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3D inversion of electromagnetic data

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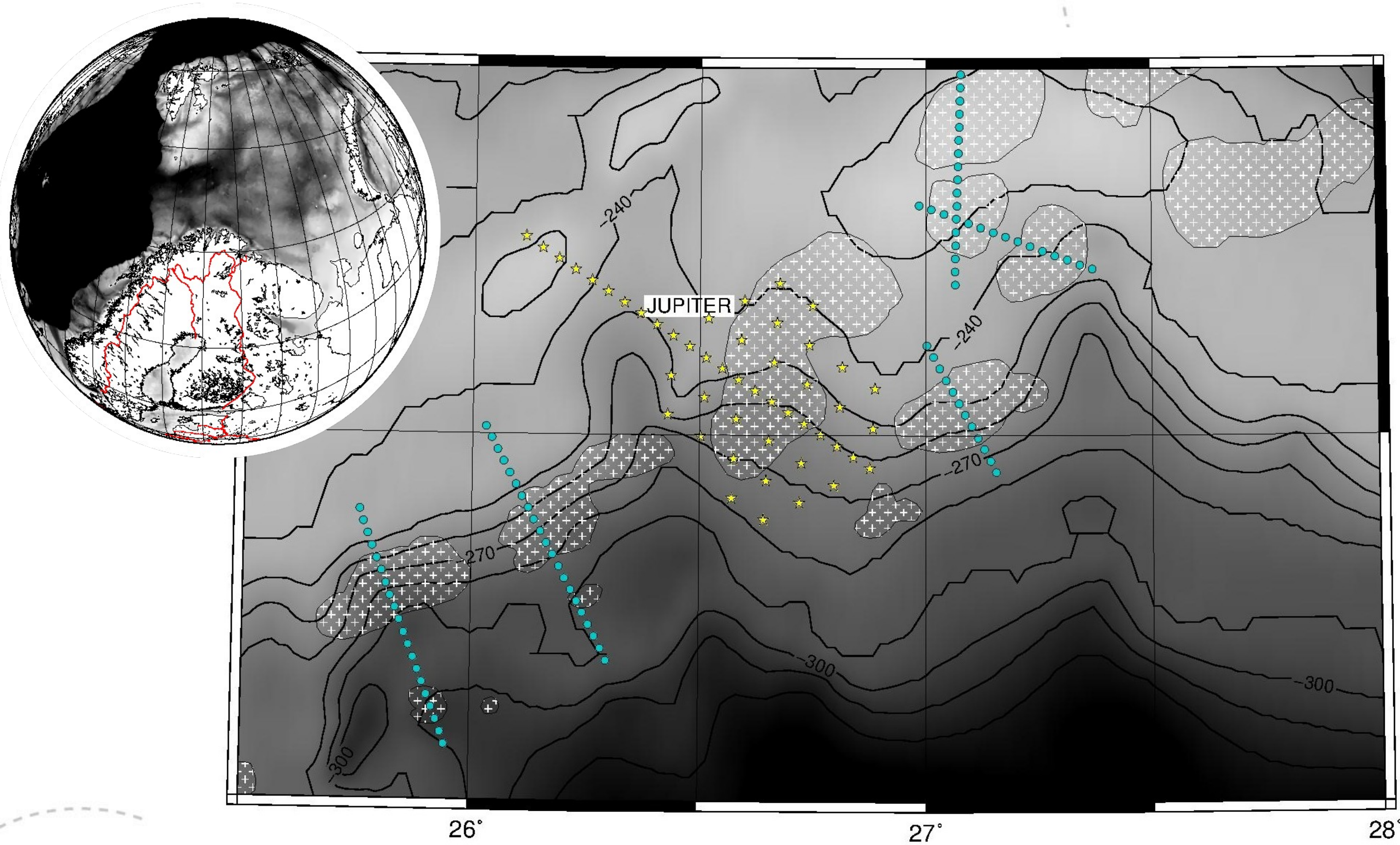
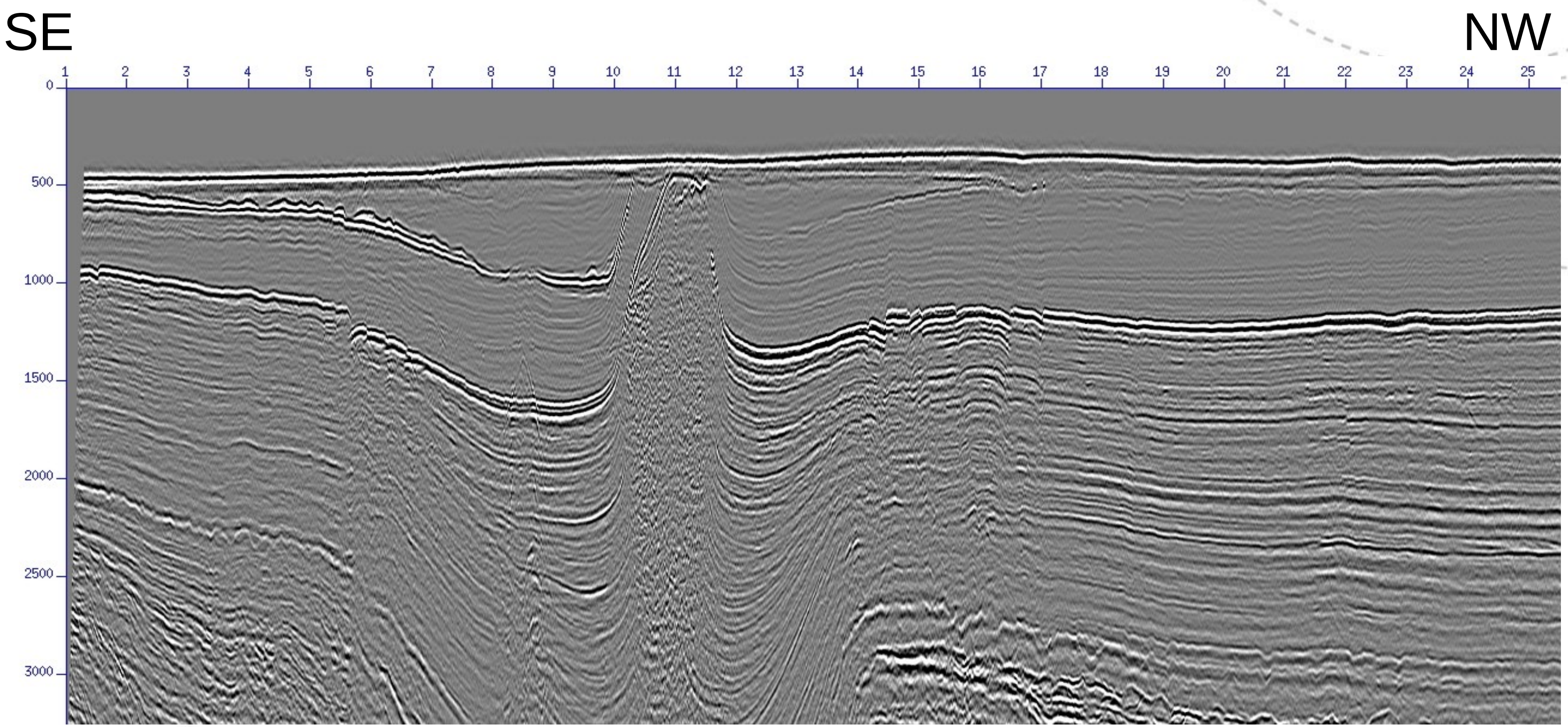
Survey Area



