effect of the sea or land surface is a well-known obstacle in seismic exploration and has been addressed since the outset of reflection seismology (Leet, 1937)
- Van Melle and Weatherburn (1953) dubbed the reflections from energy initially reflected above the level of the source, by optical analogy, 'ghosts'
- Lindsey (1960) the first (?) to present a ghost removal or deghosting solution by observing that a downgoing source signal followed by a ghost with specified time lag can be eliminated by designing a positive feedback loop
- Ghost function G can be eliminated theoretically by applying the inverse filter $D = 1/G$ to the data: $D G = 1$
- *More history in BroadBand Seismic Module*

CONTENTS

- Wave equations 3D, 2D, 1D
- Wavenumbers, slownesses, angles
- Acoustic 4C measurements
- Solution of 3D, 2D, 1D Helmholtz equation in frequency-wavenumber domain
 - Homogeneous medium
 - Homogeneous medium bounded by free surface
 - Layered medium
- Pressure and vertical component of particle velocity
- Ghosts on hydrophones and on vertical geophones
- Deghosting methods, incl GeoStreamer and IsoMetrix

Wave equations 3D, 1D, 2D

$$\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} + k^2 \right) P(x, y, z) = -a(\omega) \delta(x - x_s) \delta(y - y_s) \delta(z - z_s)$$

$$\left(\frac{d^2}{dx^2} + \frac{d^2}{dz^2} + k^2 \right) P(x, z) = -a(\omega) \delta(x - x_s) \delta(z - z_s)$$

$$\left(\frac{d^2}{dz^2} + k^2 \right) P(z) = -a(\omega) \delta(z - z_s)$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c}$$

Fourier transform over x and y to k_x and k_y reduces WEs to 1D Helmholtz equation

$$\left(\frac{d^2}{dz^2} + k_z^2 \right) p(z) = -a(\omega) \delta(z - z_s)$$

Plane wave decomposition of spherical/cylindrical wavefield

Dispersion relation

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \quad 3D$$

$$k_z = \sqrt{k^2 - k_x^2} \quad 2D$$

$$k_z = k \quad 1D$$

Acoustic 4C

$$\rho \partial_t v_x = -\partial_x p$$

$$\rho \partial_t v_y = -\partial_y p$$

$$\rho \partial_t v_z = -\partial_z p$$

In seismic (acoustic) multicomponent acquisition, the spatial derivatives of the pressure may be directly calculated from the measured particle velocities or particle accelerations (Robertsson et al Geophysics 2008)

p and vz measured on GeoStreamer

p, vz and vy measured on Nessie-6

Hydrophones measure pressure (p) changes, while geophones and accelerometers are sensitive to particle motion. The hydrophone is omnidirectional and measures the sum of upgoing and downgoing pressure waves. The vertically oriented geophone (vz) has directional sensitivity and measures their difference.

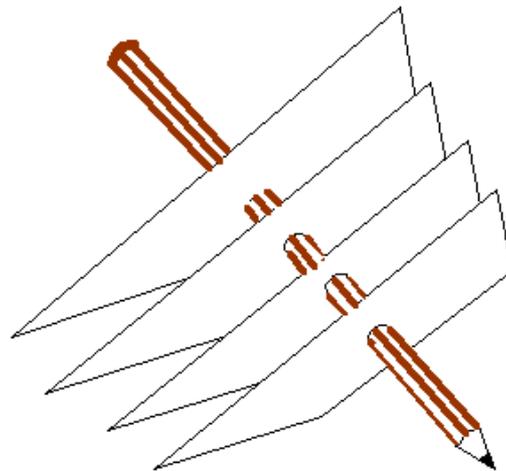
Wavenumbers, slowness, angles ...

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} \quad \text{wavenumber, } \omega \text{ circular frequency, } f \text{ frequency, } c \text{ velocity}$$

k_x, k_y horizontal wavenumbers, $k_y = 0$ in 2D

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \quad \text{vertical wavenumber}$$

In a lossless isotropic medium the direction of the wavevector $\mathbf{k}=(k_x, k_y, k_z)$ is the same as the direction of wave propagation



The pencil represents the direction of the \mathbf{k} -vector. The pencil can point in any direction in space, defined by its Cartesian components: k_x , k_y and k_z .

Wavenumbers, slowness, angles ...

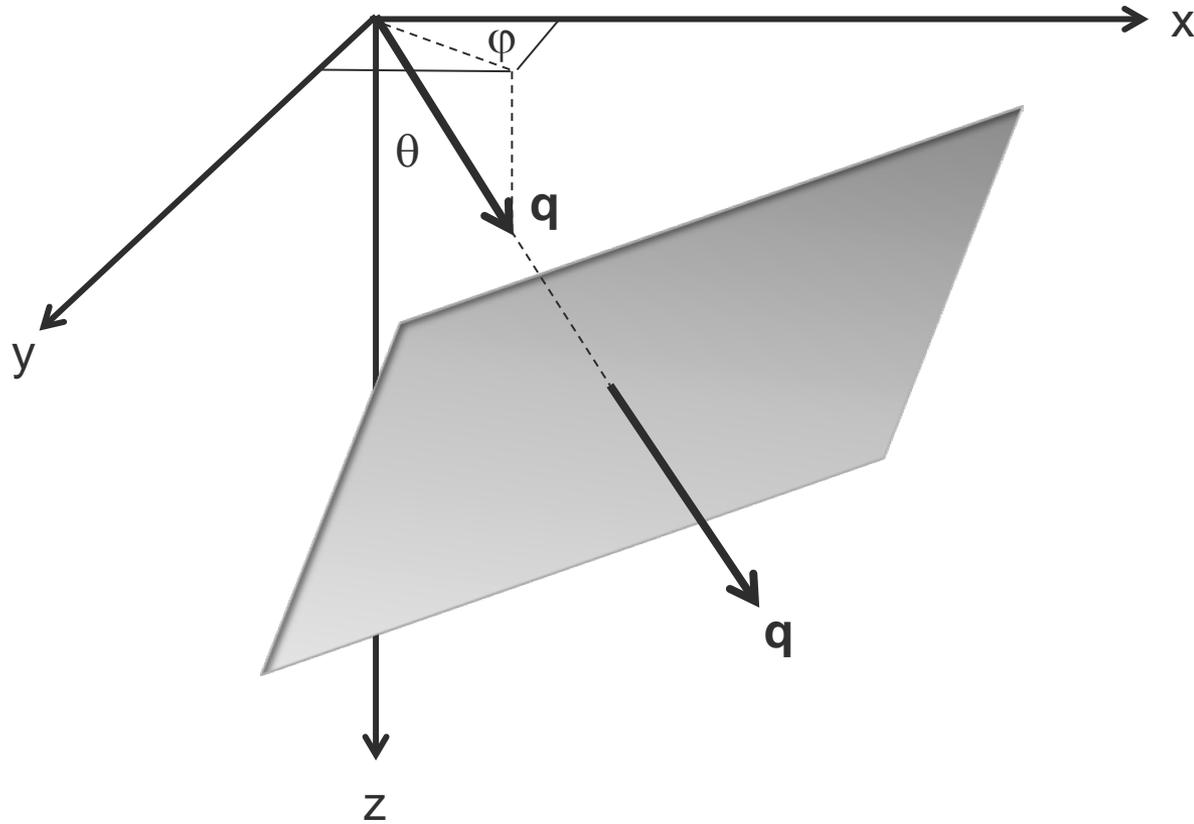
The slowness of a single plane wave is related to incidence (dip) angle θ to the vertical and azimuth φ

$$q_x = \frac{k_x}{\omega} = \frac{dt}{dx} = \frac{1}{c} \cos \varphi \sin \theta \quad \text{horizontal slowness, } q_x = \frac{1}{c} \sin \theta \text{ in 2D}$$

$$q_y = \frac{k_y}{\omega} = \frac{dt}{dy} = \frac{1}{c} \sin \varphi \sin \theta \quad \text{horizontal slowness, } q_y = 0 \text{ in 2D}$$

$$q_z = \frac{k_z}{\omega} = \frac{dt}{dz} = \sqrt{c^{-2} - q_x^2 - q_y^2} = \frac{1}{c} \cos \theta \quad \text{vertical slowness}$$

Plane wave front, perpendicular to slowness vector \mathbf{q}



Fourier transform sign convention

The sign is up to you to choose, but the inverse transform has to have the opposite sign. For the inverse Fourier transform, our choice is

$$f(t, x, y) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega dk_x dk_y \exp\left(-i(\omega t - k_x x - k_y y)\right) F(\omega, k_x, k_y)$$

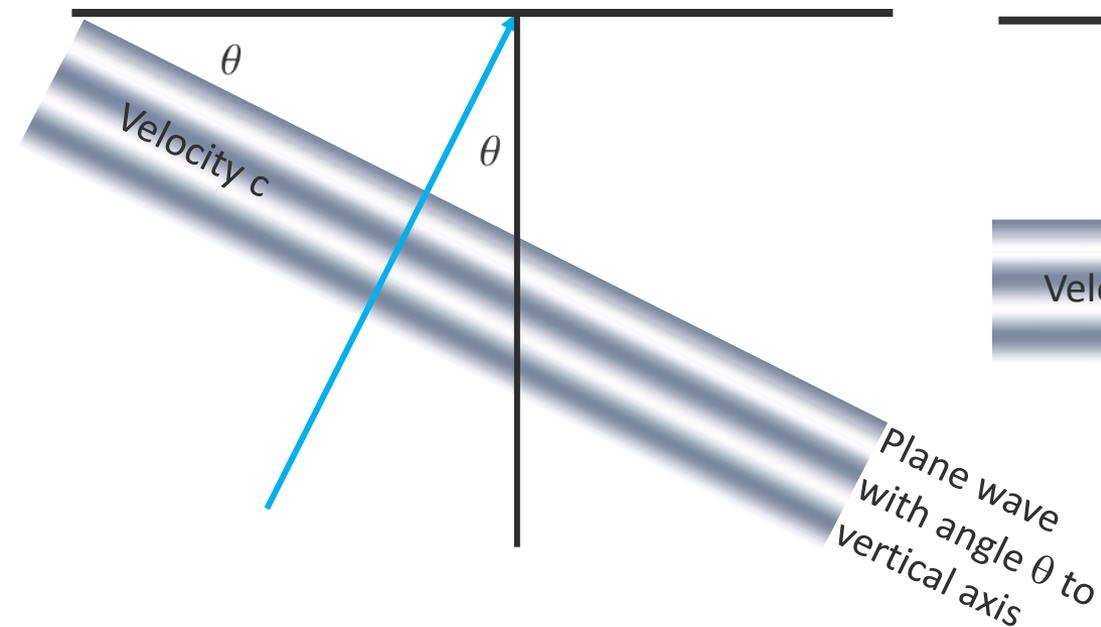
This integrand can be interpreted, in the case of positive k_x , k_y , and ω , as a wave propagating in the positive directions of x and y .

