GEOMECHANICS FOR GEOPHYSICISTS

Reservoir Geomechanics

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SINTEF NTNU

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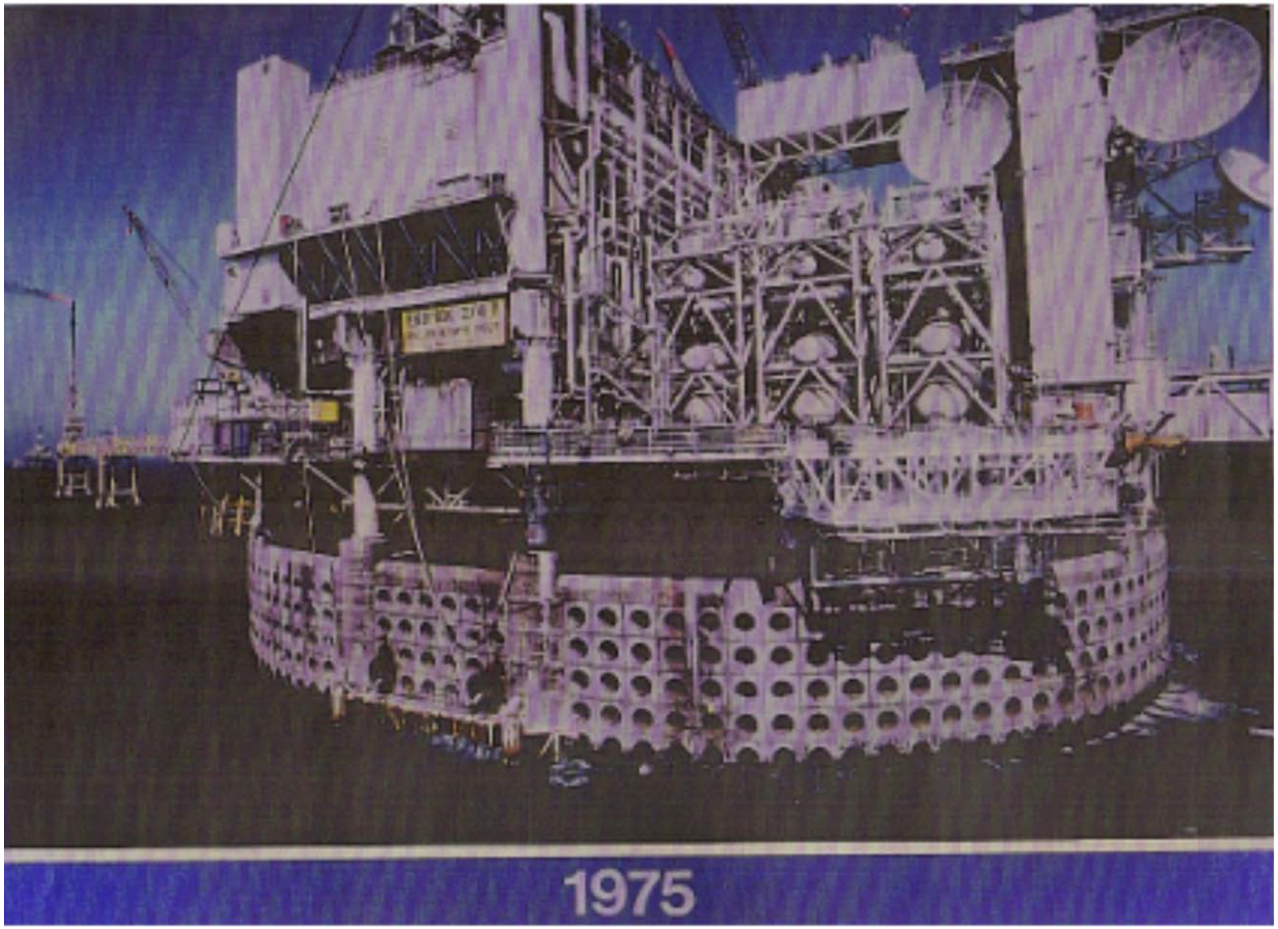

