Modeling and migration of dual-sensor marine seismic data (Work in progress)

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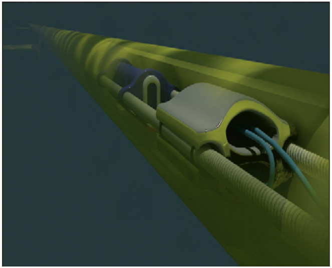
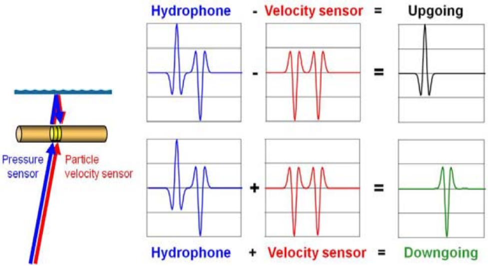
Dual-Sensor - Wavefield separation





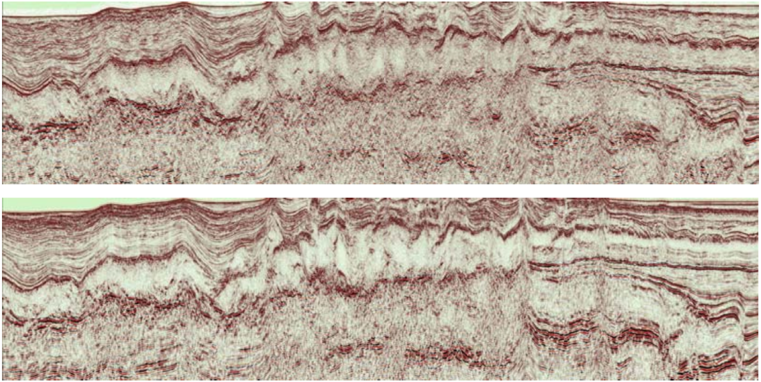
Dual-sensor towed streamer measures the pressure wavefield and the vertical velocity field, at the same spatial position.

For the pressure wavefield the two components are given as:

$$P^{up} = \frac{1}{2}(P - FV_z) \qquad \text{and} \qquad P^{down} = \frac{1}{2}(P + FV_z)$$

where F is angle-dependent scaling factor and it is required because only V_z is recorded (Widmaier et al., EAGE/SEG, 2009).

Real example: Conventional streamer versus dual-sensor streamer



Cenventional streamer (top) and dual-sensor (botton)

The combination of the sensores allows the separation of the up- and down-going wavefields and thus the removal of the ghost effect. The removal of the ghost significantly enhances frequency content and (Widmaier et al., EAGE/SEG, 2009).

Wavefield separation - Extrapolation and imaging

The upgoing and downgoing pressure wavefields in the frequency-wavenumber domain can be written in a matrix form as

$$\begin{bmatrix} P^{up} \\ P^{down} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -\frac{\rho\omega}{k_z} \\ 1 & \frac{\rho\omega}{k_z} \end{bmatrix} \begin{bmatrix} P \\ V_z \end{bmatrix}$$

Extrapolation is based on the solution of the one-way wave equation. The one way wave equation in $\omega - x$ domain is:

$$\left(\frac{\partial}{\partial z} - i\sqrt{\frac{\omega^2}{v^2} + \nabla} \right) P^+ = 0$$

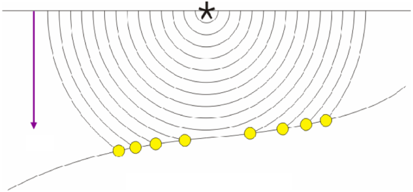
where ∇ is the Laplician operator.
For one-way algorithm, we can approximate the square-root by:

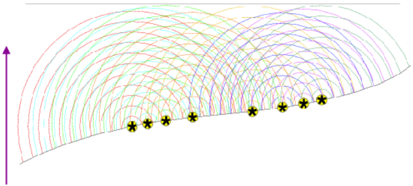
$$\sqrt{\frac{\omega^2}{v^2} + \nabla} = \text{ Phase-Shift}(k_x, \omega) + \text{Split Step }(x, \omega) + \text{FFD }(x, \omega)$$