Motivation

- Weak primaries, strong multiples, and diffraction caused by the salt
- Limited seismic imaging quality near the salt structures
- Alternative methods CSEM, MT, gravity, and magnetics



Receiver orientation analysis

Photo by K. Hokstad

Orientation of instruments at the seabed is arbitrary and is defined by three angles:

- Azimuth (z-axis, Θ angle)
- Pitch (y-axis, α angle)
- Tilt (x-axis, β angle)

Receiver orientation analysis

- Assume source of the natural EM field to be a plane wave field and utilize 3 components of the magnetic field to estimate the receiver orientation on seabed

1. Tilt and Pitch angle

$$\min_{\alpha_k \beta_k} \sum_{t_0}^{t_n} \left| H_z^{ref}(t) - \bar{H}_z^{k}(t) \right|$$

2. Separate 1D downgoing field

$$H_x^{(D)} = H_x - \left[\frac{1}{2} (H_x + \tilde{c}\tilde{\varepsilon} E_y) \right]$$

$$H_y^{(D)} = H_y - \left[\frac{1}{2} (H_y - \tilde{c}\tilde{\varepsilon} E_x) \right]$$

$$\tilde{c}\tilde{\varepsilon} = \sqrt{\frac{i\sigma}{\mu_0 \omega}}$$

(Amundsen et al. 2006)

3. Azimuth angle

$$\min_{\theta_x^k, \sigma_{seafloor}} \sum_{t_0}^{t_n} \left| \left\{ |H_x^{D,ref}(t) - \hat{H}_x^{D,k}(t)| - |H_y^{D,ref}(t) - \hat{H}_y^{D,k}(t)| \right\} \right|^2$$