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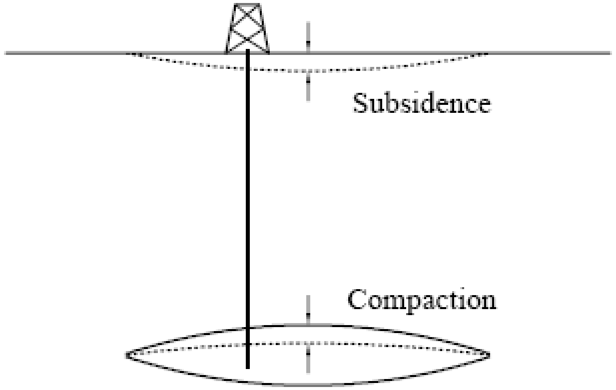
Reservoir compaction & Surface subsidence

Operational problems:



⇒ Offshore platform safety

⇒ Environmental challenges

⇒ Casing collapse in reservoir

⇒ Associated seismicity

Solutions:

•Account for possible compaction & subsidence in platform and casing design.

•Pressure maintenance.

•Platform jack-up ($$$).

Compaction is also a drive mechanism ⇒ Enhanced recovery.

Remember…

Biot-Hooke's law

- Utilizing the effective stress principle, we can use Hooke's law as for solids – but with effective stresses replacing total stresses, and frame moduli replacing solid moduli (only normal stresses shown):

$$\varepsilon_x = \frac{1}{E_{fr}}\Delta\sigma_x' - \frac{\nu_{fr}}{E_{fr}}\Delta\sigma_y' - \frac{\nu_{fr}}{E_{fr}}\Delta\sigma_z'$$

$$\varepsilon_y = -\frac{\nu_{fr}}{E_{fr}}\Delta\sigma_x' + \frac{1}{E_{fr}}\Delta\sigma_y' - \frac{\nu_{fr}}{E_{fr}}\Delta\sigma_z'$$

$$\varepsilon_z = -\frac{\nu_{fr}}{E_{fr}}\Delta\sigma_x' - \frac{\nu_{fr}}{E_{fr}}\Delta\sigma_y' + \frac{1}{E_{fr}}\Delta\sigma_z'$$

Uniaxial Reservoir Compaction

Usual assumptions:

- (Linear) Elastic rock behaviour.
- Uniaxial compaction (*no lateral strain*).
- Vertical stress fully carried by reservoir (*no arching*).
- (Often Biot's α is set =1).

Hooke's law $\Rightarrow$

$$\varepsilon_v = -\frac{\Delta h}{h} = \frac{\alpha(1-2\nu_{fr})(1+\nu_{fr})}{E_{fr}(1-\nu_{fr})}(-\Delta p_f) = \frac{\alpha(-\Delta p_f)}{H_{fr}}$$

Δh	change in reservoir thickness (<0: compaction)
ε_v	vertical strain
h	reservoir thickness
Δp_f	pore pressure change

Uniaxial Reservoir Compaction

Uniaxial compaction modulus:

$$H_{fr} = \frac{E_{fr}(1-\nu_{fr})}{(1-2\nu_{fr})(1+\nu_{fr})} = K_{fr} + \frac{4}{3}G_{fr} = \lambda_{fr} + 2G_{fr}$$

