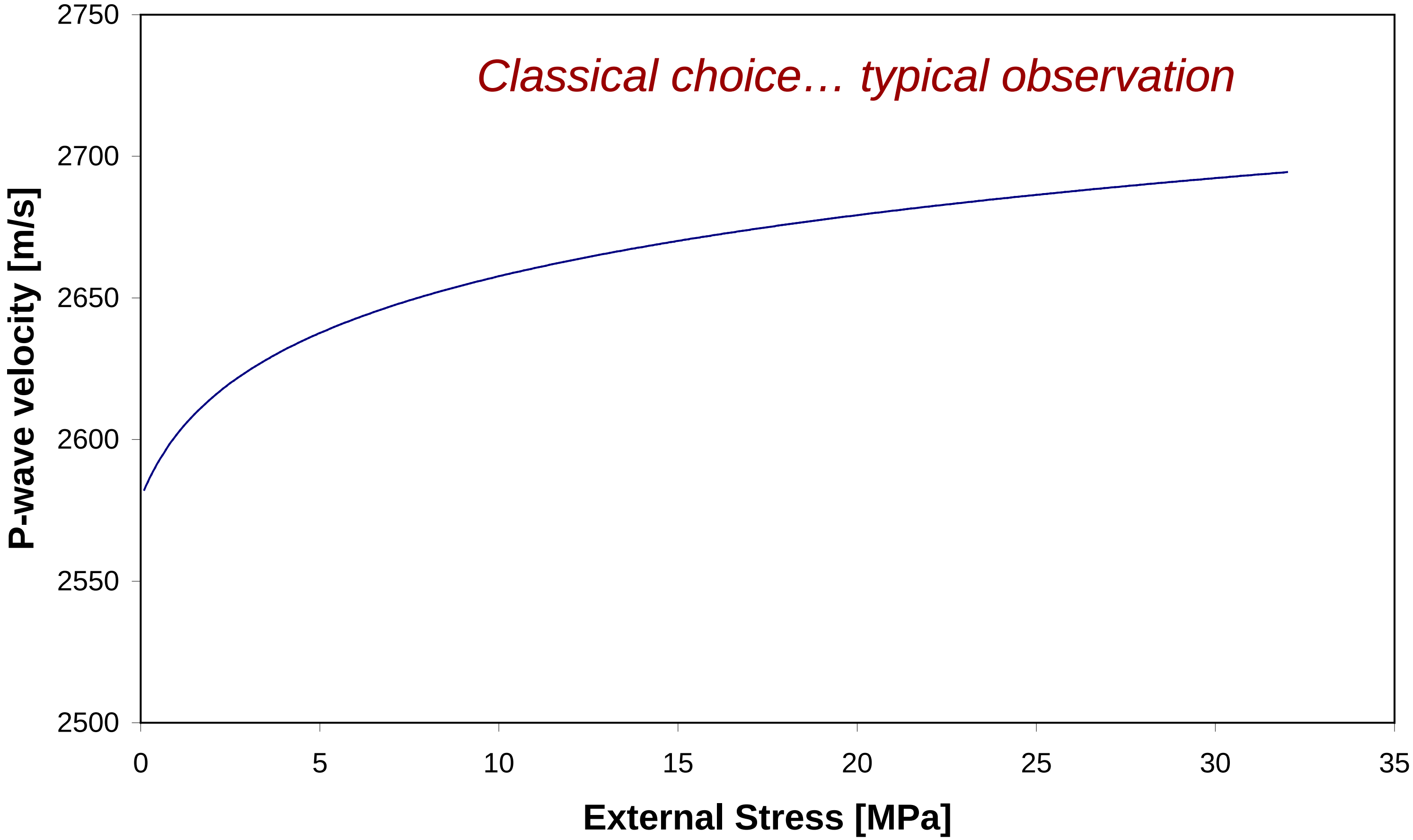
GEOMECHANICS FOR GEOPHYSICISTS

Stress Dependence of Velocities

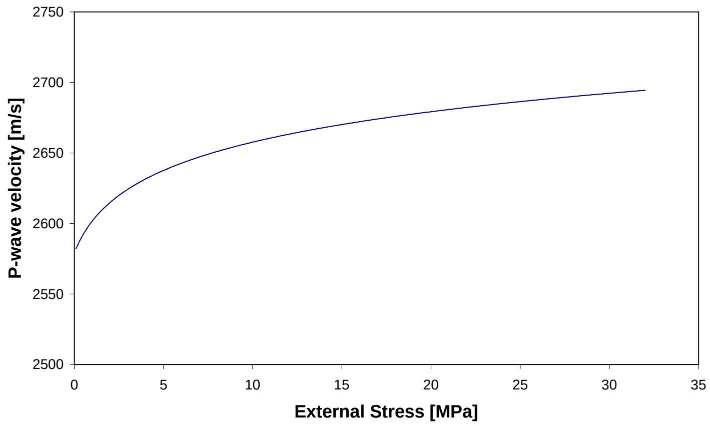
Rune M Holt

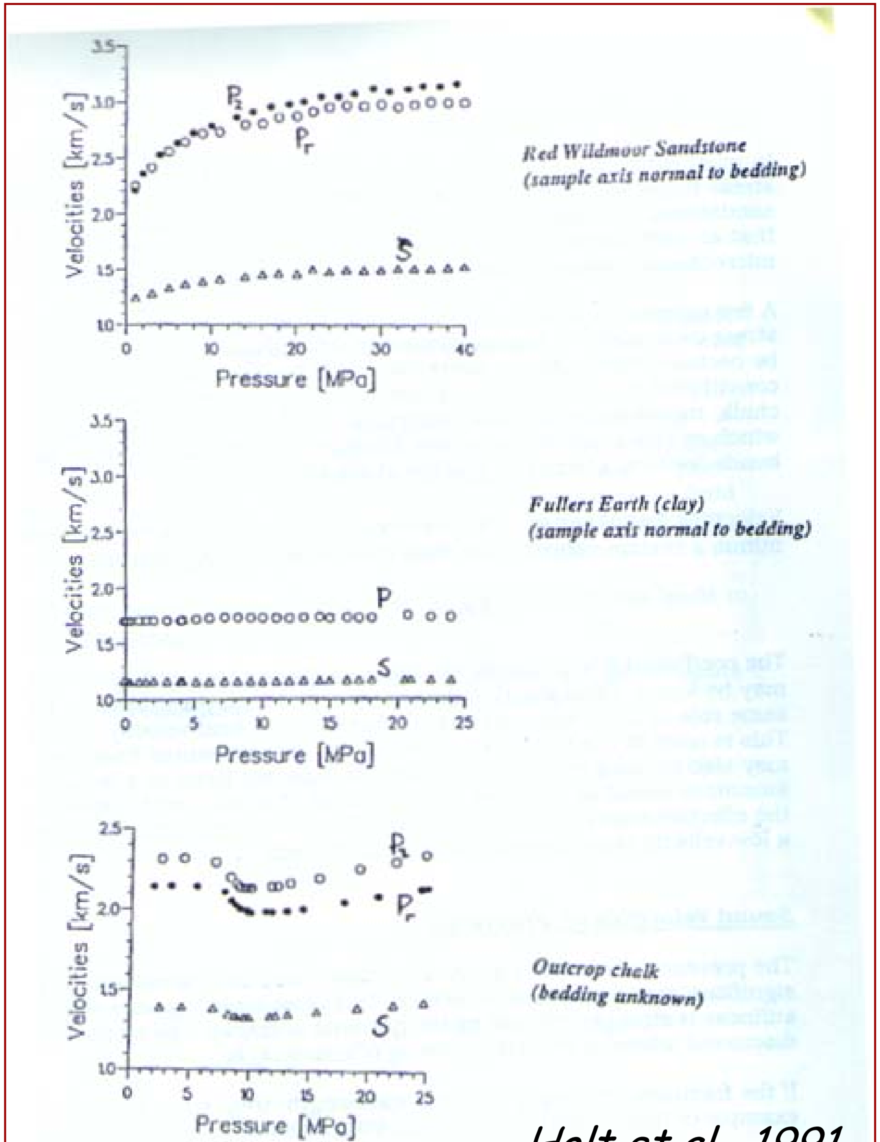
Trondheim, 25 April 2012

Stress effects on Wave velocities



Well, not always...





Holt et al., 1991

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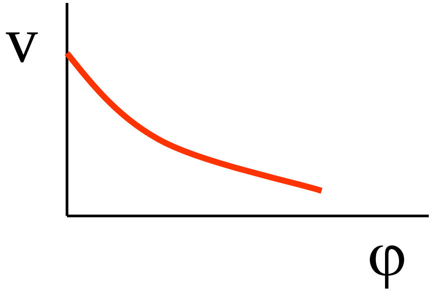
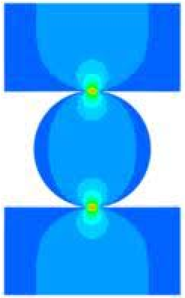
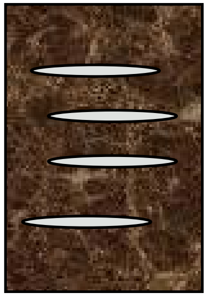
Fundamentals of Stress Dependence

- **3 main sources of stress dependence:**
  - Change in porosity with stress (can be predicted by Biot's poroelastic theory)
  - Existence of sharp (or Hertzian) grain contacts
  - Presence or generation of cracks / fractures

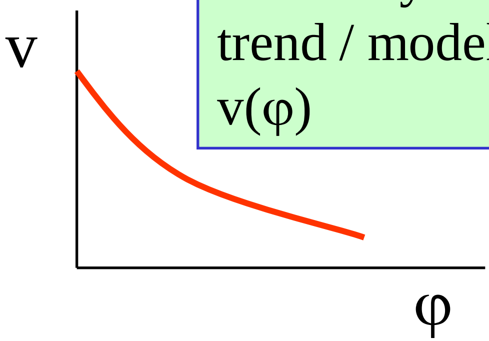






Notice: In linear elasticity, framework moduli in the Biot theory are constant by definition - thus, except for small porosity changes, stress dependence of wave velocities requires a nonlinear stress – strain relationship!

Effect of porosity change



Rock Physics trend / model v(φ)

Pore compressibility

$$\frac{\Delta v}{\Delta\overline{\sigma'}} = \frac{\Delta v}{\Delta\varphi}\cdot\frac{\Delta\varphi}{\Delta\overline{\sigma'}}$$

$$\overline{\sigma'} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} - p_f$$

Mean Net Stress

If only linked to porosity change, velocity change will depend on mean net stress