(Mütschard et al., subm. to Geophysics 2012)

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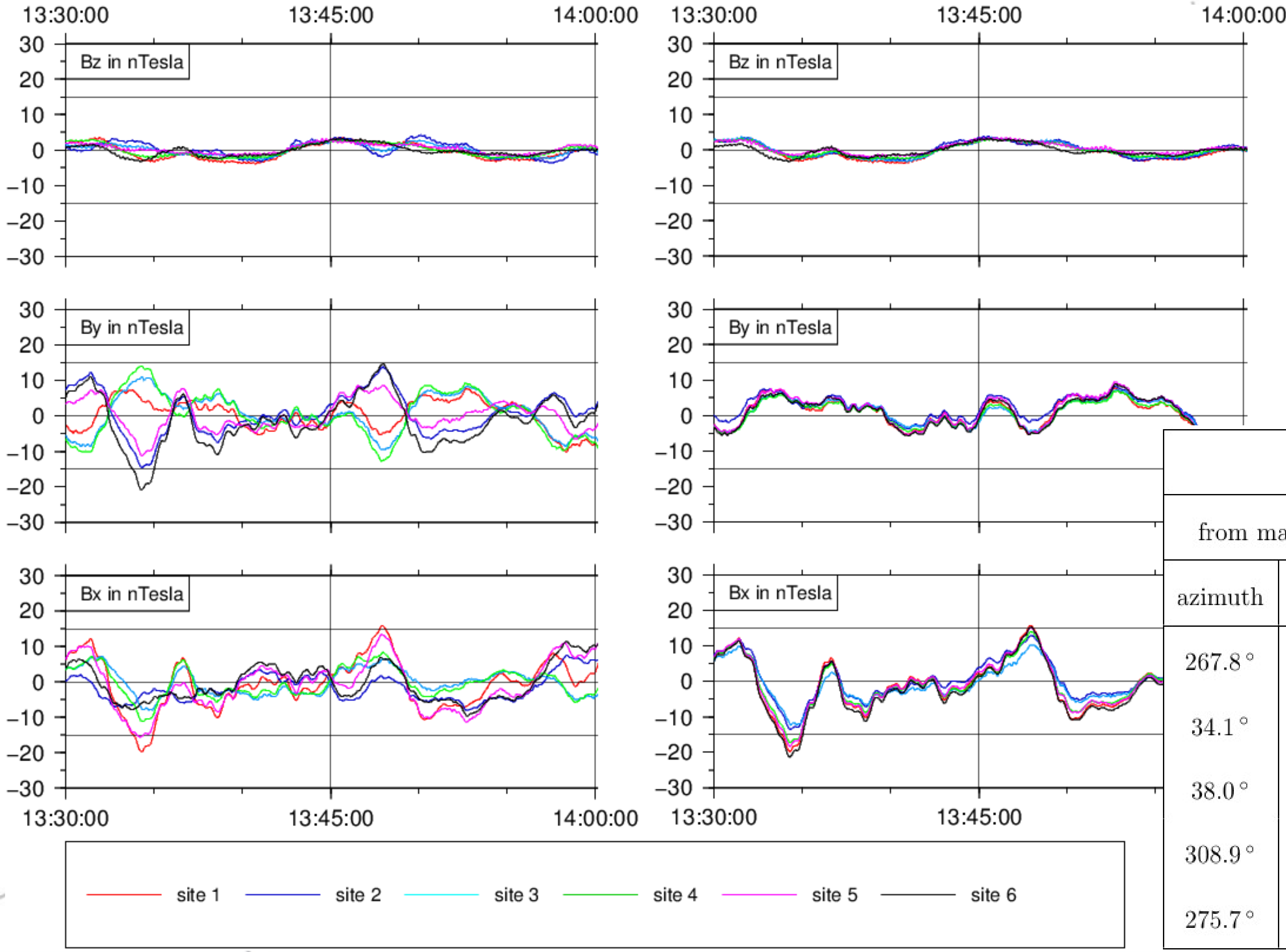
Receiver orientation analysis



Orientation			
from magnetotellurics			from CSEM
azimuth	pitch	tilt	azimuth
267.8°	10.0°	−13.0°	273.28°
34.1°	10.0°	8.0°	33.01°
38.0°	4.0°	−2.0°	41.07°
308.9°	0.0°	−8.0°	307.52°
275.7°	1.0°	0.0°	276.05°