Note also (but remember static < dynamic moduli):

$$H_{fr} = (\rho v_p^2)_{dry}$$

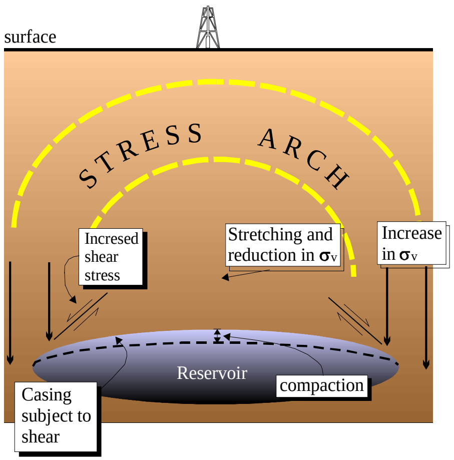
Predicted lateral stress change:

$$\frac{\Delta\sigma_h^{'}}{\Delta\sigma_v^{'}} = \frac{\nu_{fr}}{1-\nu_{fr}}$$

Reservoir Stress Path

We assumed the reservoir to carry the full weight of the overburden & uniaxial compaction during depletion – only valid if the reservoir is infinitely thin & wide (”pancake”)

In general cases, we need to define stress path coefficients (as suggested by Hettema *et al*., 2000):



$$\gamma_v = \frac{\Delta \sigma_v}{\Delta p_f}$$

$$\gamma_h = \frac{\Delta \sigma_h}{\Delta p_f}$$

Stress arching coefficient

$$\kappa = \frac{\Delta \sigma_h'}{\Delta \sigma_v'}$$

Reservoir Stress Path

- General relationship between stress path coefficients:

$$\kappa = \frac{1-\frac{\gamma_h}{\alpha}}{1-\frac{\gamma_v}{\alpha}}$$

- Effective stress path coefficients:

$$\gamma_h^{'} = \gamma_h - \alpha$$
$$\gamma_v^{'} = \gamma_h - \alpha$$

Reservoir Stress Path: Impact on Compaction

- Within limits of linear poroelasticity, reservoir compaction is given by:

$$\frac{-\Delta h}{h} = \alpha \frac{(1-\frac{\gamma_v}{\alpha}) - 2\nu_{fr}(1-\frac{\gamma_h}{\alpha})}{E_{fr}}(-\Delta p_f)$$

or

$$-\frac{\Delta h}{h} = \frac{\gamma_v^{'} - 2\nu_{fr}\gamma_h^{'}}{E_{fr}}\Delta p_f$$

h:	reservoir thickness
α:	Biot coefficient
E_{fr}, ν_{fr}:	Drained Young's modulus & Poisson's ratio for reservoir rock

Reservoir Stress Path

- **The stress path is controlled by**
  - Depleting reservoir geometry (shape; inclination)
  - Elastic contrast between reservoir and surroundings
  - Non-elastic / Failure processes

- **Models:**
  - Analytical: Rudnicki's ellipsoidal inclusion model (1999)
  - Analytical: Geertsma's Nucleus of Strain model (1973)
  - Numerical: Finite Element Method (Morita *et al*, 1989; Mulders, 2003); Discrete Element Method (Alassi PhD Thesis NTNU 2008)

Reservoir Stress Path

- **Rudnicki (1999):**
  - The reservoir is assumed to be an ellipsoidal poroelastic inclusion in an infinite solid medium (short axis || vertical).
    - Limits validity to reservoirs that are deeper than their lateral extent
  - The strains resulting from pore pressure change is calculated for a stress-free reservoir.
  - The stresses required to restore the original reservoir shape & size are calculated.
  - These stresses are added to the initial *in situ* stresses.
  - Elastic contrast between reservoir and surroundings permitted.