Let z be positive downwards. In a homogeneous medium, the solutions to the Helmholtz equation are

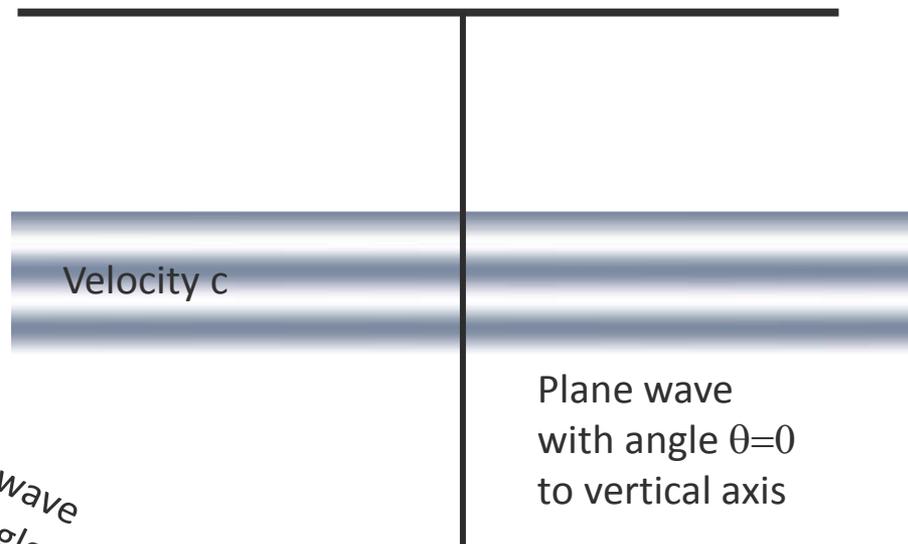
$$P(k_x, k_y, z, \omega) \propto \exp(\pm i k_z z)$$

representing downgoing (+) and upgoing (-) waves.

Plane waves (2D) traveling along the vertical axis



$$k_x = \frac{\omega}{c} \sin \theta$$
$$k_z = \frac{\omega}{c} \cos \theta$$



$$k_x = 0$$
$$k_z = \frac{\omega}{c}$$

1D wave equation (Helmholtz equation)

Solution homogeneous medium with source

1D WE

$$\left(\frac{d^2}{dz^2} + k_z^2 \right) P(z) = -a(\omega) \delta(z - z_s)$$

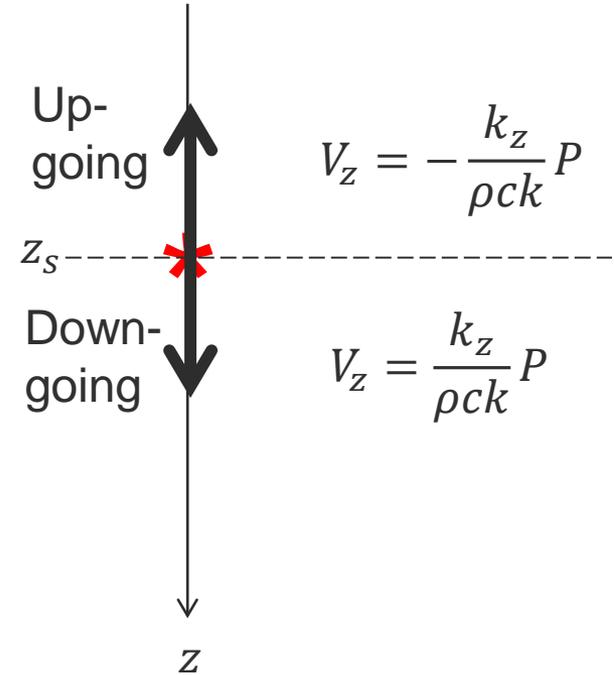
Pressure field for monopole point source with wavelet $a(\omega)$ is omni-directional.

$$P(z) = \exp(ik_z|z - z_s|) s$$

$$s = -\frac{a(\omega)}{2ik_z} \text{ source radiation pattern}$$

Associated particle velocity

$$V_z(z) = (i\omega\rho)^{-1} \frac{dP(z)}{dz} = \frac{k_z}{\rho ck} \text{sign}(z - z_s) P(z)$$



Solution homogeneous medium bounded by free surface $P(z=0)=0$

$$P(z) = [\exp(ik_z|z - z_s|) - \exp(ik_z(z + z_s))] s$$

Incident wave

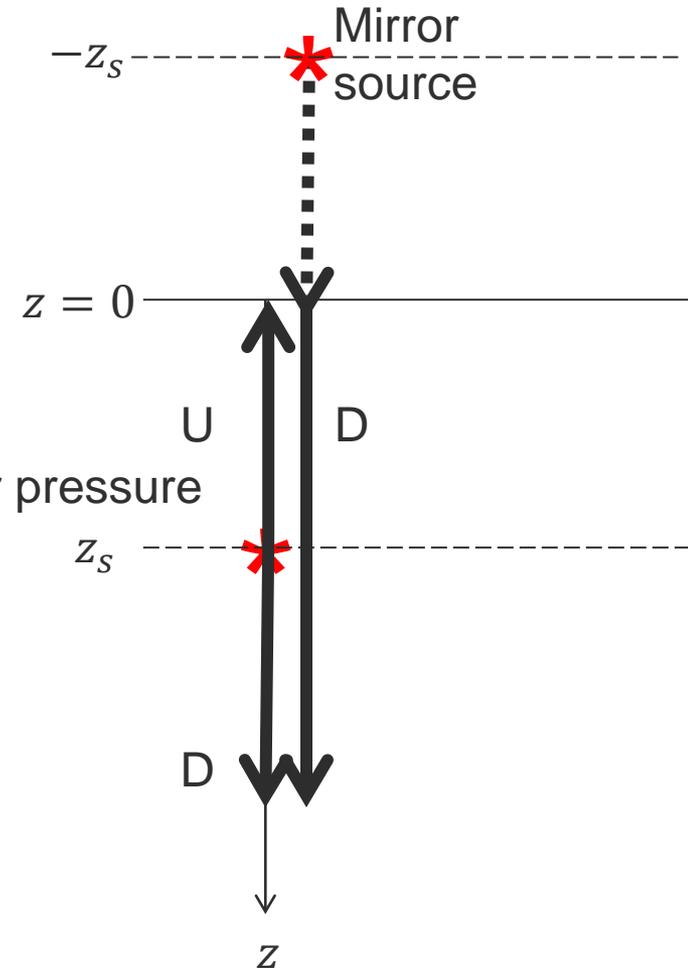
Ghost

$$0 < z < z_s: P(z) = \exp(ik_z(z_s - z)) G_-(z) s$$

$G_-(z) = 1 - \exp(2ik_z z)$ receiver ghost function for pressure

$$z > z_s: P(z) = \exp(ik_z(z - z_s)) G_-(z_s) s$$

$$G_-(z_s) = 1 - \exp(2ik_z z_s) \text{ source ghost function}$$



Solution homogeneous medium bounded by free surface $P(z=0)=0$

Consider $0 < z < z_s$

$$P(z) = \exp(ik_z(z_s - z)) G_-(z) s$$

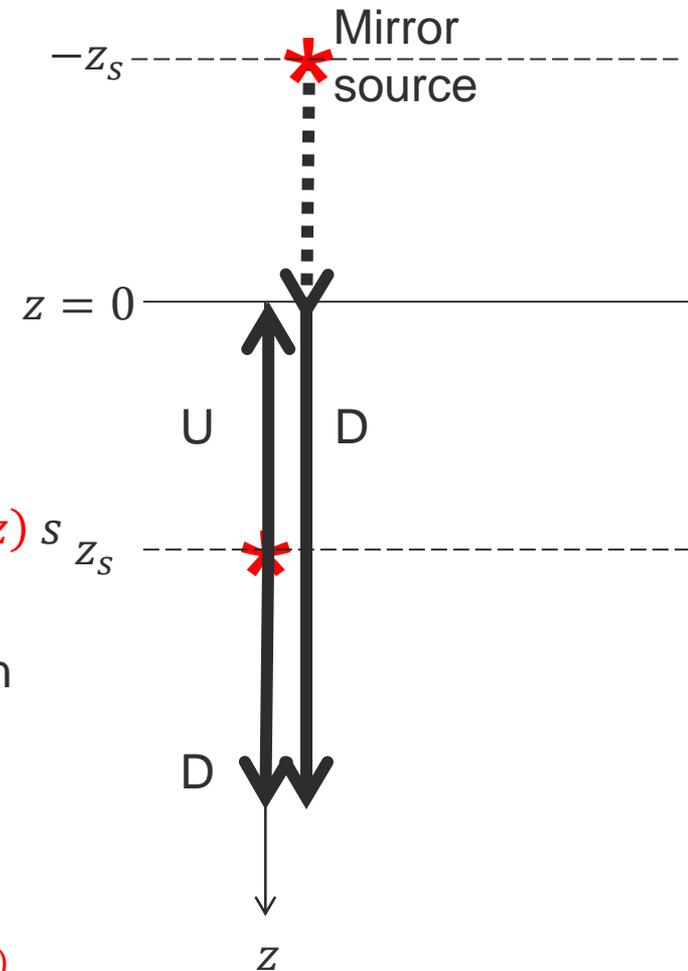
$G_-(z) = 1 - \exp(2ik_z z)$ receiver ghost function for pressure

$$V_z(z) = (i\omega\rho)^{-1} \frac{dP(z)}{dz} = -\frac{k_z}{\rho ck} \exp(ik_z(z_s - z)) G_+(z) s$$