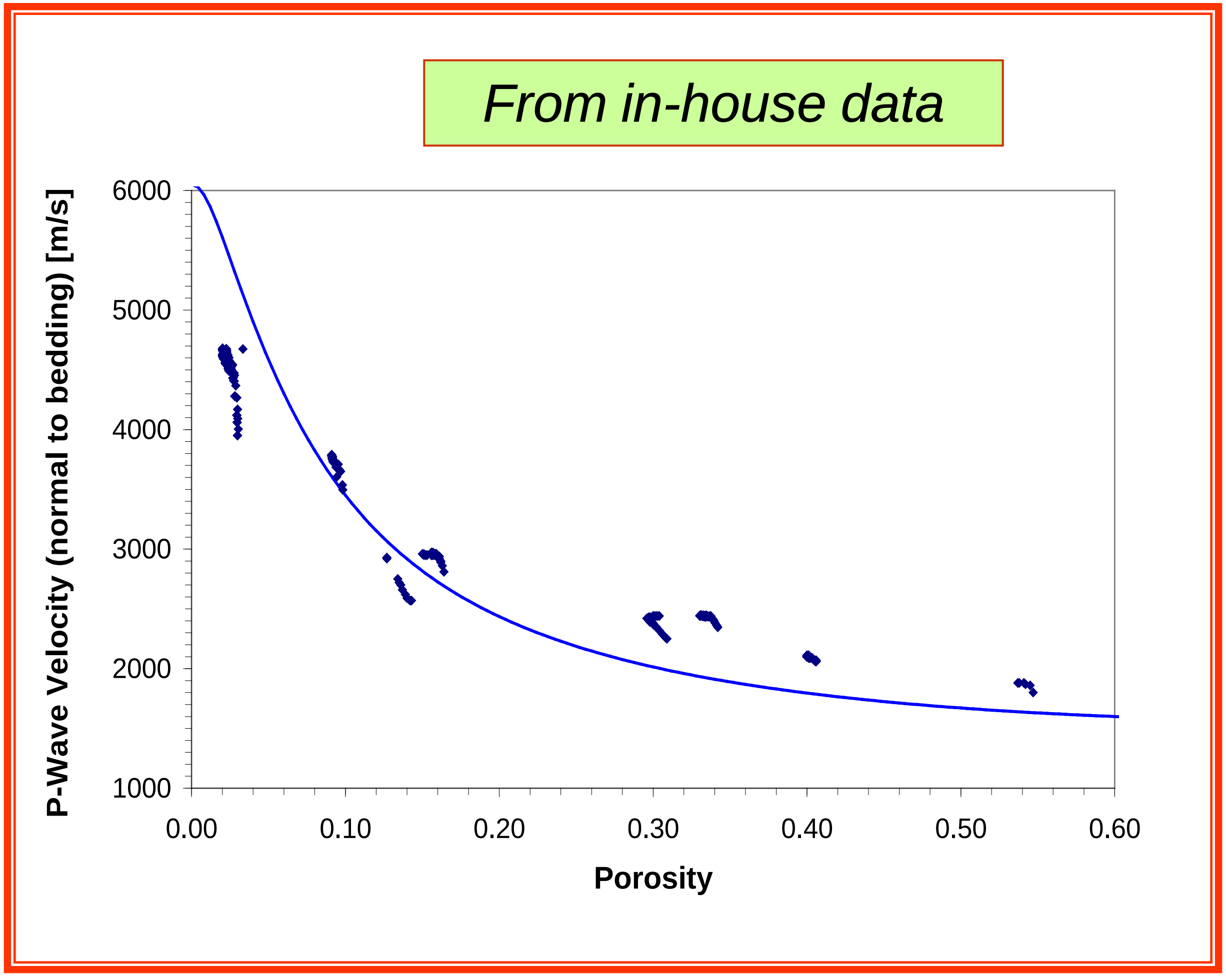
Following Biot:

$$\frac{\Delta\varphi}{\varphi} = \left(\frac{\alpha}{\varphi}-1\right)\frac{\Delta\overline{\sigma'}}{K_{fr}}$$

$\alpha = 1 - K_{fr}/K_s$

In most cases, this leads to only small velocity changes

Overburden Shales: Porosity Dependent Stress Sensitivity



The data include both hydrostatic & trixial loading

Porosity appears to be a main factor controlling stress dependence of wave velocitites in shales –

BUT NOT THE ONLY ONE...

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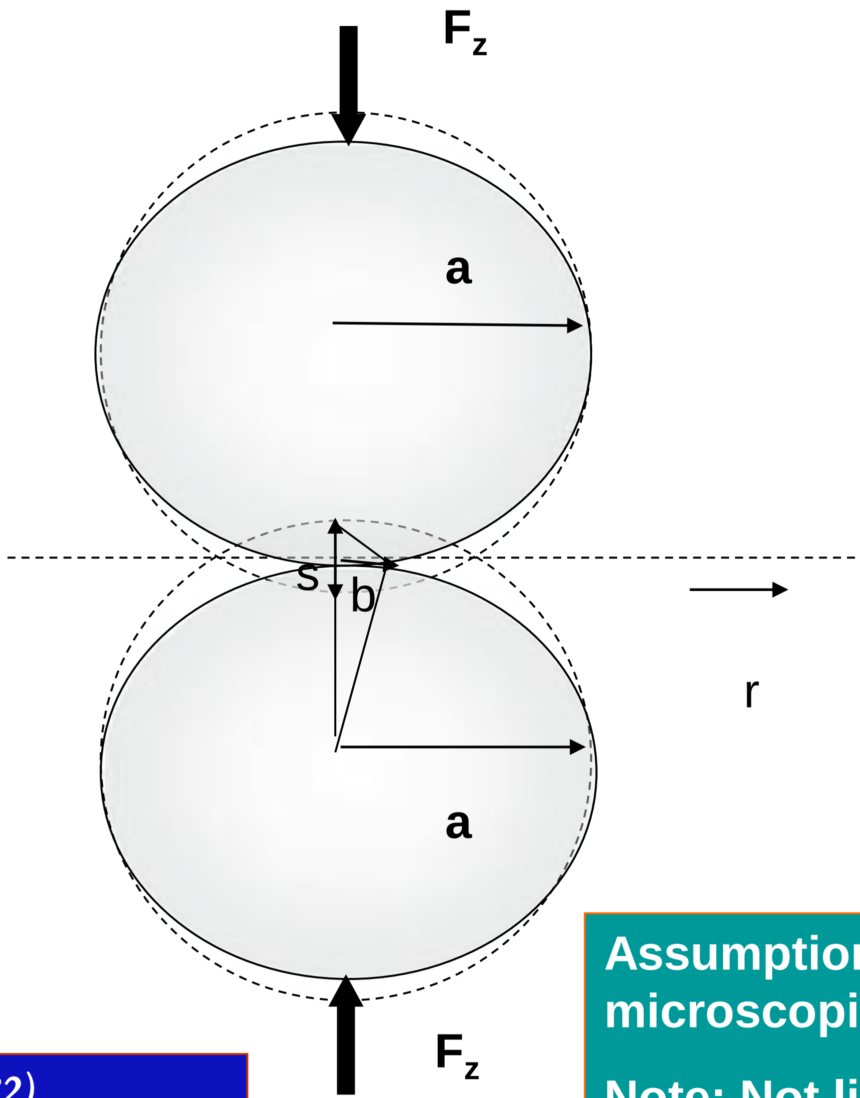
Hertzian contact theory:
Normal compression of two spheres