Reservoir Stress Path

- Solutions are expressed in terms of the aspect ratio *e=h/2R* (reservoir thickness divided by diameter).
- Note that h and R refer to the dimensions of the zone where pore pressure actually changes (e.g. depleting zone).

$$\gamma_h = \alpha \frac{1-2\nu_{fr}}{1-\nu_{fr}} \left[1 - \frac{e}{2\sqrt{(1-e^2)^3}} \left(\arccos e - e\sqrt{1-e^2} \right) \right]$$

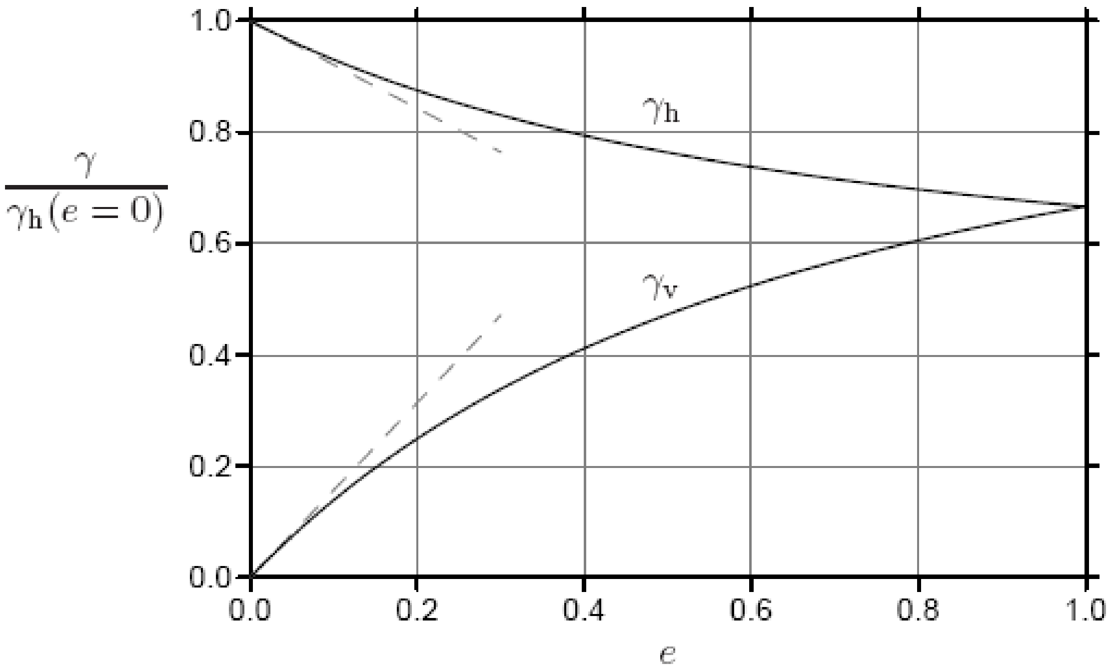
$$\gamma_v = \alpha \frac{1-2\nu_{fr}}{1-\nu_{fr}} \frac{e}{\sqrt{(1-e^2)^3}} \left(\arccos e - e\sqrt{1-e^2} \right)$$

- For small values of *e*, these equations can be approximated as:

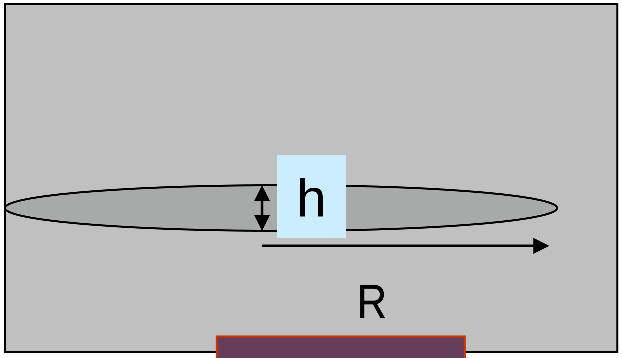
$$\gamma_h = \alpha \frac{1-2\nu_{fr}}{1-\nu_{fr}} \left(1 - \frac{\pi}{4} e \right)$$