Two-way propapator is a simples extension of the one-way propagation.
The two-way acoustic wave equation in the $\omega - x$ domain becomes (Zhang et at.,2005) then

$$\left(\frac{\partial}{\partial z} - i\sqrt{\frac{\omega^2}{v^2} + \nabla}\right)P^- = \Gamma P^+$$

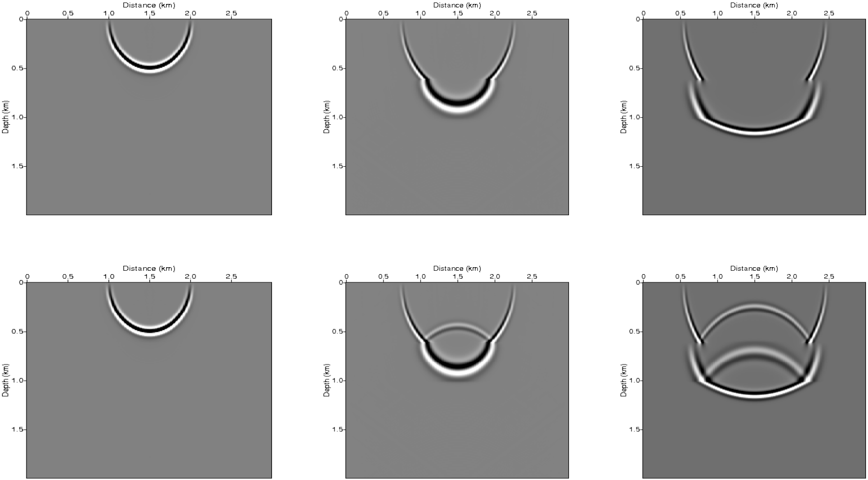
where Γ is, in some sense, equivalent to a reflection coffecient and P^- is the upgoing wavefield.





One-way and two-way simulation

Two layer model - Snapshots



Imaging

After proper extrapolation of the decomposed monochromatic wavefields to the desired depth level, an estimate of the reflection coefficient(s) can be obtained through the application of the classical P^{up}/P^{down} imaging condition.

- Cross-correlation

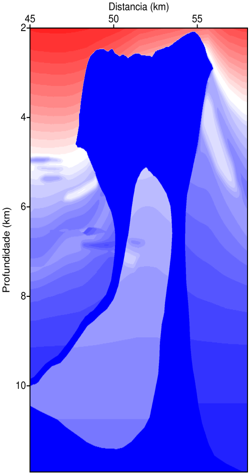
$$I_1(x,h) = \sum_{x_s}\sum_{\omega}(P^{down}(x-h,\omega;x_s))^{*}\,P^{up}(x+h,\omega;x_s)$$

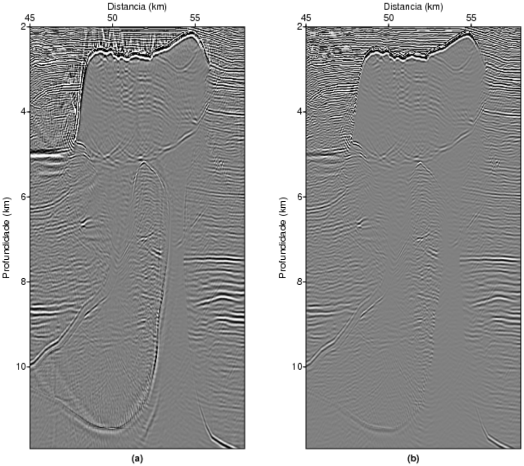
- Deconvolution

$$I_2(x,h) = \sum_{x_s}\sum_{\omega}\frac{(P^{down}(x-h,\omega;x_s))^{*}\,P^{up}(x+h,\omega;x_s)}{\langle(P^{down}(x-h,\omega;x_s))^{*}\,P^{down}(x-h,\omega;x_s)\rangle+\epsilon}$$

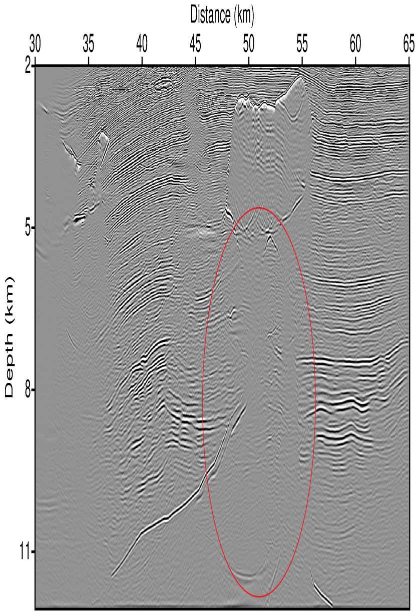
where $\langle.\rangle$ stands for the smoothing in the image space in the x direction. Note, we obtain zero offset subsurface images when $h=0$.