Normal force F_z creates a contact area between the two particles

a: radius of undeformed spheres ($a=R_1=R_2$)

b: radius of contact area between deformed spheres

s: relative displacement of sphere centers

Assumption: Particles are macroscopically and microscopically smooth

Note: Not limited to spheres

Hertz (1882)

Hertzian contact theory:
Normal compression of two spheres

- Contact stress:

(Derivation inspired by deGennes, 1996)

$$\sigma_{z,contact} \propto \frac{F_z}{b^2} \propto M_s \frac{b}{a} \propto M_s \frac{s}{b}$$

$$\Rightarrow$$

$$F_z \propto M_s sb \propto M_s s^{\frac{3}{2}} a^{\frac{1}{2}} \propto M_s a^2 \left(\frac{s}{a}\right)^{\frac{3}{2}}$$

M_s is an appropriate elastic modulus of the solid particle material

$\Rightarrow$ Macroscopic stress:

$$\sigma_{z,macro} \propto \frac{F_z}{a^2} \propto M_s \varepsilon_{z,macro}^{\frac{3}{2}}$$

$$\frac{d\sigma_z}{d\varepsilon_z} \propto \sigma_z^{\frac{1}{3}}$$

A stress dependent elastic modulus!

The Hertzian contact is a source of nonlinear elasticity

Hertzian contact theory:
Normal compression of two spheres

- The full equations (equal spheres):

$$b = \left[\frac{3F_z a}{4M_s} \right]^{\frac{1}{3}}$$

$$s = \left[\frac{9F_z^{\ 2}}{2M_s^{\ 2} a} \right]^{\frac{1}{3}}$$

$$M_s = \frac{E_s}{1 - \nu_s^2}$$

E_s & ν_s are Young's modulus & Poisson's ratio of the solid

Hertzian contact theory:
Normal compression of two spheres

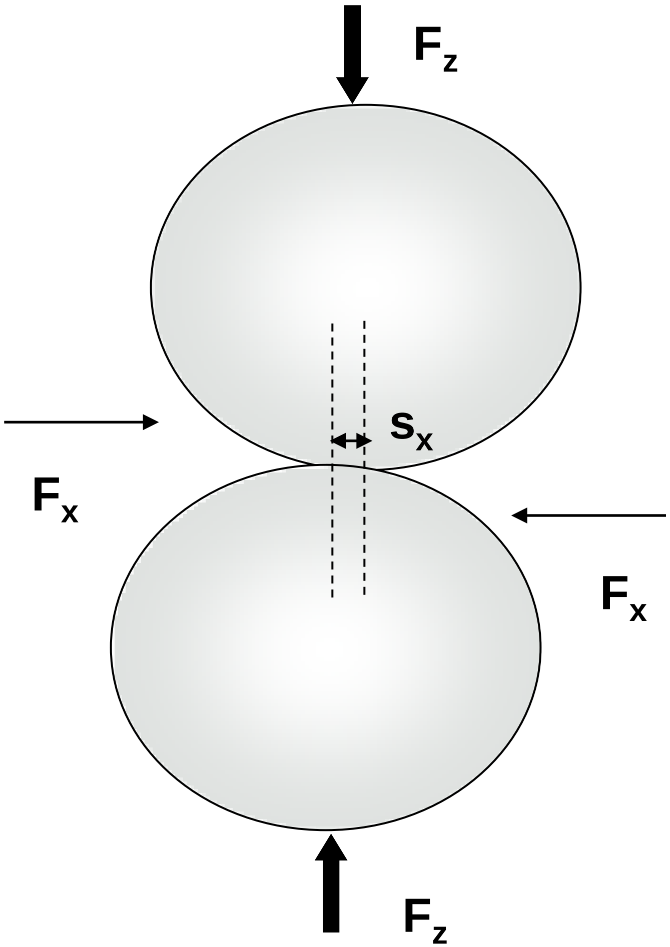
- The normal force coefficient

$$D_n = \frac{dF_z}{ds} = \left[\frac{3M_s^{\,2} F_z a}{4} \right]^{\frac{1}{3}} = \frac{E_s}{1-\nu_s^{\,2}} b = \frac{2G_s}{1-\nu_s} b$$

where G_s is the shear modulus of the solid particles.