$$\gamma_v = \alpha \frac{1-2\nu_{fr}}{1-\nu_{fr}} \frac{\pi}{2} e$$

Reservoir Stress Path



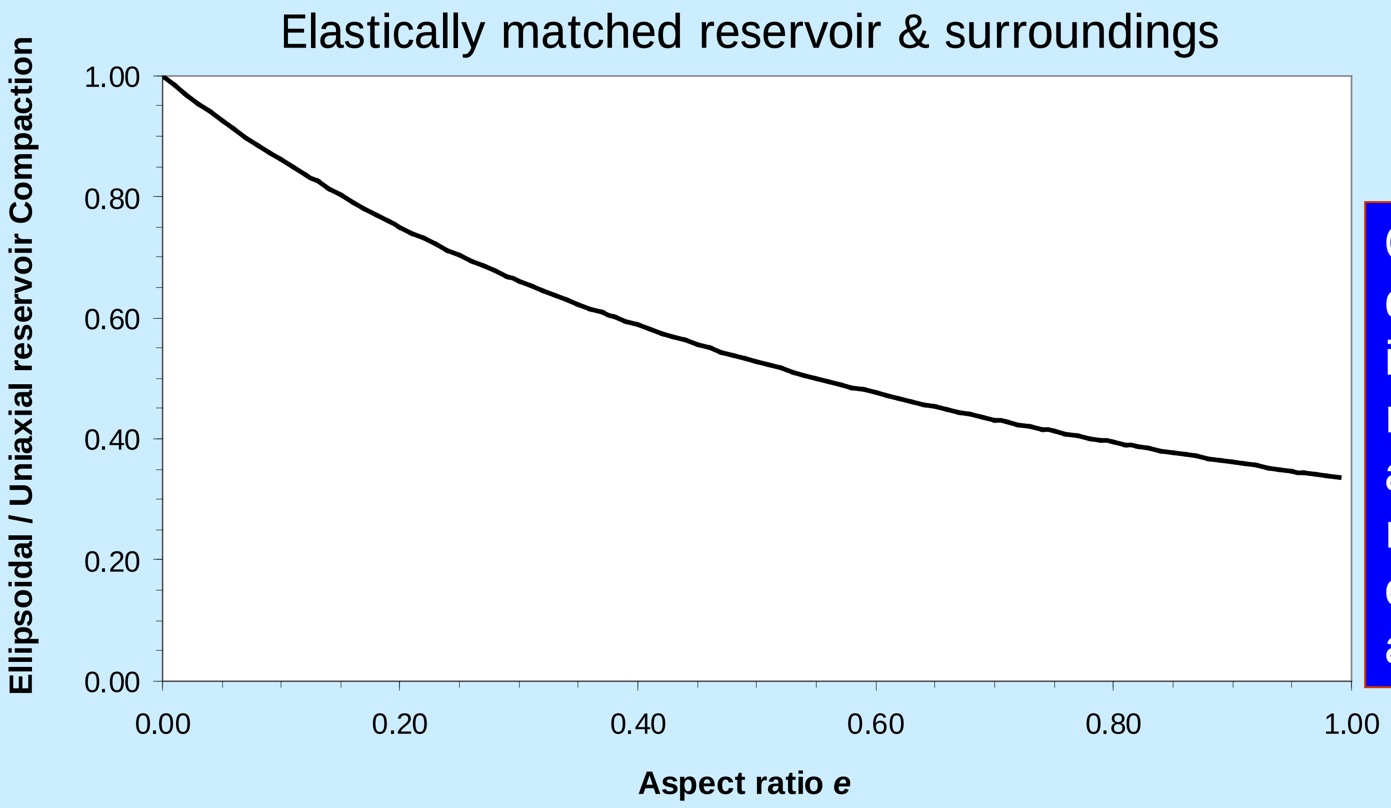
Only for [European] pancake shaped reservoir (e=0) is the uniaxial strain & no arching assumption fulfilled.



e=h/2R

Stress path coefficients from Rudnicki's model; Reservoir is elastically matched to the surroundings (Poisson's ratio = 0.20)