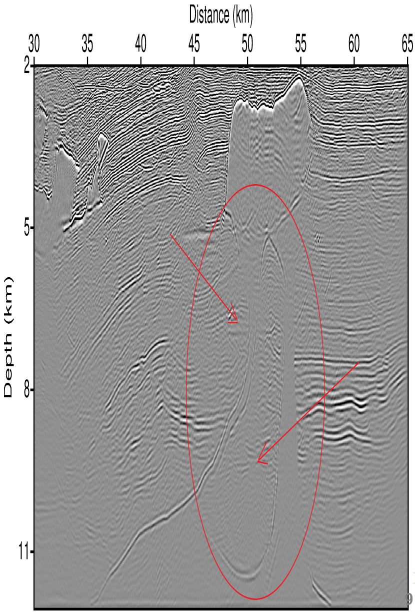
BP-Dataset: One-way and two-way results





BP Dataset: One-way (PSPI) and RTM results





Coupled wave equation system

The wave equation can be transformed into two coupled first-order differential equations in the depth z, and one obtains:

$$\frac{\partial}{\partial z}\begin{pmatrix} P \\ \frac{\partial P}{\partial z} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -(\frac{\omega^2}{c^2} + \frac{\partial^2}{\partial x^2}) & 0 \end{pmatrix}\begin{pmatrix} P \\ \frac{\partial P}{\partial z} \end{pmatrix}$$

We can write in a compact notation the wave equation for the field vector $\psi = (P, \frac{\partial P}{\partial z})^T$ as

$$\frac{\partial \psi(\mathbf{x}, \omega)}{\partial z} = A\,\psi(\mathbf{x}, \omega)$$

where the matrix A is given by:

$$A = \begin{pmatrix} 0 & 1 \\ -(\frac{\omega^2}{c^2} + \frac{\partial^2}{\partial x^2}) & 0 \end{pmatrix}$$

Assuming that the velocity c is constant as a function of depth with each layer z to $z + \Delta z$, the solution of this equation is given by:

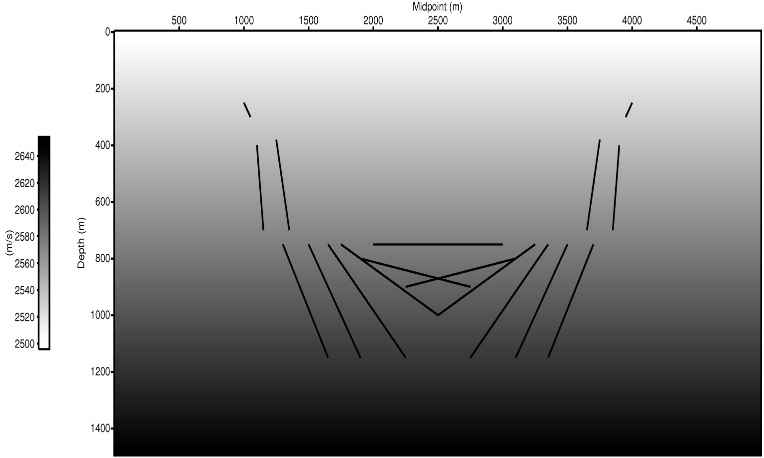
$$\psi(x, y, z + \Delta z, \omega) = e^{A\Delta z}\, \psi(x, y, z, \omega)$$

For general case, the exponential term can be computed using Chebyshev expansion according to