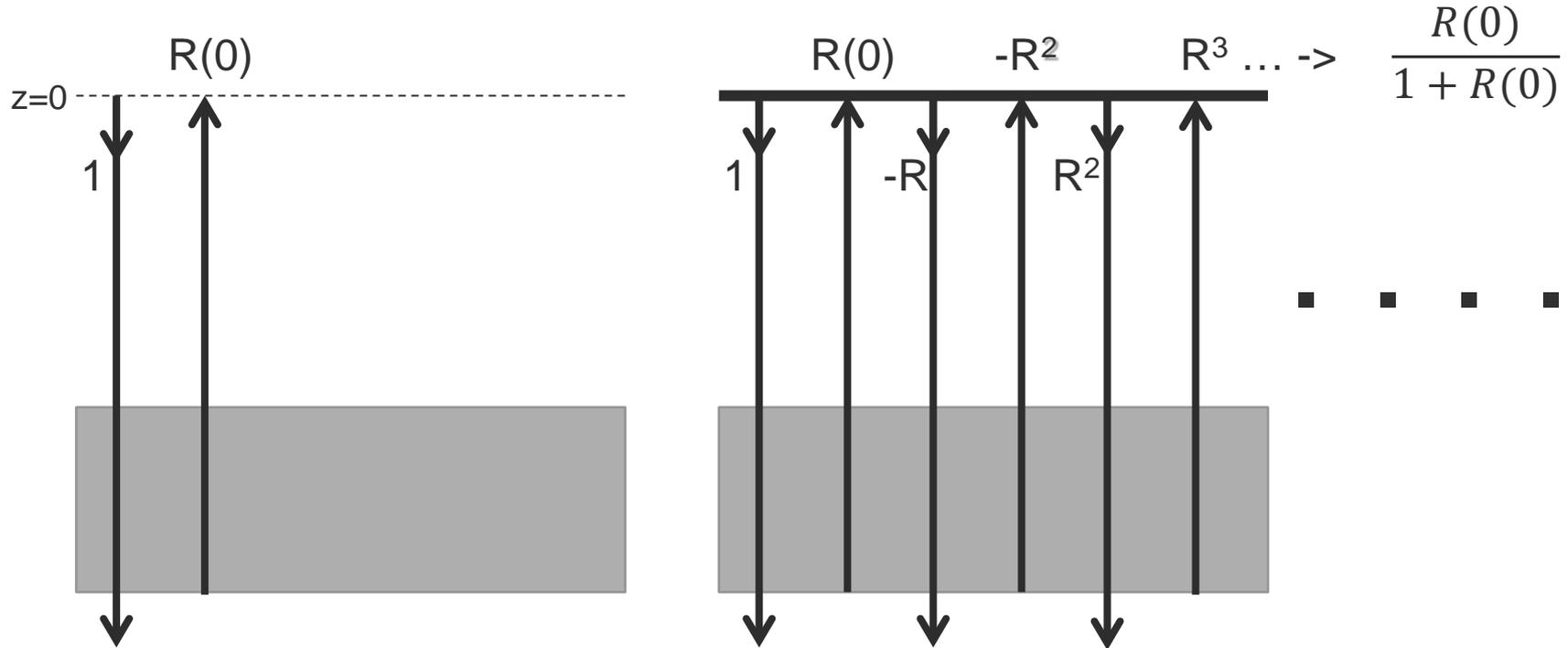
$G_+(z) = 1 + \exp(2ik_z z)$ receiver ghost function for particle velocity

Note + sign in ghost function, since:

$$\frac{d}{dz} [\exp(-ik_z z) - \exp(ik_z z)] = -ik_z [\exp(-ik_z z) + \exp(ik_z z)] = -ik_z \exp(-ik_z z) G_+(z)$$



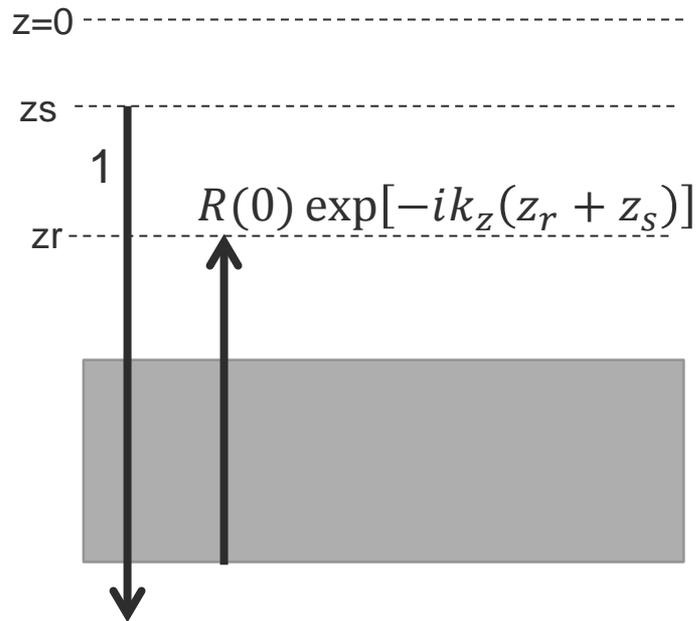
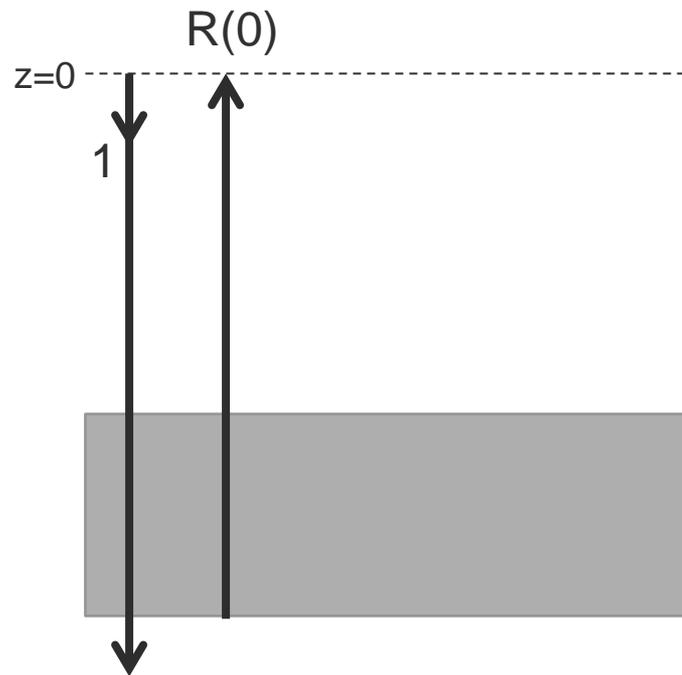
Layered medium: reflection response at $z=0$



Free surface absent

Free surface present

Layered medium: reflection response at receiver depth z_r from source at depth z_s



Layered medidum: pressure response

Pressure = Direct wave (incident + ghost)
+ Reflected wavefield (due to layering)

Reflected pressure wavefield = Reflection response $R(0)$ adusted to z_s and z_r
x free-surface related multiples
x receiver ghost function
x source ghost function
x source radiation pattern

Layered medium, $z_r > z_s$

Pressure

$$P(z_r) = P_d(z_r) + P_r(z_r)$$

$$P_d(z_r) = \exp[ik_z(z_r - z_s)] G_-(z_s) s \quad s = -\frac{a(\omega)}{2ik_z}$$

$$P_r(z_r) = \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_-(z_r) G_-(z_s) s$$

Particle velocity (neglect direct part)

$$V_z(z_r) = -\frac{k_z}{\rho\omega} \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_+(z_r) G_-(z_s) s$$

$$V_z(z_r) = -\frac{k_z}{\rho\omega} \frac{G_+(z_r)}{G_-(z_r)} P(z_r)$$

$V_z - P$ relation (Amundsen, 1993, 1995)

(for low frequencies, not very sensitive to errors in receiver depth ... reflection coefficient close to -1)

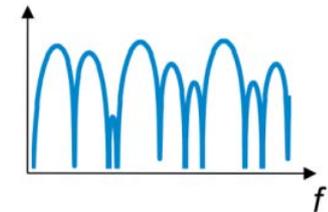
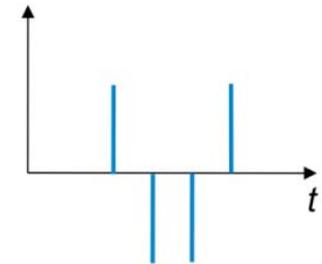
Layered medium, Ghosts 1D