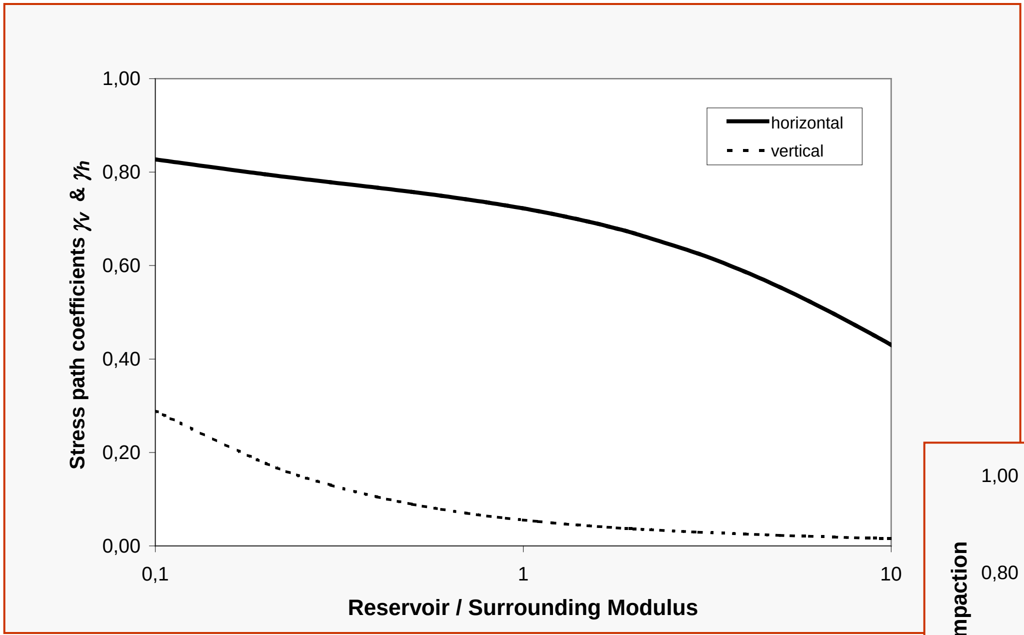
Reservoir Stress Path: Impact on Compaction



Compaction decreases with increasing reservoir aspect ratio, reflecting enhanced arching

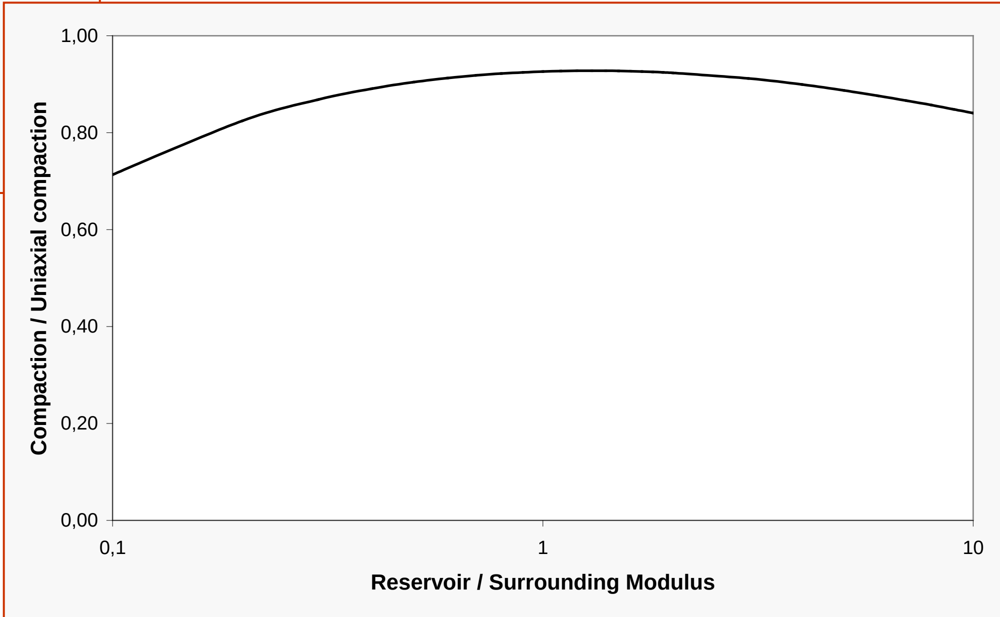
Based on Rudnicki (1999)