Mindlins's approach:

The influence of a shear force



- Applying a small tangential stress to a loaded grain contact gives a tangential force coefficient:

$$D_t = \frac{dF_x}{ds_x} = \frac{\left[6(1-\nu_s^2)E_s^2 F_z a\right]^{\frac{1}{3}}}{(2-\nu_s)(1+\nu_s)} = \frac{(1-\nu_s)}{(2-\nu_s)}\left[6M_s^2 F_z a\right]^{\frac{1}{3}};$$

$$D_t = \frac{4G_s}{2-\nu_s} b$$

Mindlin (1948)

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Analytical modelling of uncemented granular media

- Walton (1987) calculated the effective elastic moduli for a random dense packing of equally sized spheres (porosity $\cong$ 0.36).
  - Assumptions: The granular assembly is in a pre-set strain state (isotropic or uniaxial strain) and the wave-induced stresses are small (Hertz-Mindlin contact law applies to all contacts).
  - No new contacts, no contacts lost during loading or unloading.
  - The incremental stiffnesses are computed by summation over all contacts between spheres.
  - Spheres may be
    - infinitely rough (no slip), or
    - infinitely smooth (perfect slip; i.e. zero friction)

Analytical modelling of uncemented granular media

- **Walton's results for isotropic (hydrostactic) stress:**

$$K = \frac{n(1-\varphi)}{6\pi a} D_n = \left(\frac{n^2 (1-\varphi)^2 G_s^2}{18\pi^2 (1-\nu_s)^2} \sigma \right)^{\frac{1}{3}}$$

$$G = \frac{n(1-\varphi)}{10\pi a} (D_n + \frac{3}{2} D_t) \rightarrow$$

$$G_{noslip} = \frac{5-4\nu_s}{5(2-\nu_s)} \left(\frac{3n^2 (1-\varphi)^2 G_s^2}{2\pi^2 (1-\nu_s)^2} \sigma \right)^{\frac{1}{3}}$$ Rough

$$G_{nofriction} = \frac{1}{10} \left(\frac{12 n^2 (1-\varphi)^2 G_s^2}{\pi^2 (1-\nu_s)^2} \sigma \right)^{\frac{1}{3}}$$ Smooth

n: Coordination number (= average number of contacts per particle)

The "rough" limit is also known as the Hertz-Mindlin theory

Analytical modelling of uncemented granular media

- The coordination number:
  - 6 for simple cubic, 12 for hcp & fcc; ~ 9 for random dense pack.
  - Approximate porosity dependence: $n \approx 22(1-\varphi)^2$

- The v_p/v_s ratio in a random grain pack:

$$\frac{v_P}{v_S} = \sqrt{3\frac{D_n + \frac{2}{3}D_t}{D_n + \frac{3}{2}D_t}}$$

$$\frac{v_P}{v_S} = \sqrt{\frac{10-7\nu_s}{5-4\nu_s}}$$ Rough

$$\frac{v_P}{v_S} = \sqrt{3}$$ Smooth

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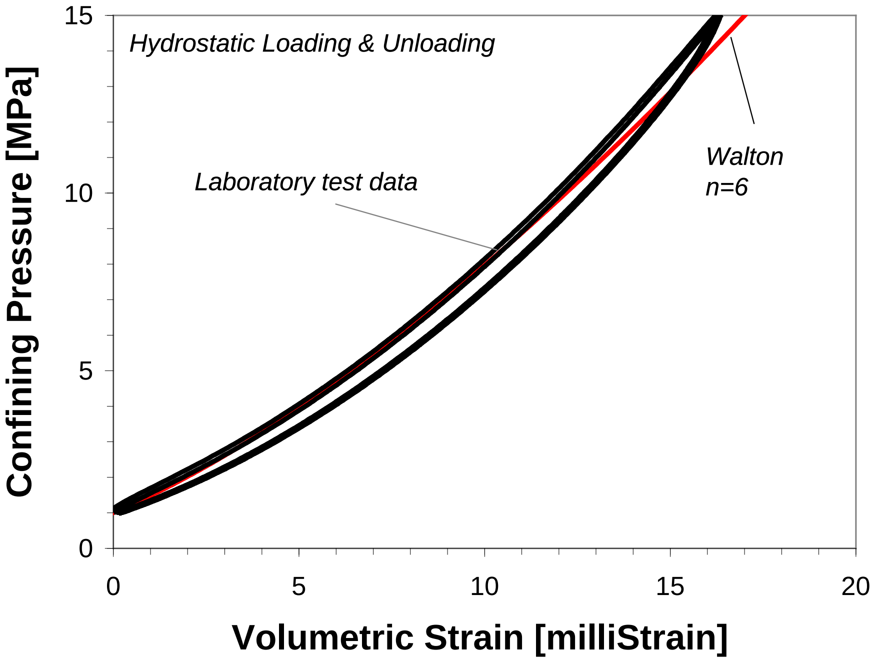
Experimental data on glass beads – Comparison with Walton's Hertz-Midlin theory