Frequency domain

$$G_-(z_r)G_-(z_s) = 1 - \exp(2ikz_r) - \exp(2ikz_s) + \exp(2ik(z_r+z_s))$$

Time domain

$$s(t) - s(t - 2z_r/c) - s(t - 2z_s/c) + s(t - 2(z_r + z_s)/c)$$



Effect of sea surface may modify ghosts (in particular for higher frequencies)

$$G_{\pm}(z_r) = 1 \pm |r(\omega)| \exp(2ik_z z_r)$$

The receiver ghost model depends on

- Receiver depth z_r
- Velocity c
- Sea surface reflection coefficient; for flat sea $r(\omega)=-1$

Ghost effect on hydrophones (1D)

Ghost function

$$G_-(z) = 1 - \exp(2ikz)$$

Frequency spectrum has zeroes or 'notches' at frequencies

$$|G_-(z)| = 2 \sin(kz) = \mathbf{2} \sin(2\pi fz/c)$$

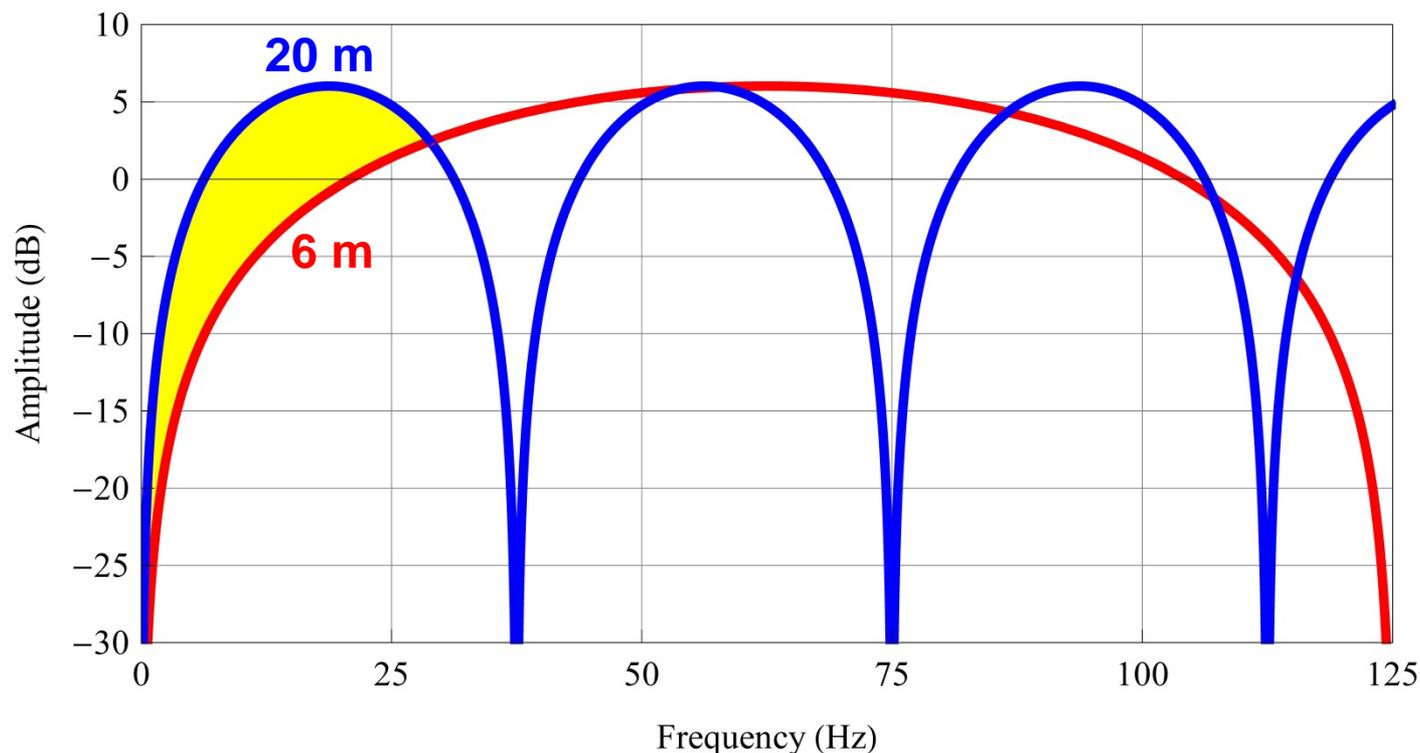
$$f_n = \frac{nc}{2z} \quad (n = 0, 1, 2, 3, \dots)$$

Ghosts interfere constructively or destructively with the primary reflections

$$\frac{d|G_-(z)|}{df} \propto z \quad \text{for low frequencies}$$

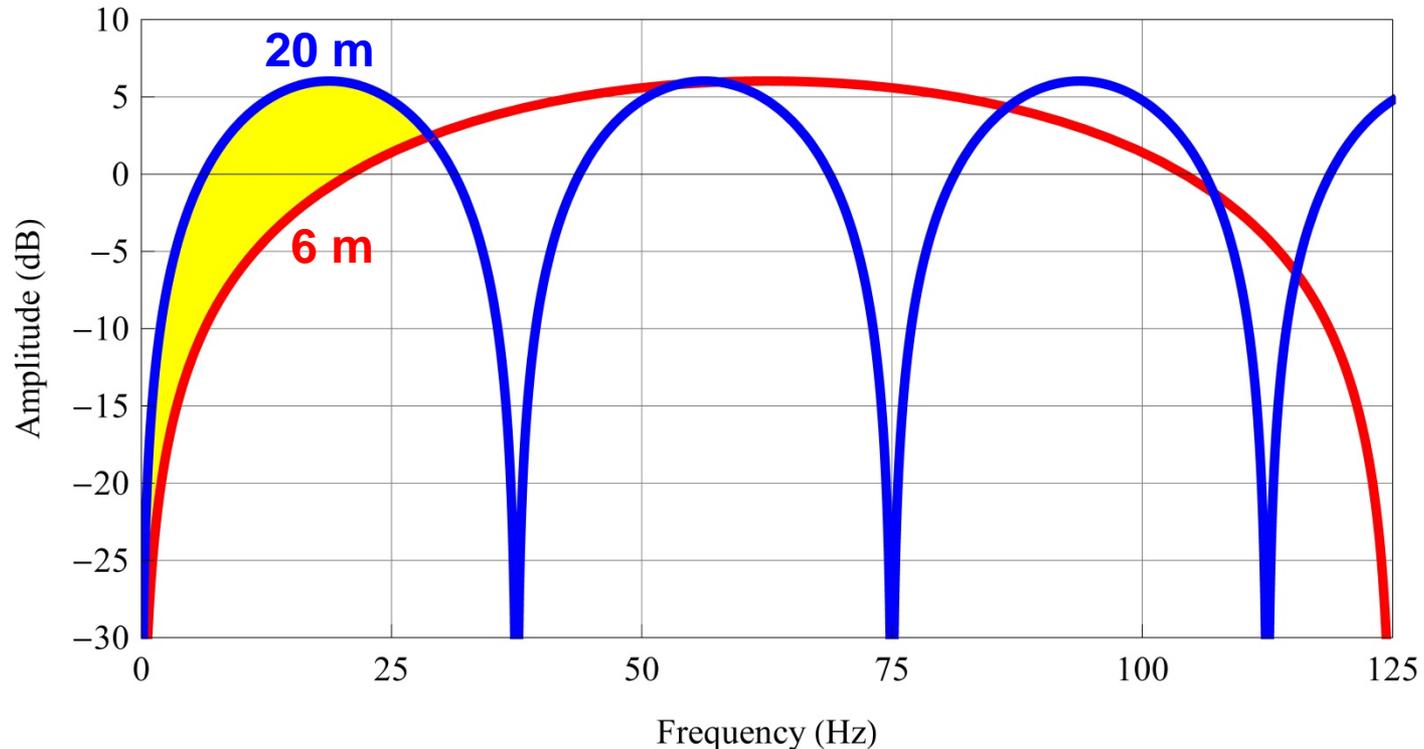
The inverse of the ghost function, $1/G_-$, has simple poles at fn .

Ghost effect on hydrophones (1D)



The ghost modulates the pressure spectrum, reducing energy at the notch frequencies. The first notch is at $f=0$, resulting in strong loss of useful low-frequency energy. Similar losses at the second and higher notch frequencies. The useable seismic pressure bandwidth is normally between the first and second notch. Towing streamers shallowly favors the higher frequencies at the expense of attenuating the low frequencies, while towing streamers deeper favors the lower frequencies, at the expense of attenuating frequencies within the seismic bandwidth.

Ghost effect on hydrophones (1D)



Ghost responses for deep-tow at 20m and shallow-tow at 6m. The ghost amplifies some frequencies (amplitude >0 dB) and attenuates other frequencies (amplitude <0 dB). By towing deeper the pressure signal is improved below ~30 Hz. Although deep-tow yields nice low-frequency characteristics, the second notch at 37.5 Hz has a detrimental effect on resolution.