Reservoir Stress Path: Impact on Compaction



Effect of elastic (shear modulus) mismatch:

- Soft reservoir: Enhanced arching
- Stiff reservoir: Reduced horizontal stress change



The classical approach (uniaxial strain + no elastic contrast) is the most conservative

Compaction Drive

- **Pore compressibility from laboratory tests (Zimmerman, 1991):**

$$C_{pc} = -\frac{1}{V_p}\left(\frac{\Delta V_p}{\Delta \sigma}\right)_{p_f = const} = \frac{1}{\phi}\left(\frac{1}{K_{fr}} - \frac{1}{K_s}\right)$$

$$C_{pp} = \frac{1}{V_p}\left(\frac{\Delta V_p}{\Delta p_f}\right)_{\sigma = const} = \frac{1}{\phi}\left(\frac{1}{K_{fr}} - \frac{1}{K_s}\right) - \frac{1}{K_s}$$

φ:	Porosity
K_{fr}:	Drained rock bulk modulus
K_s:	Solid mineral bulk modulus

- P

$$C_{pp}{}^{\gamma} = \frac{1}{V_p}\frac{\Delta V_p}{\Delta p_f} = \frac{1-\bar{\gamma}}{\phi}\left(\frac{1}{K_{fr}} - \frac{1}{K_s}\right) - \frac{1}{K_s}$$

$$\bar{\gamma} = \frac{1}{3}\left(\gamma_v + \gamma_H + \gamma_h\right)$$

Compaction drive

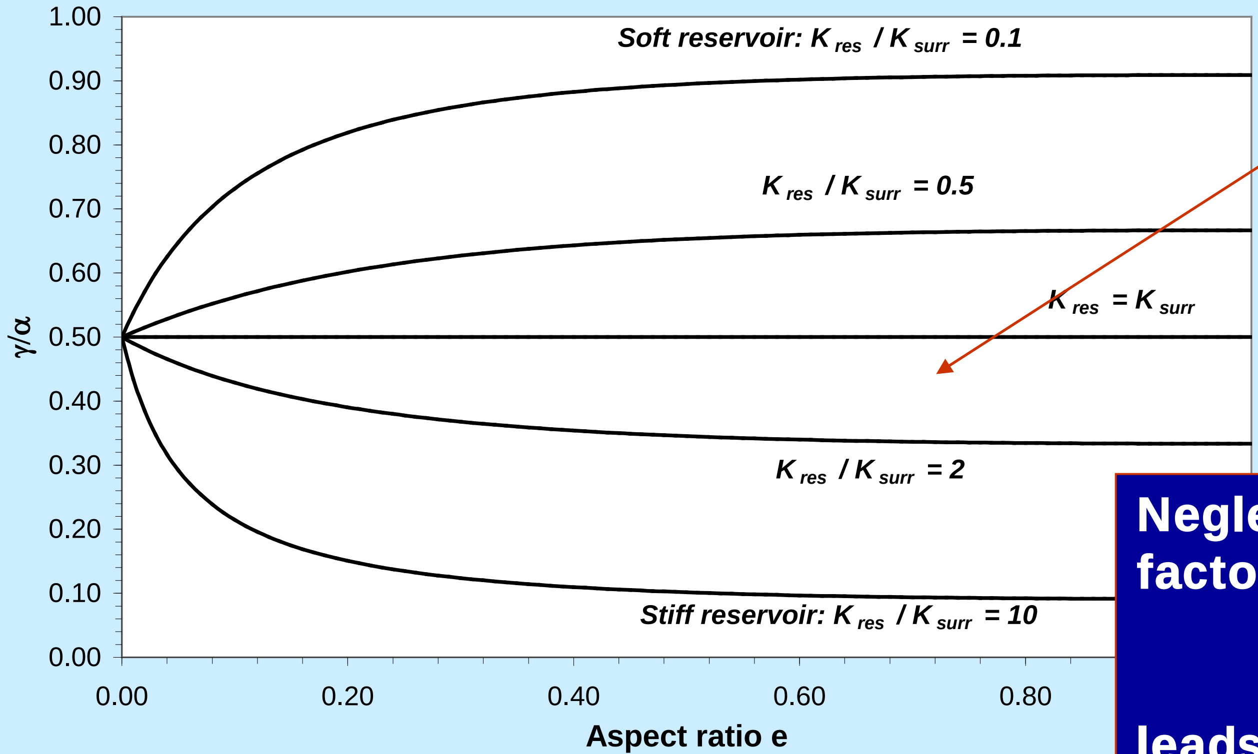
$$\zeta = -\phi\left(\frac{\Delta V_p}{V_p} + \frac{\Delta p_f}{K_f}\right)$$