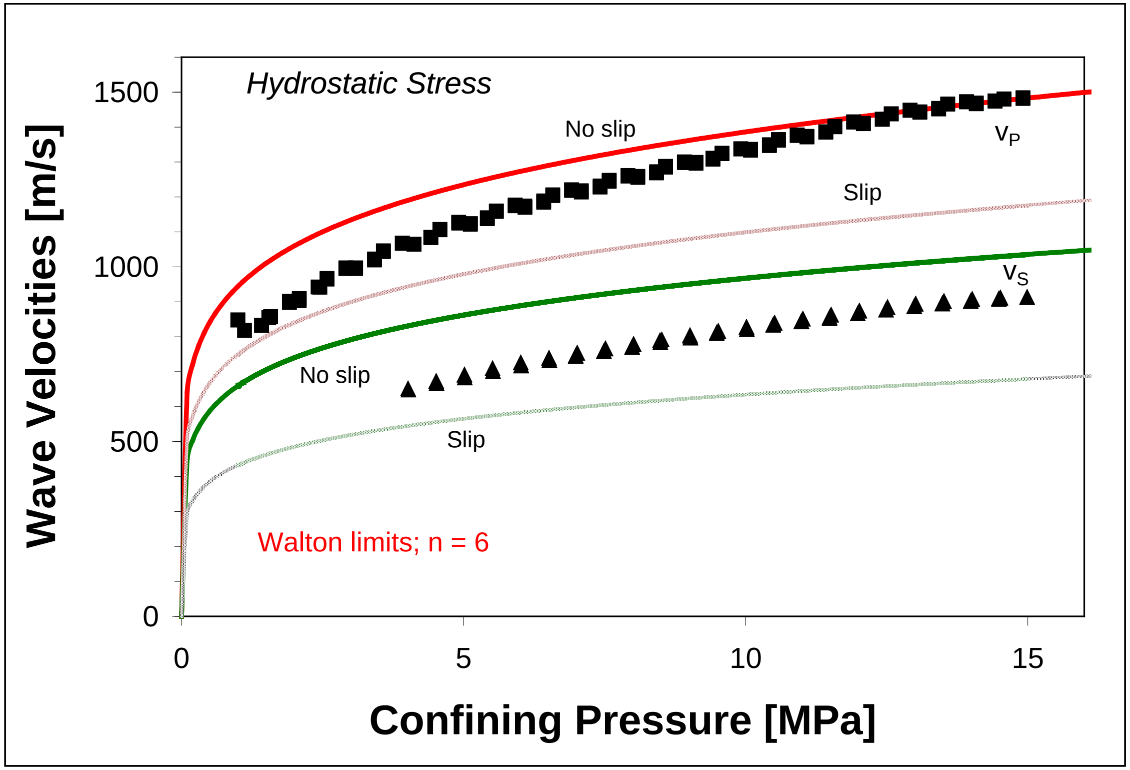
Analytical Modelling: Hydrostatic Behaviour



- ❑ Analytical model fits experimental curve well with coordination number n=6

Analytical Modelling: Wave Velocities

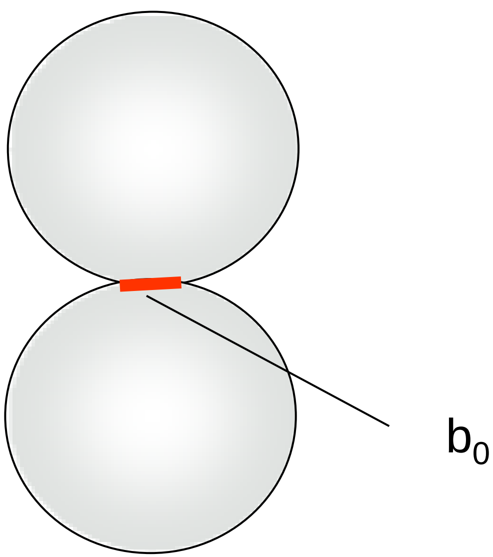
- Walton (Hertz-Mindlin) theory predicts velocities to increase with $\sigma^{1/6}$ and n $^{1/3}$ (n: coordination number)
- Experiments show velocities increase with $\sigma^{0.20 - 0.25}$
  - *Transition from slip to non-slip?*
  - *Increasing effective coordination number with stress?*
- Velocities in sands are often significantly lower than predicted by theory



From Holt et al., 2007

The effect of cementation

Classical approach: Digby (1981): Two spheres are bonded at an adhesive contact with radius b_0. Outside the bonded area, a Hertzian approach is used.



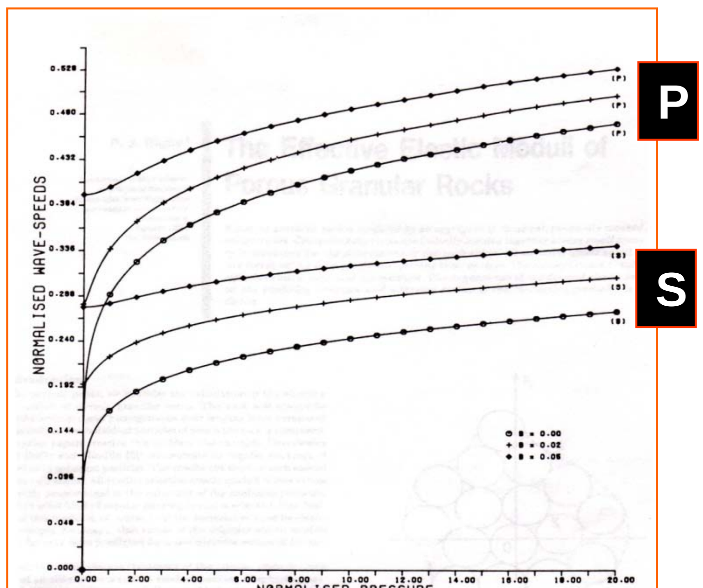


Fig. 5 Normalized wave speeds plotted as functions of the normalized pressure for some fixed values of the normalized adhesion radius $B = b/R$

In real cemented rocks, we may expect stress dependence

- ❖ If rock is soft, so many grain contacts are not cemented
- ❖ If there are pre-existing cracks / fractures

Pragmatic stress sensitivity...