Ghost effect on hydrophones (3D)

Ghost function

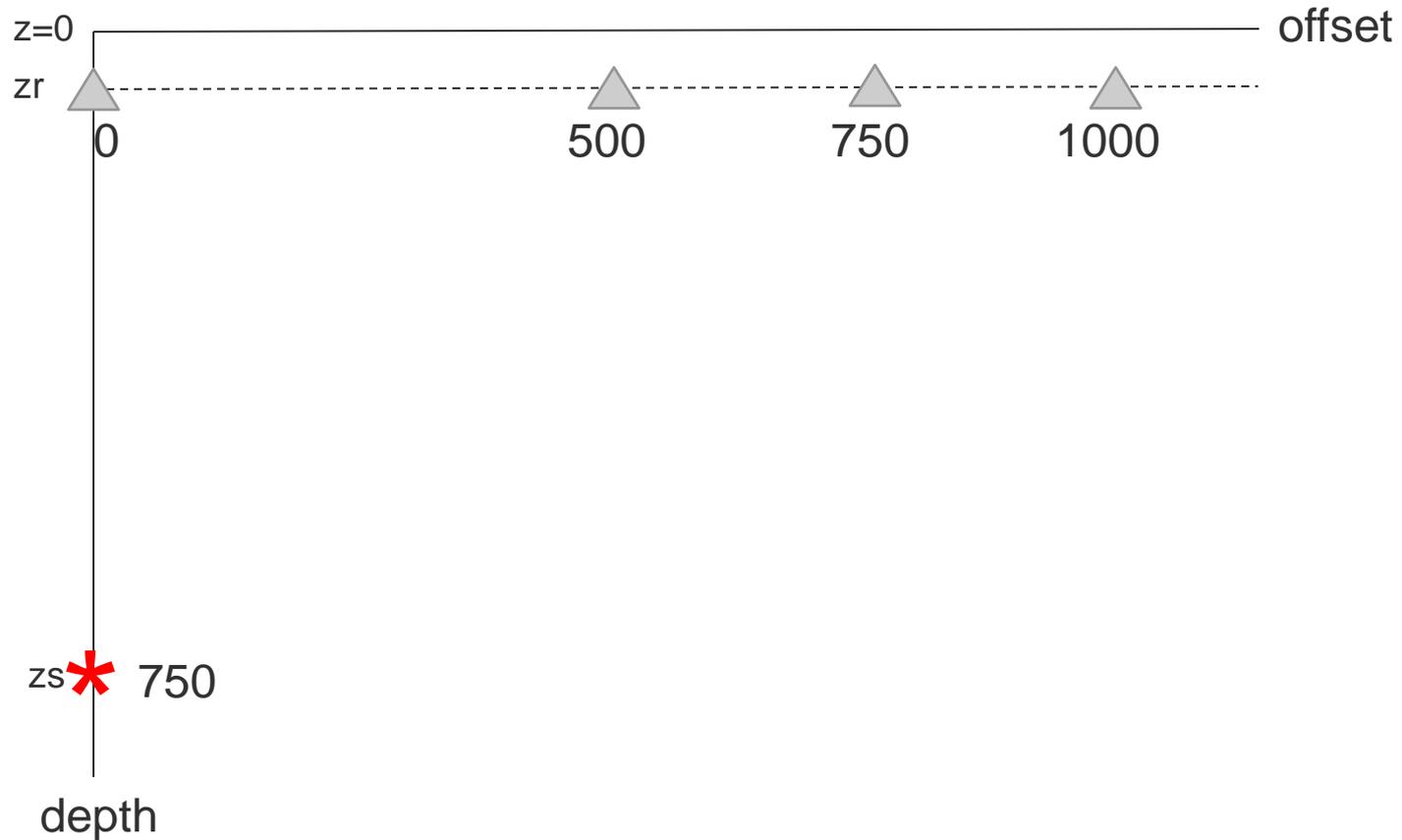
$$G_-(z) = 1 - \exp(2ik_z z)$$

$$k_z = k \cos \theta$$

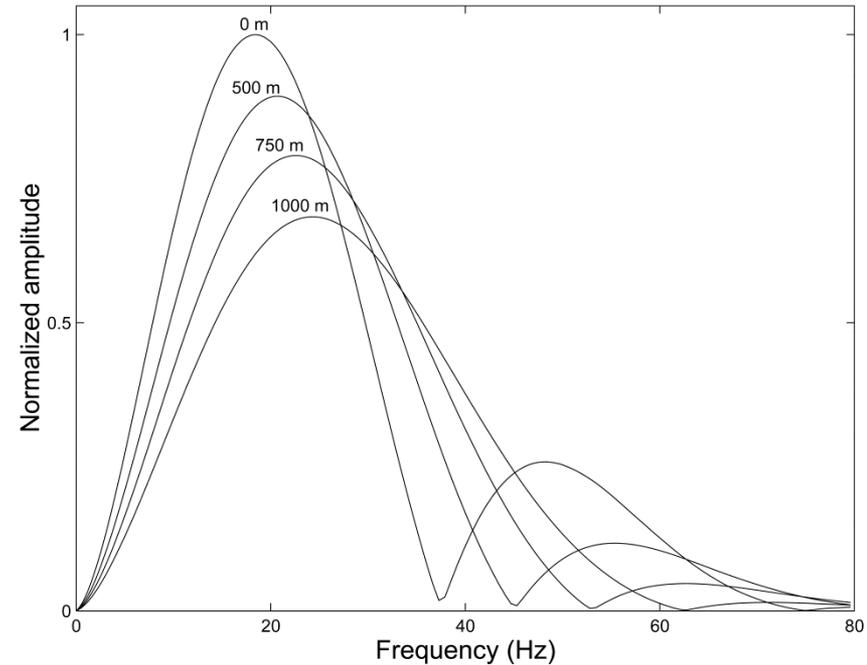
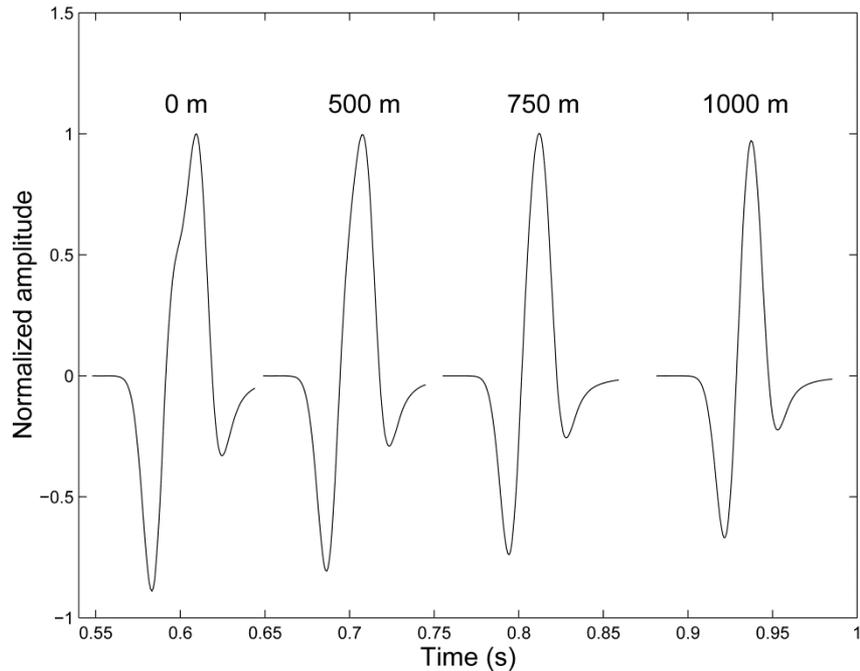
Frequency spectrum has notches at frequencies

$$f_{n,\theta} = \frac{nc}{2z \cos \theta} = \frac{f_n}{\cos \theta} \geq f_n \quad (n = 0, 1, 2, 3, \dots)$$

Simple test model



Ghost effect vs offset/angle (2D)



Pressure data consisting of incident wave and ghost at $z=20$ m for a few offsets. For zero offset the (second) notch frequency is at $f_1 = 37.5$ Hz. Observe that this notch frequency increases with increasing offset.

Ghost effect on geophones (3D)

Ghost function

$$G_+(z) = 1 + \exp(2ik_z z)$$

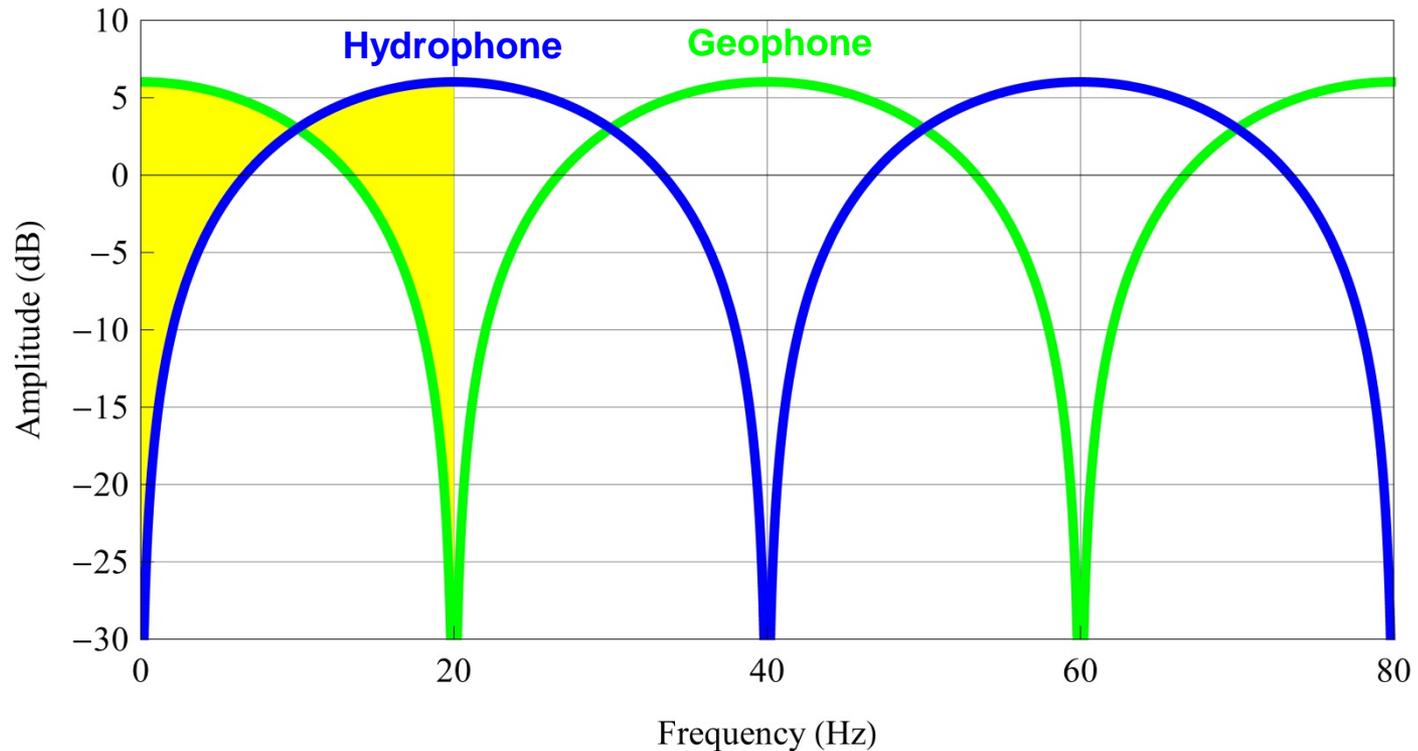
Frequency spectrum has notches at frequencies