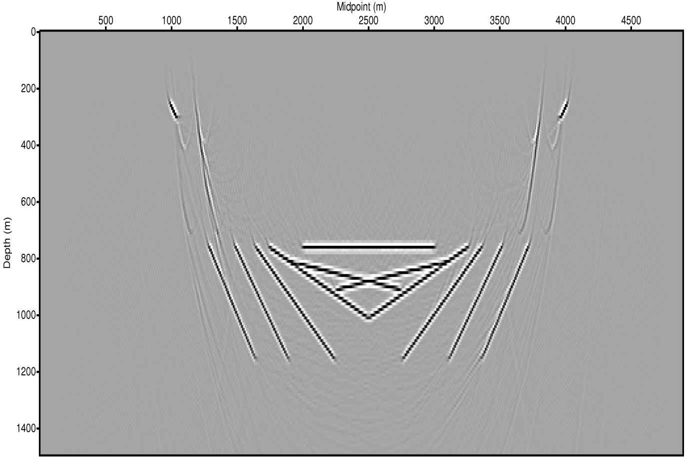
$$e^{A\Delta z} = \sum_{k=0}^{M} C_k\, J_k(R)\, T_k\left(\frac{A\Delta z}{R}\right)$$

where $R = (\omega\Delta z)/c_{min}$ and M $>$ R (Tal-Ezer, 1984).

Depth model and velocity in the background

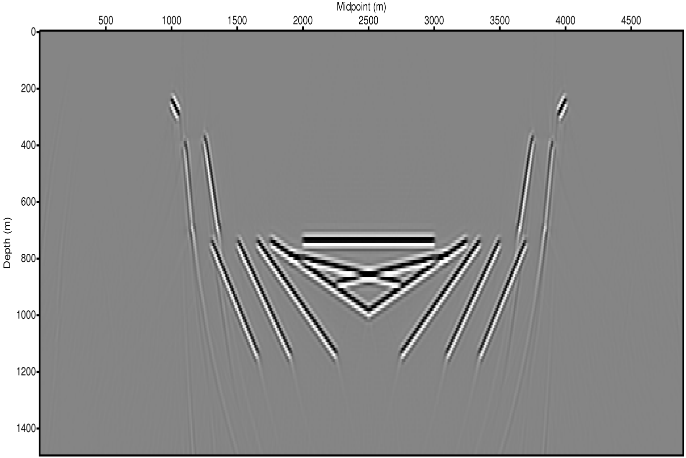


One-way PSPI method



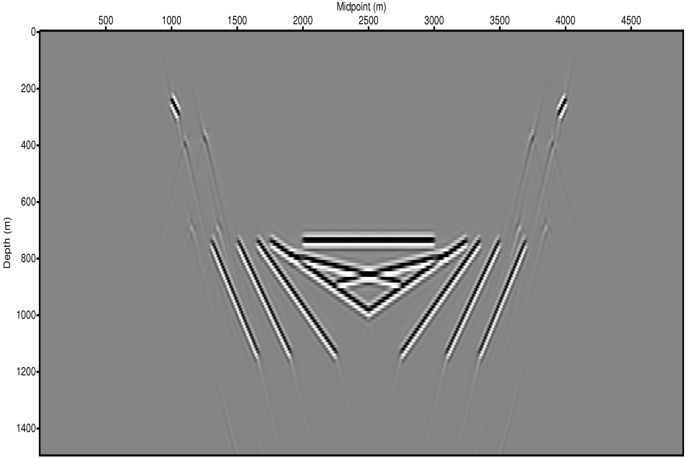
Tal-Ezer method - Coupled wave equation system

High-cut filter $k_{cut} > \omega / c_{max}$



Tal-Ezer method - Coupled wave equation system

High-cut filter $k_{cut} > \omega / c_{max}$ and taper filter



Differential system in space-frequency domain

The linerized acoustic wave equation in the space-frequency domain can be written as the following system of coupled equations

$$\begin{cases} \nabla P(\mathbf{x},\omega) - i\omega\rho\mathbf{V}(\mathbf{x},\omega) = 0 \\ \nabla\cdot\mathbf{V}(\mathbf{x},\omega) - \frac{i\omega}{\rho c^2(\mathbf{x})}P(\mathbf{x},\omega) = 0 \end{cases}$$

where ω denotes the temporal frequency, $\mathbf{x} = (x, y, z)$ the Cartesian coordinates, $\mathbf{V} = (V_x, V_y, V_z)$ is the particle velocity vector and P is the pressure.