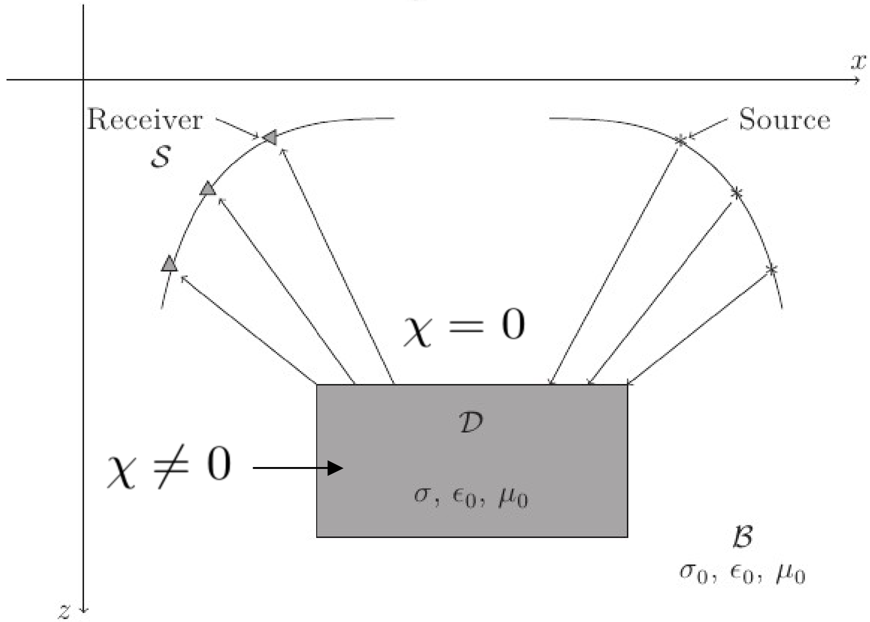
Joint inversion of CSEM and MT data

CSEM	Magnetotellurics
Active method, periodically alternating electric dipole	Passive method, natural occuring plane wave EM source field
0.1Hz - 20Hz	0.001Hz - 10Hz
Skin depth 5km in marine environm.	Skin depth up to 50km
High resolution, number of source positions	Low resolution, receiver spacing
Sensitive to resistors in a conductive background	Sensitive to conductors in a resistive background
Vertical + horizontal resistivity	Horizontal resistivity

Joint inversion of CSEM and MT data

3D Contrast source inversion of the scattered electric field

Model devided into a background B and anomalous D region



$$e_i(\mathbf{x}) = e_i^{\mathrm{inc}}(\mathbf{x}) + \int_{\mathcal{D}} G_{ij}^{E}(\mathbf{x}, \mathbf{x}')\, \sigma_{0,v}(\mathbf{x}')\, \chi_{jj}(\mathbf{x}')\, e_j(\mathbf{x}')\, \mathrm{d}\mathbf{x}', \quad \mathbf{x} \in \mathcal{D}$$

Total field = background field + scattered field (Lippmann – Schwinger equation)

χ is the contrast $\quad \mathcal{W} = \chi e$ is the contrast source

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Joint inversion of CSEM and MT data

Definition of:

1) Contrast sources

$$\mathcal{W}_{\mathrm{CSEM}} = \left\{\mathbf{w}^{j,k}_{\mathrm{CSEM}}\right\}_{j=1\ldots N_s}^{k=1\ldots N_{f_{\mathrm{CSEM}}}} = \left\{\chi\mathbf{e}^{j,k}_{\mathrm{CSEM}}\right\}_{j=1\ldots N_s}^{k=1\ldots N_{f_{\mathrm{CSEM}}}}$$

$$\mathcal{W}_{\mathrm{MT}} = \left\{\mathbf{w}^{k}_{\mathrm{MT}}\right\}^{k=1\ldots N_{f_{\mathrm{MT}}}} = \left\{\chi\mathbf{e}^{k}_{\mathrm{MT}}\right\}^{k=1\ldots N_{f_{\mathrm{MT}}}},$$

2) Data operators

$$\mathbf{e} = \mathbf{e}^{\mathrm{inc}} + G^{E,\mathcal{D}}\mathbf{w}$$

$$\mathbf{f}^{E} = G^{E,\mathcal{S}}\mathbf{w}$$

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Joint inversion of CSEM and MT data