$$|G_+(z)| = 2 \cos(k_z z) = 2 \cos(2\pi f z \cos \theta / c)$$

$$f_{n,\theta} = \frac{(2n + 1)c}{4z \cos \theta} \quad (n = 0, 1, 2, 3, \dots)$$

These notches are lying mid-between the notches in pressure recordings

Hydrophones and geophones measure complementary information



Receiver ghost responses for hydrophone and geophone at 18.75m depth. The geophone has maximum response (+6 dB) where the hydrophone has notch, and vice versa. For the low frequencies $f < 20$ Hz, the real geophone signal is too noisy, and deghosting is achieved using the geophone-hydrophone model.

Deghosting of pressure by inverse filtering

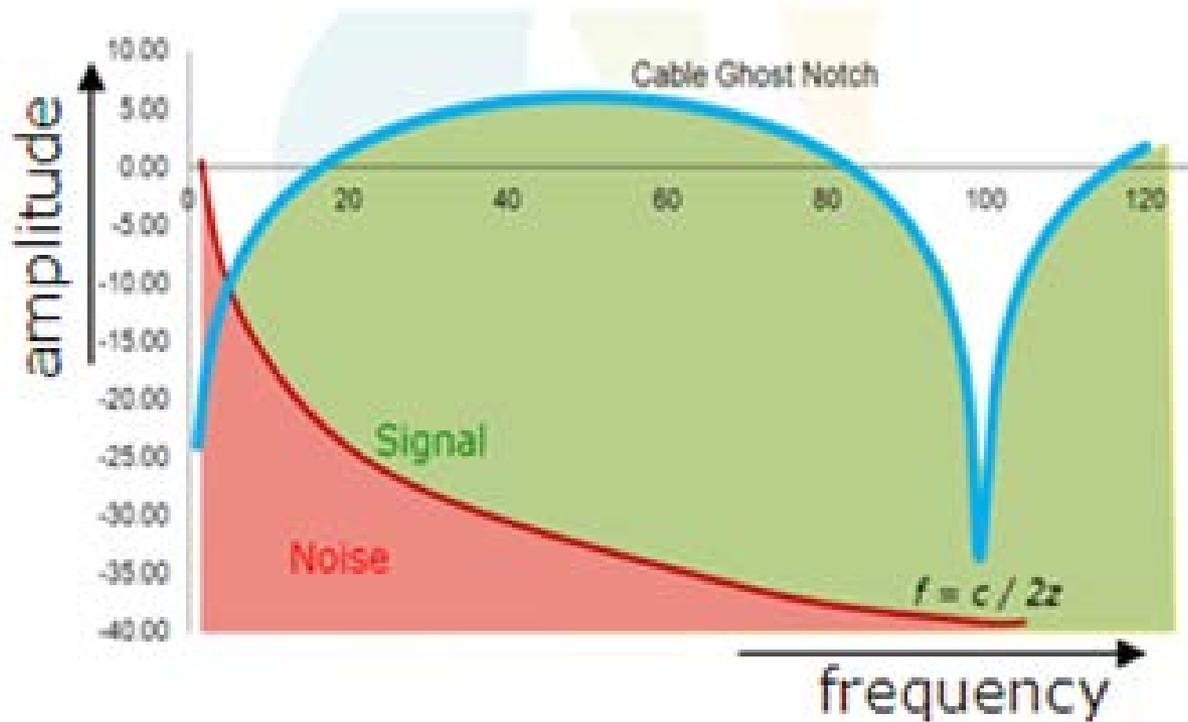
$$P_r(z_r) = \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_-(z_r) G_-(z_s) s$$

Inverse filtering

$$P_r^{dg}(z_r) = P_r(z_r) \frac{1}{G_-(z_r)} = \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_-(z_s) s$$

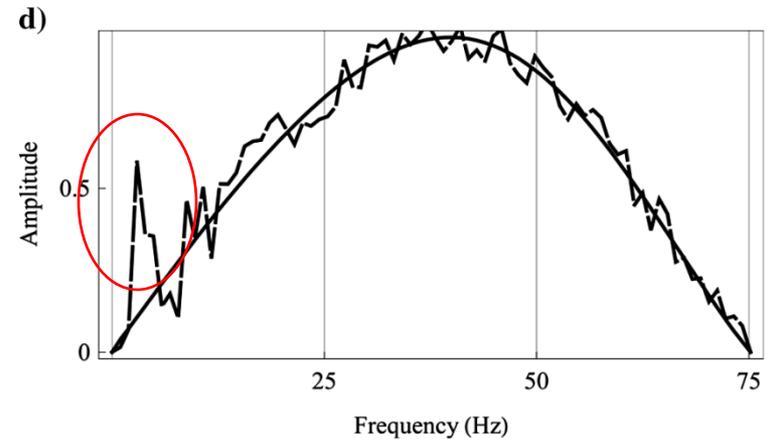
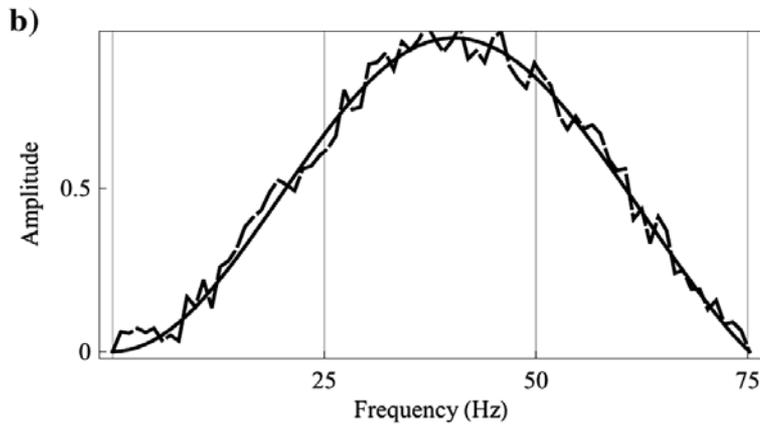
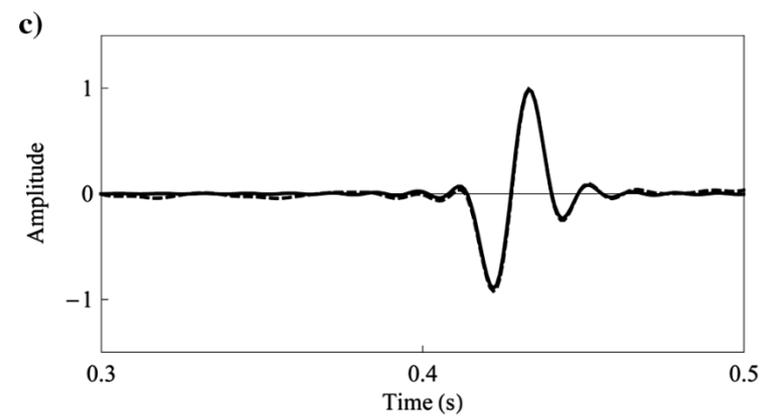
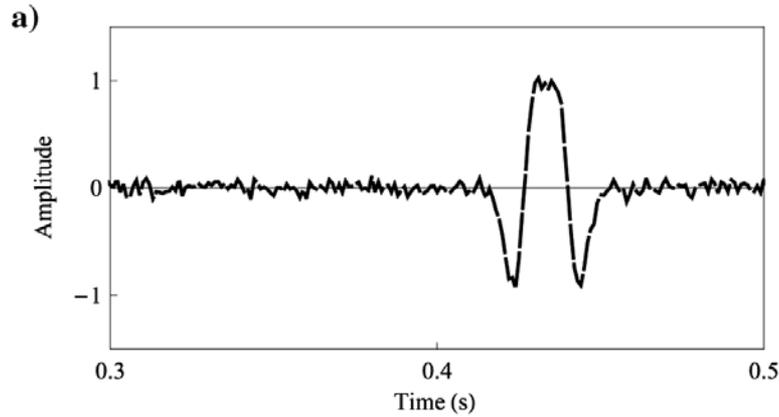
$$\frac{1}{G_-(z_r)} = \frac{1}{1 - \exp(2ik_z z_r)}$$

For deghosting by inverse filtering, $S/N > 1$



www.cgg.com

Inverse filtering «impossible» close to notch when $S/N < 1$



Signal and signal + noise (noise to 150 Hz)

Deghosted signal and signal + noise (highcut 75Hz)
Noise down-weighted below 3 Hz

Low-frequency trace deghosting (LFD)

$$G_-(z_r) = 1 - \exp(2ikz_r)$$

$$\frac{1}{G_-(z_r)} = \frac{1}{1 - \exp(2ikz_r)} \approx \frac{1}{2} \left[1 - \frac{c}{i\omega z_r} + \frac{i\omega z_r}{3c} + \dots \right]$$

Significant contribution for low frequencies

$$P_r^{dg}(x, t) = \frac{1}{2} \left[P_r(x, t) + \frac{c}{z_r} \int_0^t dt' P_r(x, t') + \frac{z_r}{3c} \partial_t P_r(x, t) \right]$$

For $kz_r=k$, the low-frequency deghosted pressure field can be computed trace-by-trace as the sum of the pressure field and its scaled temporally integrated and temporally differentiated fields. LFD deghosts data up to a frequency that is typically half of the second notch frequency. LFD can be appropriate to apply to the part of seismic data that have penetrated and reflected beneath complex and attenuating overburdens such as basalt, salt, and chalk.