For the depth extrapolation problem it is useful to write these equations in terms of the vertical velocity V_z and pressure P.

$$\begin{cases} \frac{\partial P}{\partial z} & = i\,\omega\,\rho\,V_z \\ \frac{\partial V_z}{\partial z} & = \frac{i\omega}{\rho c^2(\mathbf{x})} P - \frac{\partial}{\partial x}\left(\frac{1}{i\omega\rho}\frac{\partial P}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{1}{i\omega\rho}\frac{\partial P}{\partial y}\right) \end{cases}$$

In a simplified notation with the operator H_2 as:

$$H_2 = \left(\frac{\omega}{c(\mathbf{x})}\right)^2 + \rho\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial}{\partial x}\right) + \rho\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial}{\partial y}\right)$$

We can write in a compact notation the wave equation for the field

vector $\psi = (P, V_z)^T$ as

$$\frac{\partial \psi(\mathbf{x},\omega)}{\partial z} = A\,\psi(\mathbf{x},\omega)$$

The wave equation can be written as

$$\frac{\partial \psi(\mathbf{x},\omega)}{\partial z} = M\,\psi(\mathbf{x},\omega) + Q\,\psi(\mathbf{x},\omega)$$

where the matrix M is chosen independent of z over the interval $z + \Delta z$,

$$M = \begin{pmatrix} 0 & i\omega\rho \\ -\frac{1}{i\omega\,\rho} H_2^0 & 0 \end{pmatrix}$$

with

$$H_2^0 = \left(\frac{\omega}{c_o}\right)^2 + \rho\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial}{\partial x}\right) + \rho\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial}{\partial y}\right)$$

where c_o and ρ are constant,

and Q is given as:

$$Q = \begin{pmatrix} 0 & 0 \\ -\frac{w}{i\rho}\left(\frac{1}{c(\mathbf{x})^2} - \frac{1}{c_o^2}\right) & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ q & 0 \end{pmatrix}$$

Since M and e^{-Mz} commute we have (Maji and Kouri, 2011)

$$\frac{\partial}{\partial z}\Big(\exp(-M\,z)\,\psi(\mathbf{x},\omega)\Big) = \exp(-M\,z)\,Q\,\psi(\mathbf{x},\omega)$$

Next, integrating from z to $z + \Delta z$, the exact solution is given by:

$$\psi(x,y,z+\Delta z,\omega) = \exp(M\Delta z)\,\psi(x,y,z,\omega) + \int_z^{z+\Delta z} dz' \exp[(z+\Delta z - z')\;M]\,Q(x,y,z',\omega)\,\psi(x,y,z',\omega)$$

If the simple trapezoidal rule is used, we obtain

$$\psi(x, y, z + \Delta z, \omega) = \left(I - \frac{\Delta z}{2} Q(x, y, z + \Delta z, \omega) \right)^{-1}$$
$$\exp(M \Delta z) \left(I + \frac{\Delta z}{2} Q(x, y, z, \omega) \right) \psi(x, y, z, \omega)$$

where I is the 2x2 identity matrix.

We note that due to the structure of Q, the computation of the inverse is trivial:

$$\left(I - \frac{\Delta z}{2} Q(x, y, z + \Delta z, \omega) \right)^{-1} = \begin{pmatrix} 1 & 0 \\ \frac{\Delta z}{2} q(x, y, z + \Delta z, \omega) & 1 \end{pmatrix}$$

Using P and V_z, we obatin that

$$\begin{pmatrix} P \\ V_z \end{pmatrix}_{z+\Delta z} = \begin{pmatrix} 1 & 0 \\ \frac{\Delta z}{2} q(x,y,z+\Delta z,\omega) & 1 \end{pmatrix} \exp(M\,\Delta z) \begin{pmatrix} 1 & 0 \\ \frac{\Delta z}{2} q(x,y,z) & 1 \end{pmatrix} \begin{pmatrix} P \\ V_z \end{pmatrix}_{z}$$

When $P(x,y,z,\omega)$ and $V_z(x,y,z,\omega)$ are known, $P(x,y,z+\Delta z,\omega)$ and $V_z(x,y,z+\Delta z,\omega)$ can be obtained from this equation in a step by step process.