Objective function:

$$
\begin{aligned}
F_1\left(\mathcal{W}_{\mathrm{CSEM}}, \mathcal{W}_{\mathrm{MT}}, \chi\right) = \; & \alpha_{1,\mathrm{CSEM}}^{E} \sum_{k=1}^{N_{\mathrm{f_{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \Xi_{\mathrm{CSEM}}^{E,j,k} \left(\mathbf{f}_{\mathrm{CSEM}}^{E,j,k} - G^{E,\mathcal{S},k} \mathbf{w}_{\mathrm{CSEM}}^{j,k} \right) \right\|_{\mathcal{S}}^{2} \\
+ \; & \alpha_{2,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f_{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \chi \mathbf{e}_{\mathrm{CSEM}}^{\mathrm{inc},j,k} - \mathbf{w}_{\mathrm{CSEM}}^{j,k} + \chi G^{E,\mathcal{D},k} \mathbf{w}_{\mathrm{CSEM}}^{j,k} \right\|_{\mathcal{D}}^{2} \\
+ \; & \alpha_{1,\mathrm{MT}}^{E} \sum_{k=1}^{N_{\mathrm{f_{MT}}}} \left\| \Xi_{\mathrm{MT}}^{E,k} \left(\mathbf{f}_{\mathrm{MT}}^{E,k} - G^{E,\mathcal{S},k} \mathbf{w}_{\mathrm{MT}}^{k} \right) \right\|_{\mathcal{S}}^{2} \\
+ \; & \alpha_{2,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f_{MT}}}} \left\| \chi \mathbf{e}_{\mathrm{MT}}^{\mathrm{inc},k} - \mathbf{w}_{\mathrm{MT}}^{k} + \chi G^{E,\mathcal{D},k} \mathbf{w}_{\mathrm{MT}}^{k} \right\|_{\mathcal{D}}^{2}
\end{aligned}
$$

CSEM data fidelity

Lippmann Schwinger

MT data fidelity

Lippmann - Schwinger

+ regularization

(Wiik, to be subm. to Geophys. Prospecting 2012)

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Joint inversion of CSEM and MT data

Input: Initial contrast and contrast sources

foreach *iteration* **do**
- **foreach** *CSEM frequency* **do**
  - **foreach** *source* **do**
    - Minimize equation F_1 with respect to $\mathbf{w}^{j,k}_{\mathrm{CSEM}}$
  - **end**
- **end**
- **foreach** *MT frequency* **do**
  - Minimize equation F_1 with respect to $\mathbf{w}^{k}_{\mathrm{MT}}$
- **end**
- Minimize equation F_1 with respect to $\boldsymbol{\chi}$
- **if** *stop criterion is true* **then**
  - stop iterations
- **else**
  - proceed to next iteration
- **end**