Amundsen and Zhou Geophysics 2013

Deghosting by P-Vz summation!

- You can exorcize the receiver ghost by “summing” P and scaled V_z .
- Receiver side deghosting, equivalent to computing the upgoing component of the pressure field, can be done from P - V_z measurements.
- This is a fundamental basis of both PGS’s GeoStreamer solution and WesternGeco’s IsoMetrix solution
- **Independent of the sea surface**

$$P_r(z_r) = \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_-(z_r) G_-(z_s) s$$

$$-\frac{\rho\omega}{k_z} V_z(z_r) = \frac{R(0)}{1 + R(0)} \exp[-ik_z(z_r + z_s)] G_+(z_r) G_-(z_s) s$$

$$P_r^{dg}(z_r) = \frac{1}{2} \left[P(z_r) - \frac{\rho\omega}{k_z} V_z(z_r) \right]$$

$$k_z = \frac{\omega}{c} \cos \theta$$

Vz-P relation

$$V_z(z_r) = -\frac{k_z G_+(z_r)}{\rho\omega G_-(z_r)} P(z_r)$$

- For low frequencies where V_z is noisy, V_z can be estimated from P by deghosting the pressure (multiplying P by $1/G_-$) and ghosting the result (multiplying by G_+)
- This V_z estimation is used at low frequencies whereas the V_z measurements are used at higher frequencies in GeoStreamer deghosting (Anthony Day, Tilman Klüver, Walter Söllner, Hocine Tabti, and David Carlson, Geophysics 2013)
- V_z - P model is used also in 3D deghosting of IsoMetrix measurements, not necessarily to replace low-frequency V_z data but rather to further constrain the cross-line reconstruction problem
- P - V_z sensor streamers have no direct benefits for low frequency recording as they use only the hydrophone at low frequencies; the geophones are used to infill the higher frequency ghost notches
- To get high-quality pressure measurements at low frequencies, the cables must be towed deep where the pressure ghost notch has minimum effect and the S/N ratio is good

Simultaneous deghosting and crossline wavefield reconstruction (GMP)

$$\begin{aligned}P(k_y) &= G_-(k_y)P^{dg}(k_y) = H_1(k_y)P^{dg}(k_y) = S_1(k_y) \\ \partial_y P(k_y) &= ik_y G_-(k_y)P^{dg}(k_y) = H_2(k_y)P^{dg}(k_y) = S_2(k_y) \\ \partial_z P(k_y) &= ik_z G_+(k_y)P^{dg}(k_y) = H_3(k_y)P^{dg}(k_y) = S_3(k_y)\end{aligned}$$

Note: Ghost models are used to relate each input to the unknown deghosted pressure

Model the deghosted pressure at position yn as a sum of complex exponential basis functions, each described by amplitude, wavenumber and phase

$$P^{dg}(y_n) = \sum_m A_m \exp(ik_{y,m}y_n + \varphi_m)$$

Expand multi-measurement input signals ($i=1,2,3$) at position yn as a combination of the same basis functions

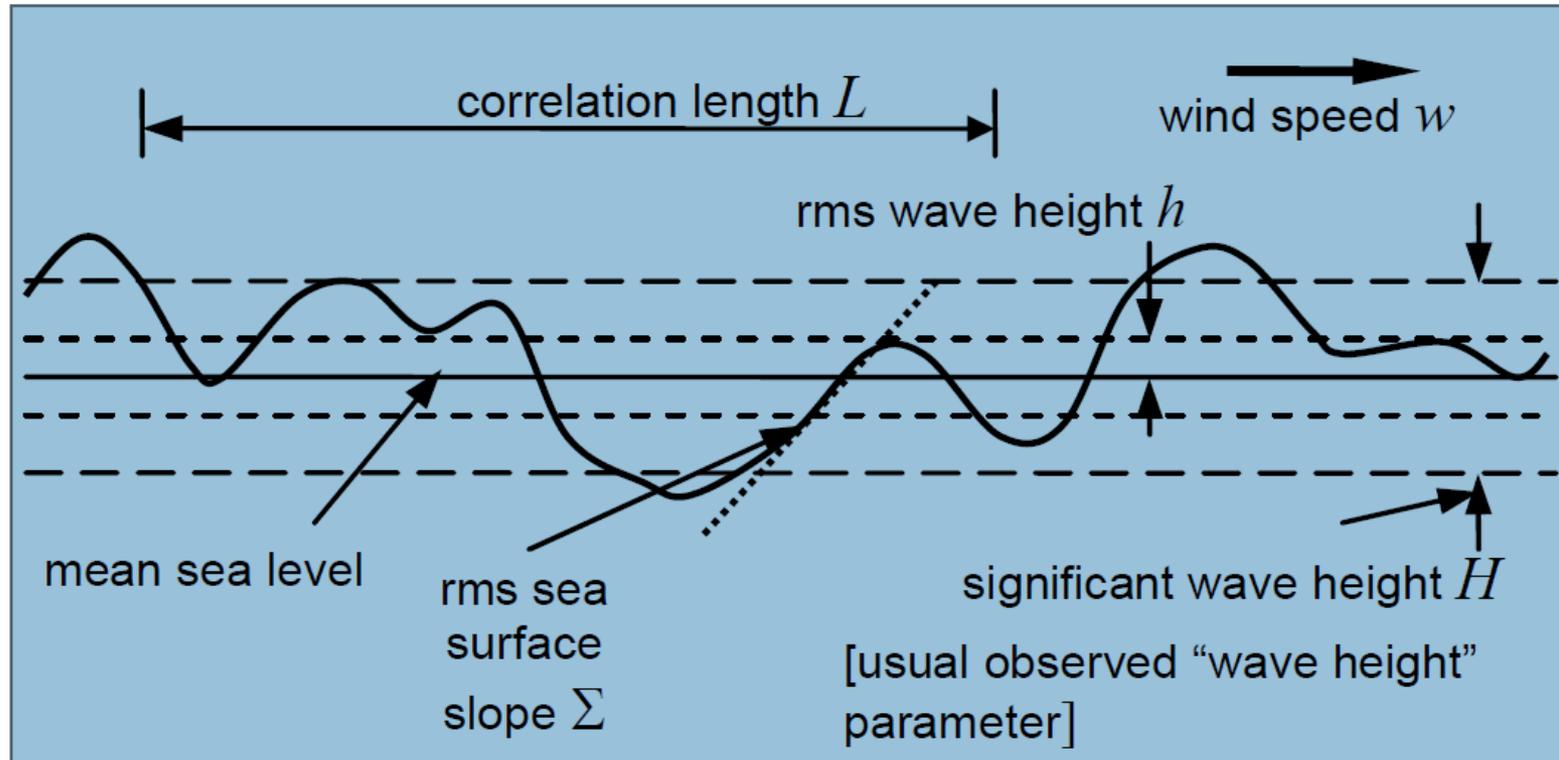
$$S_i(y_n) = \sum_m H_i(k_{y,m})A_m \exp(ik_{y,m}y_n + \varphi_m)$$

GMP (Generalized Matching Pursuit) iteratively looks for a set of basis functions that match all input signals at the measured positions (Özbek et al Geophysics 2010).

Effect of sea surface may modify ghosts (for higher frequencies)

$$G_-(z_r) = 1 - |r(\omega)| \exp(2ik_z z_r)$$

Parameters for a rough surface



Jones et al 2009

Kirchhoff model

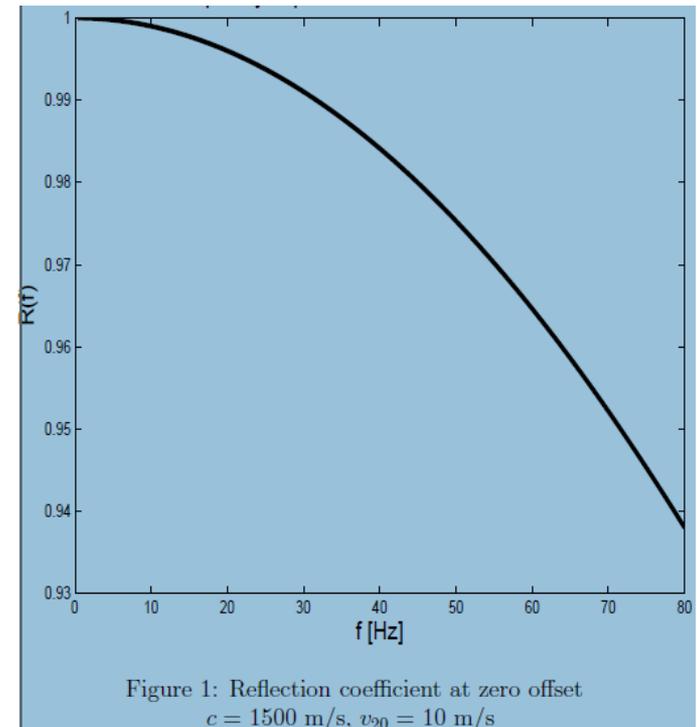
For randomly rough surface, the Kirchhoff approximation to scattering and the assumption of a Gaussian probability of surface elevations of standard deviation h (rms roughness) gives

$$r(\omega) = -\exp(-2(kh \cos \theta)^2)$$

Relation to wind velocity v_{20} measured 20 m above the sea surface

$$h^2 = Dv_{20}^4$$

The ghost notch effect is less in rough weather ... good or bad?