$$\Delta V_{prod} = V_p \left|\Delta p_f\right| (C_f + C_{pp}^{\gamma})$$

Pore compressibility will lead to enhanced production. This is what we call *compaction drive*.

- **Relevant in soft rock reservoirs.**
- **Irrelevant in gas reservoirs.**

Compaction Drive:
Effect of Stress Path



Note: For elastically matched reservoir & surroundings, the multiplying factor

$$\bar{\gamma} = \alpha \frac{2(1-2\nu_{fr})}{3(1-\nu_{fr})} = const.$$

Neglecting the multiplying factor in pore compressibility

$$1-\bar{\gamma}$$

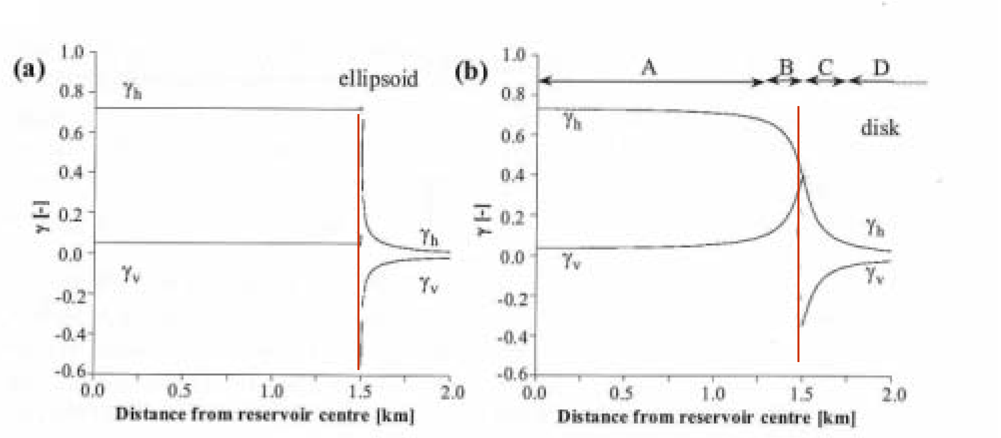
leads to overestimated compaction drive for soft reservoirs and underestimated compaction drive for stiff reservoirs

Based on Rudnicki (1999)

Reservoir Stress Path

- **Real reservoirs are not ellipsoidal –**
  - Simulations need to be done with numerical models, incorporating geometry and heterogeneity.

FEM simulations by Mulders (2003) give the same result as Rudnicki's solution near the centre of a disk shaped reservoir, but the stress path coefficients will vary with distance from the center of the reservoir.



  - The pore pressure distribution in a producing reservoir is heterogeneous (and so is the reservoir…)!