end

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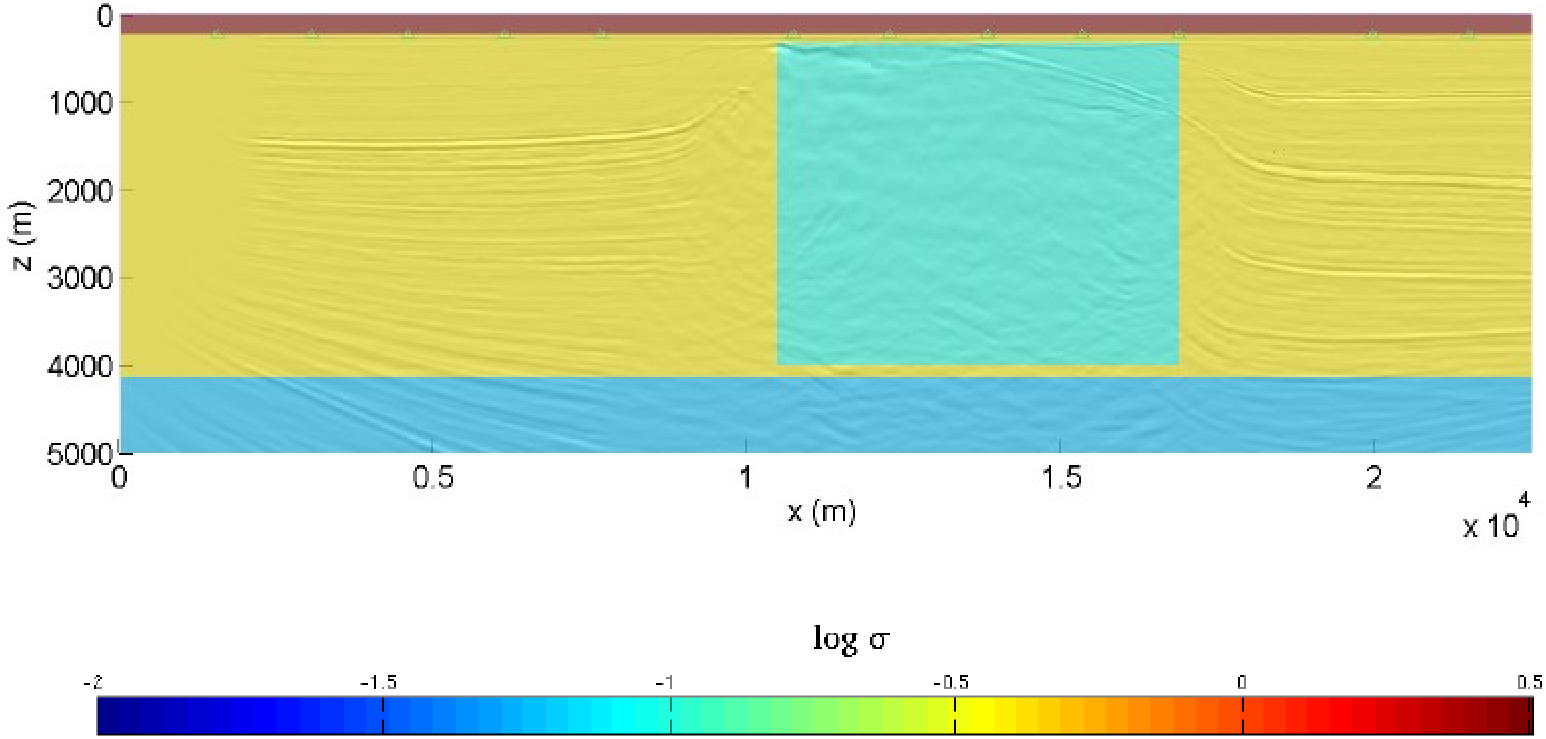
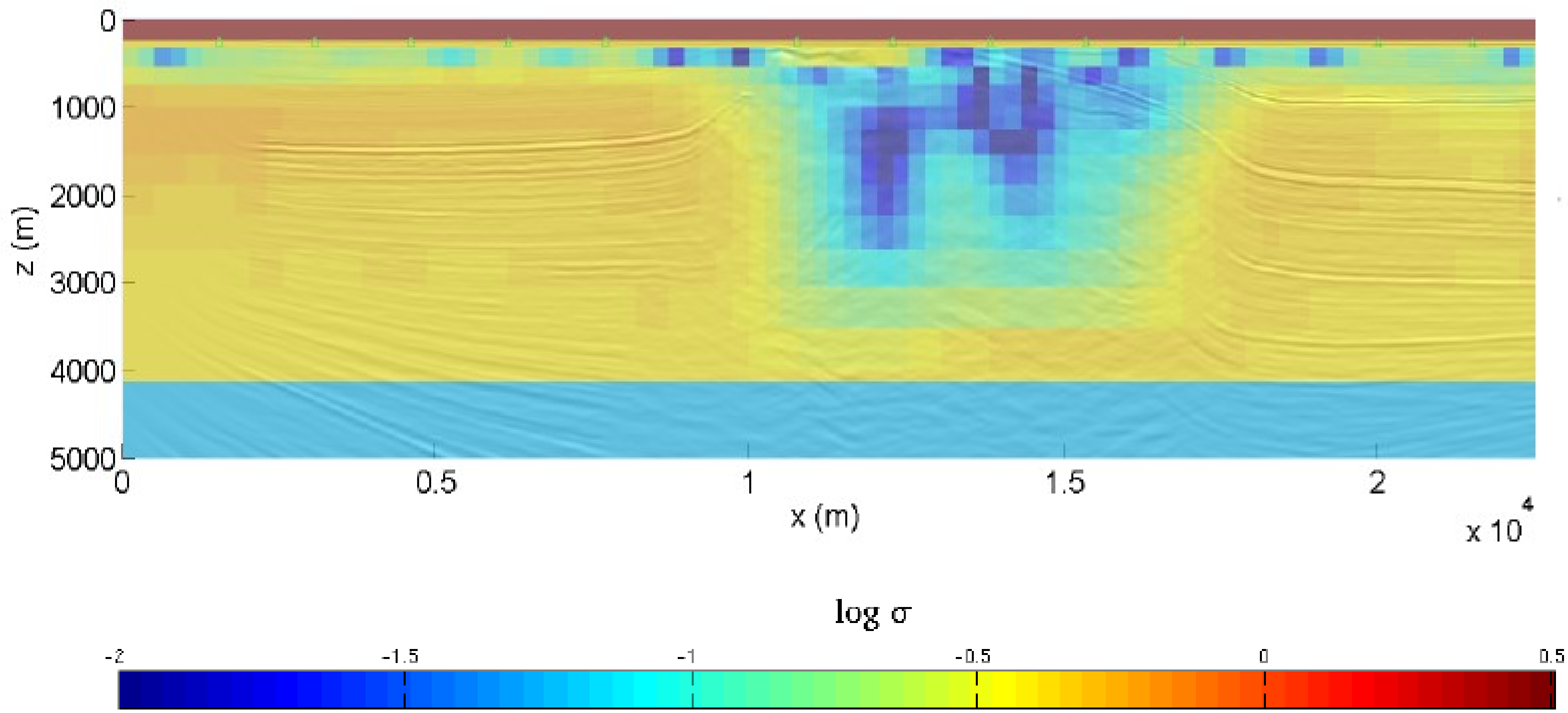
Joint inversion of CSEM and MT data