- Lab measured velocities *vs*. hydrostatic stress may often be fitted to an equation like:

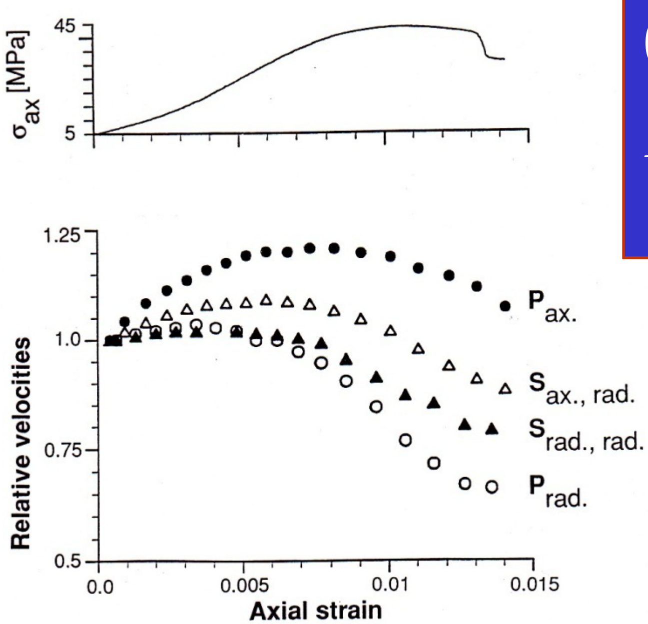
$$\mathrm{v} = \mathrm{v}_0 (\sigma + \sigma_0)^m$$

  - The exponent *m* (typically $\subset$ 0.05, 0.25) may represent grain or fracture roughness, or crack aspect ratio distribution.
  - The parameter σ_0 ensures non-zero velocity at zero stress, and may be seen as a measure of the degree of rock cementation (~ tensile strength).

Stress-Induced Anisotropy

Effects of Anisotropic Stress

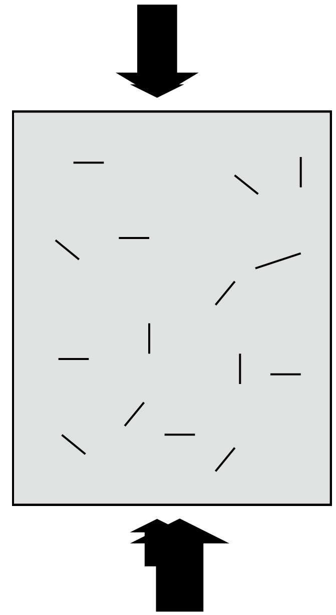
SANDSTONE (Red Wildmoor)



Low confining pressure (5 MPa)

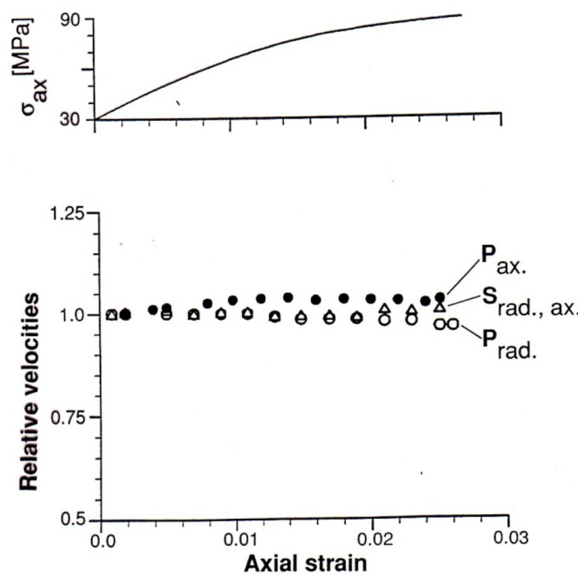
Triaxial tests with Red Wildmoor Sandstone (25 % porosity)

Holt et al., 1991



Effects of Anisotropic Stress

SANDSTONE (Red Wildmoor)



High confining pressure (30 MPa)

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Overview: Sources of stress sensitivity

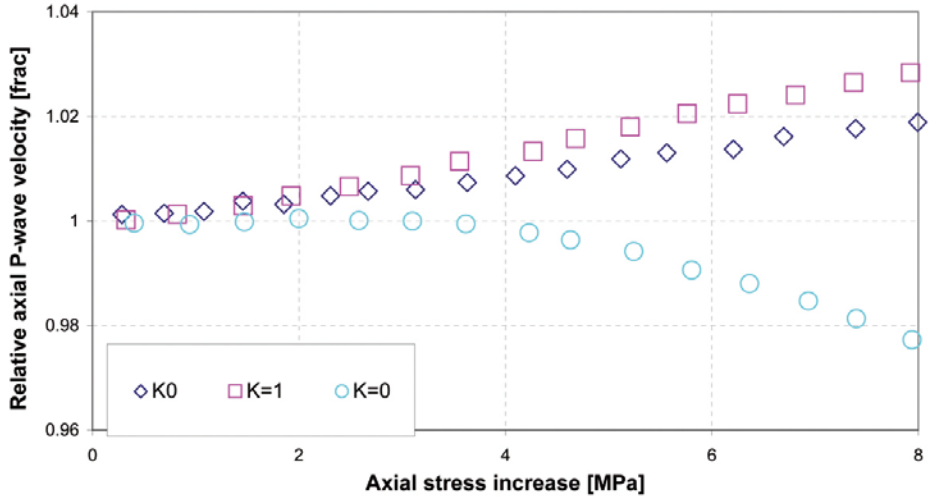
Table 1. *Various sources of in-situ stress sensitivity for velocities in a depleting reservoir*

Mechanism	Controlling factor	Wave velocity change	Effect on velocity anisotropy	Conditions
Porosity decrease	Increase of mean effective stress	⇑ (small effect)		Most efficient near critical porosity (suspension threshold)
Grain contact compression	Increase of effective stresses	⇑	⇓	Requires uncemented grains
Closure of cracks/ fractures	Increase of effective stress	⇑	⇓	Fractured reservoir
Generation of cracks/fractures	Reservoir stress path (deviatoric versus normal effective stress)	⇓	⇑	Rock brought beyond yield onset (primarily in decompression)/initially fractured rock
Decrease of pore fluid bulk modulus	Pore pressure reduction	⇓ (small effect)	⇑ (small effect)	Above bubble point of fluid; constant amount of dissolved gas

From Holt *et al*., 2006 (TLE)

Role of the Stress Path

- Wave velocities depend on stress – and on the stress path!