Elimination of exponentially increasing evanescent energy

In order to eliminate the unstable evanescent component in the extrapolation procedure, we rewrite the M matrix via an eigen decomposition in the following way:

$$M = L\,\Lambda\,L^{-1}$$

where L represents the eigenvectores, and Λ is the eigenvalue matrix

$$\Lambda = \begin{pmatrix} -ik_z & 0 \\ 0 & ik_z \end{pmatrix}.$$

The vertical wavenumber k_z is

$$k_z = \begin{cases} \sqrt{(\frac{\omega}{c})^2 - (k_x^2 + k_y^2)}, & \text{if} \quad \sqrt{(k_x^2 + k_y^2)} \leq |\frac{\omega}{c}|, \\ i\sqrt{(k_x^2 + k_y^2) - (\frac{\omega}{c})^2}, & \text{if} \quad \sqrt{(k_x^2 + k_y^2)} > |\frac{\omega}{c}|. \end{cases}$$

Elimination of exponentially increasing evanescent energy

The eigenvector matrix of M may be chosen as (Claerbout, 1976; Ursin et al., 2012)

$$L = \begin{pmatrix} 1 & 1 \\ -\frac{1}{Z} & -\frac{1}{Z} \end{pmatrix}.$$

with inverse eigenvector matrix

$$L^{-1} = \begin{pmatrix} 1 & -Z \\ 1 & Z \end{pmatrix}.$$

Here,

$$Z = \frac{\rho\omega}{k_z}$$

Then, for k_z real,

$$e^{M\Delta z} = L\, e^{\Lambda\,\Delta z}\, L^{-1} = \begin{pmatrix} \cos(k_z\Delta z) & Z\,i\sin(k_z\Delta z) \\ \frac{i}{Z}\,\sin(k_z\Delta z) & \cos(k_z\Delta z) \end{pmatrix}$$

This is stable if k_z is real.

For k_z imaginary, $k_z = i|k_z|$, we remove the unstable mode by a projection operator (Maji and Kouri, 2011; Sandberg and Beylkin, 2009), and we use

$$e^{M\Delta z} = L \begin{pmatrix} 0 & 0 \\ 0 & e^{-|k_z|\,\Delta z} \end{pmatrix} L^{-1} = \frac{1}{2} \begin{pmatrix} 1 & Z \\ \frac{1}{Z} & 1 \end{pmatrix} e^{-|k_z|\,\Delta z}$$

where now

$$Z = \frac{\rho\omega}{i|k_z|}$$

Source wavefields

For seismic modeling we may start with a point source where the downgoing wavefield is

$$D_0' = \frac{-2\pi S(\omega)}{ik_z} e^{-i\,(k_x x_s + k_y y_s)}$$

Here the $S(\omega)$ is the source signature and (x_s, y_s) is the source position.
For seismic imaging we follow Arntsen et al. (2012) and start with

$$D_0 = \frac{1}{(D_0')^*} = \frac{ik_z}{2\pi S(\omega)^*} e^{i\,(k_x x_s + k_y y_s)}$$

where $*$ denotes complex conjugate.
In both cases the initial values of the modeling equations are

$$\begin{pmatrix} P \\ V_z \end{pmatrix}_0 = L \begin{pmatrix} D_0 \\ 0 \end{pmatrix} = \begin{pmatrix} D_0 \\ \frac{-k_z}{\rho\omega} D_0 \end{pmatrix}$$

Full wavefield imaging

Next, we start with the recorded data P_0 and V_{z0} which we downward continue to get P_d and V_{zd}.

At a certain depth level we compute the upgoing wavefield from the data

$$U_d = \frac{1}{2}(P_d - Z\,V_{zd})$$

and the downgoing wavefield from the source

$$U_s = \frac{1}{2}(P_s + Z\,V_{zs})$$

A common-image gather for a single shot is

$$R(\mathbf{p},\mathbf{x},z) = \int\int U_d(\omega,\mathbf{x}+\frac{\mathbf{h}}{2},z)\,D_s^*(\omega,\mathbf{x}-\frac{\mathbf{h}}{2},z)\;e^{-i\omega\mathbf{p}\cdot\mathbf{h}}\,d\mathbf{h}\,d\omega,$$

where $\mathbf{h} = (h_x, h_y, 0)$ is the horizontal offset coordinate and $\mathbf{p}.\mathbf{h} = p_x h_x + p_y h_y$.

Conclusions

- We are proposing a two-way propagation method for P and V_z wavefields (Generalized phase shift method).
- Elimination of unstable evanescent component using a an eigen decomposition.
- This method is very promise and more work is needed.

Acknowledgments

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