Initial model for MT and CSEM inversion

Joint inversion of CSEM and MT data

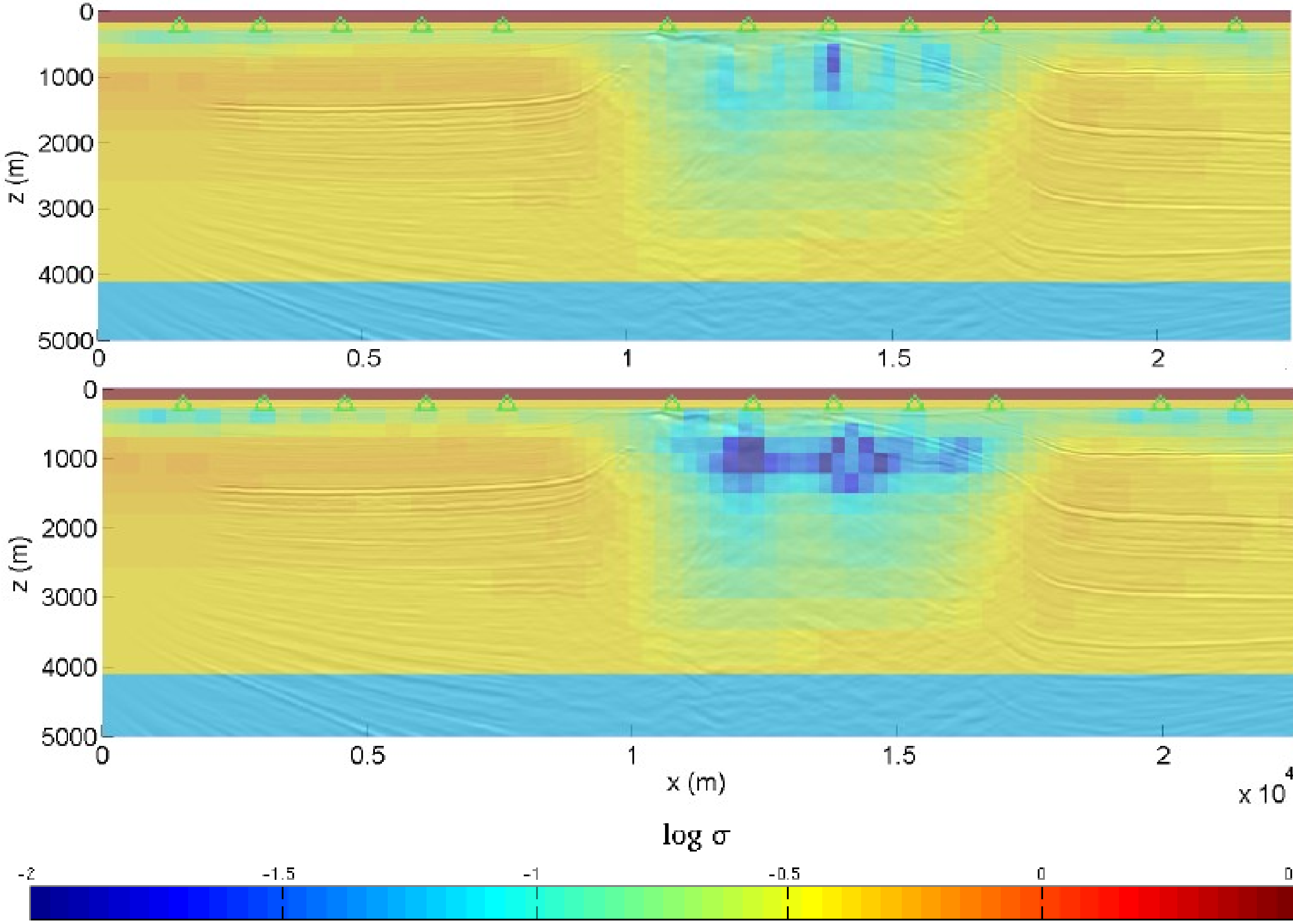


Magnetotelluric data only

Joint inversion of CSEM and MT data

Horizontal CSEM

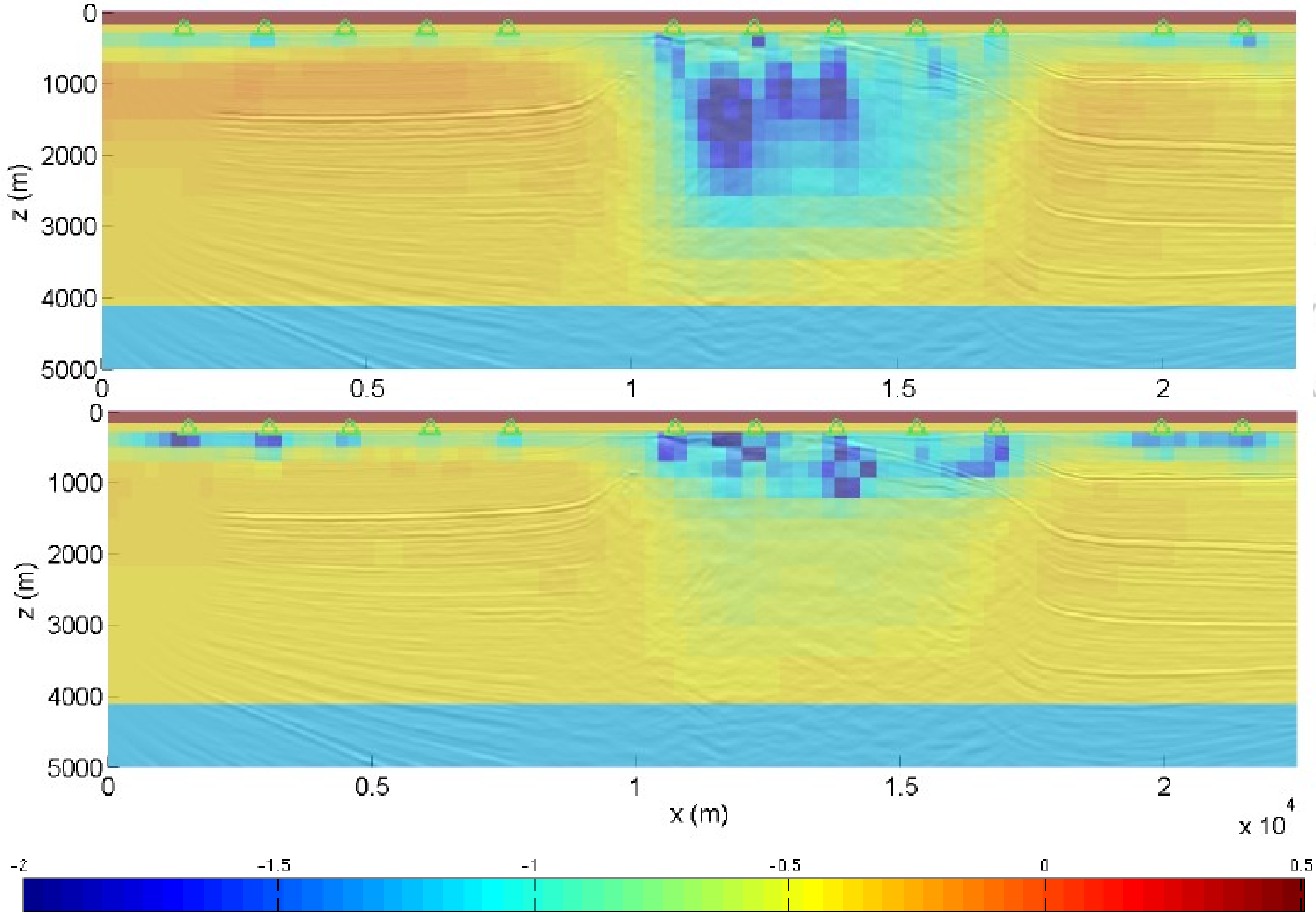
Vertical CSEM



Joint inversion of CSEM and MT data

Joint horizontal

Joint vertical



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Forskningsrådet

Statoil

GDF SUEZ

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$$\mathbf{R_z} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R_y} = \begin{pmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

$$\mathbf{R_x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{pmatrix}$$

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}' = \mathbf{R_z}(\theta)\mathbf{R_y}(\alpha)\mathbf{R_x}(\beta) \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$$