Reitan, Landrø, Amundsen 2013
(work in progress)

The kz problem

We don't have properly sampled data in cross-line (y) direction: streamer separation is 75-100 m. Processing data on each streamer Individually, we implicitly assume that $k_y=0$, and as a result, k_z is systematically overestimated.

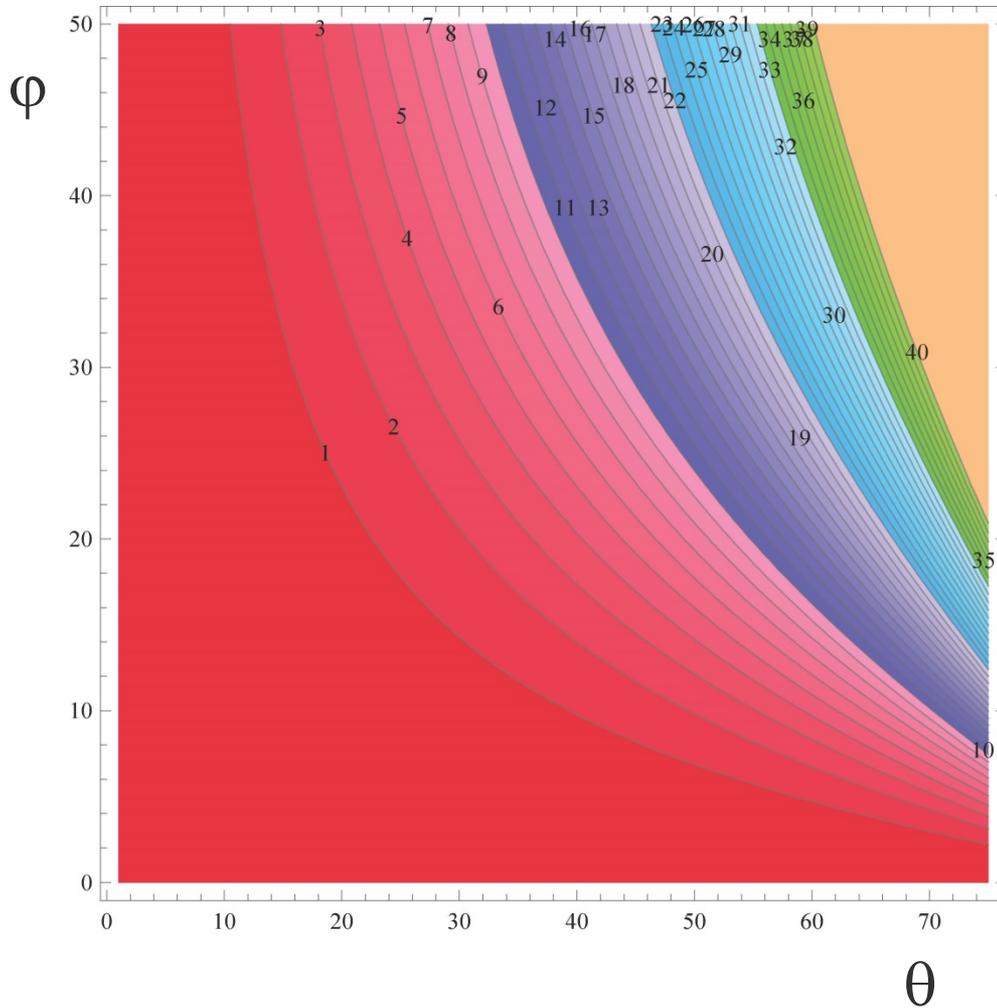
The 3D vertical wavenumber can be expanded in the k_y direction. The 2D inline direction part can be implemented accurately in the in-line direction. If the second spatial derivative of data in the cross-line direction is measured, one may achieve better 3D deghosting (Robertsson 2005)

$$k_z^{2D} = \sqrt{k^2 - k_x^2}$$

$$k_z^{3D} = \sqrt{k^2 - k_x^2 - k_y^2} = k_z^{2D} \left(1 - \frac{k_y^2}{2(k_z^{2D})^2} + O(k_y^4) \right)$$

One may derive slownesses (and 3D vertical wavenumber) from 4C acoustic measurements (Vassallo 2009; Amundsen, Ikelle, Robertsson, Westerdahl SEG 2013), using an assumption of non-overlapping primaries/multiples

$$(1/q_z^{3D} - 1/q_z^{2D}) / (1/q_z^{3D})$$



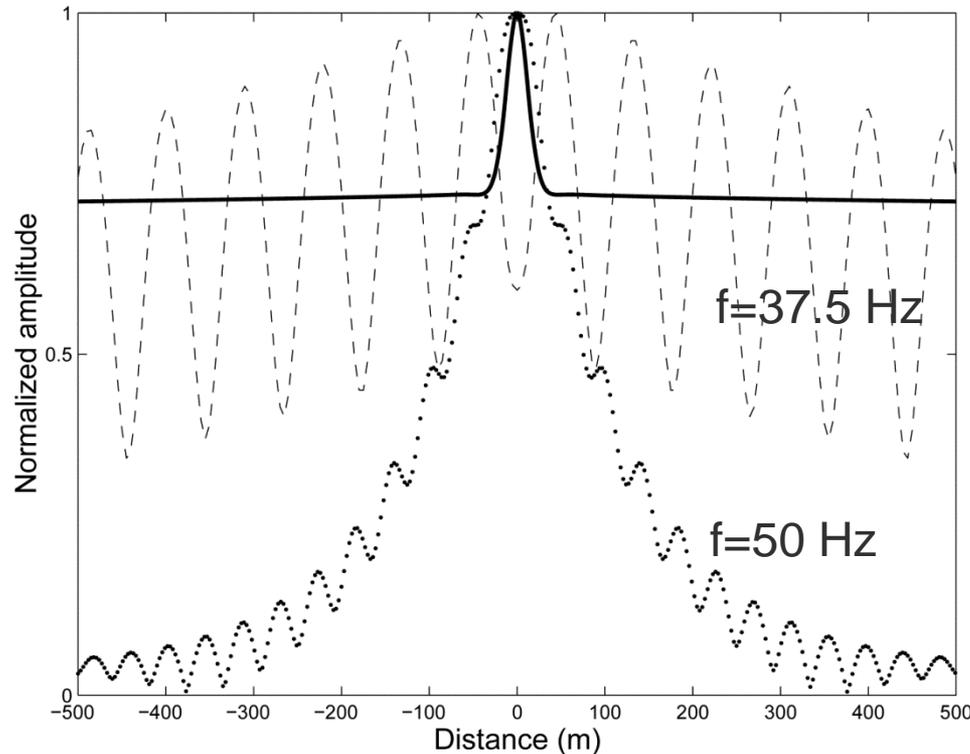
$$q_x = \frac{1}{c} \cos \varphi \sin \theta$$

$$q_y = \frac{1}{c} \sin \varphi \sin \theta$$

$$q_z^{3D} = \sqrt{c^{-2} - q_x^2 - q_y^2} = \frac{1}{c} \cos \theta$$

$$q_z^{2D} = \frac{1}{c} \sqrt{1 - (\cos \varphi \sin \theta)^2}$$

Deghosting filters in space domain are very long



f=20 Hz

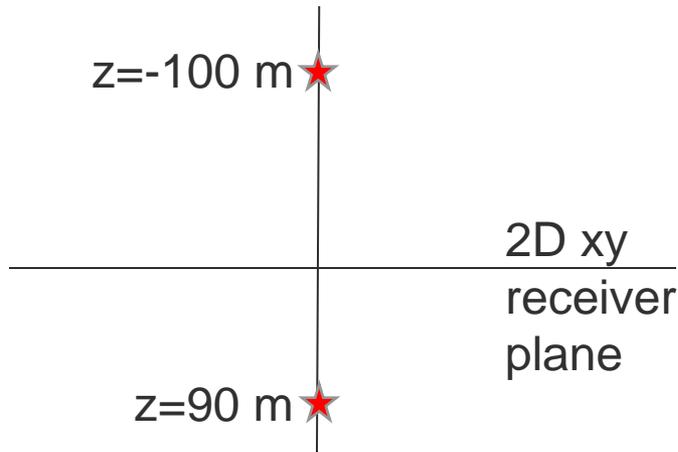
f=37.5 Hz

f=50 Hz

Amundsen,
Weglein and
Reitan,
Geophysics 2013

Amplitude spectra of deghosting filters with offset (in the range ± 500 m) for $z=20$ m. For fundamental frequencies $f_n = nc/(2z)$ the deghosting filter has logarithmic singularities or “peaks”. These fundamental frequencies correspond to the notch frequencies in the ghost function for plane waves that travel vertically ($\theta = 0$).

Test of deghosting error 1D vs 3D (OBS seismic)



Modeling in homogeneous medium upgoing and downgoing waves with opposite wavelet polarities gives traces with ghost on xy receiver plane.
 Error plots showing the relative error (rms of the difference over traces normalized by the rms of reference solution trace).

Amundsen, Ikelle, Robertsson, Westerdahl, SEG 2013