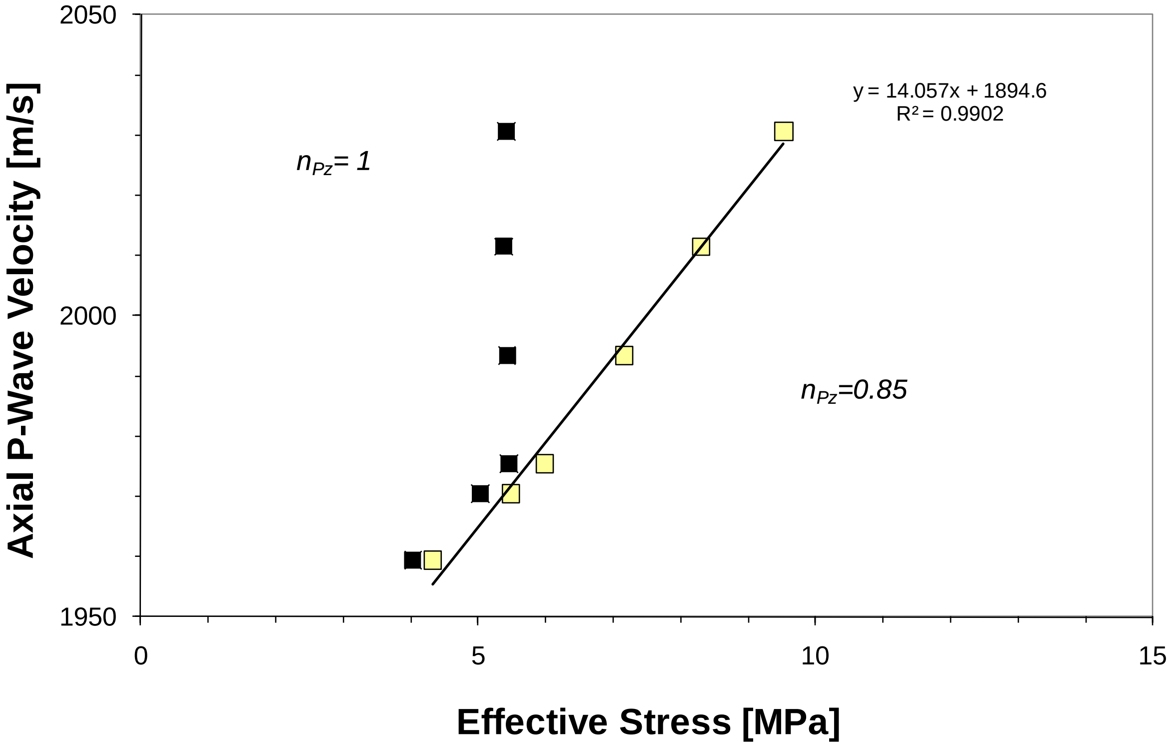
Synthetic Sandstone

- K=1: Hydrostatic stress increase
- $K=K_0$: Uniaxial strain
- K=0: Uniaxial stress increase

$$\Delta\sigma_h' = K\Delta\sigma_v'$$

Effective Stress for Velocities

- Ultrasonic Lab data with North Sea overburden Field Shale: 40 % porosity; 35-50 % clay (smectite + kaolinite)



Common observations with many rocks:

$n_P < n_S$

(n_S may occasionally >1)

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Effective Stress coefficients

Rock Type	ϕ %	Pc MPa	Pp MPa	n_p	n_s	α	Reference study
Chelmsford granite	0.5	-	0-10	0.5-0.8	-	-	Todd & Simmons (1972)
Australian SST	20.4 20.6 23.7 24.1	15-65	5-55	0.6-1.0 0.7-1.0 0.8-1.0 0.8-1.0	-	_	Siggins & Dewhurst (2003)
Berea SST drained	-	0.5 5 5 10 15 20 20 25 60 100	0 ≠0 0 ≠0 ≠0 ≠0 0 ≠0 0 0	0.990 0.946 0.930 0.986 0.969 0.858 0.89 0.776 0.84 -	- - 1.02 - - - 1.06 - 1.07 1.17	- - - - - - - - - -	Christensen & Wang (1985) Prasad & Manghani (1997) Christensen & Wang (1985) Prasad & Manghani (1997) Prasad & Manghani (1997) Prasad & Manghani (1997) Christensen & Wang (1985) Prasad & Manghani (1997) Christensen & Wang (1985) Christensen & Wang (1985)
Michigan SST		5 10 15 20 25	≠0	0.977 0.928 0.850 0.831 0.615	-	-	Prasad & Manghani (1997)
Limestone		0* 10*	- -	1.02 1.35	1.01 1.09	0.96 0.92	Ringstad & Fjær (1997)
Limestone + oil		0* 10*	- -	0.95 0.67	0.89 0.41	0.95 0.94	Ringstad & Fjær (1997)
Limestone	0.5	-	0-10	0.5-0.9	-	-	Todd & Simmons (1972)
Epidosite	0.5	100-240	100	0.95	-	-	Gangi & Carlson (1996)

From Ojala & Fjær, 2007

Effective Stress for Velocities

- Stress changes lead to changes in
  - Framework stiffness (by grain contacts, cementation); $f(\sigma - p_f)$?
  - Porosity; $f(\sigma - p_f)$
  - Free pore fluid; $f(p_f)$
  - Soft grain coatings, such as cla on sand or adsorbed /bound water in clay; $f(p_f)$
  - \+ Frequency dependent processes where relaxation time may depend on either net stress or pore pressure or something else…
  - A simple model can be constructed by sorting processes that depend on net stress vs. processes that depend only on pore pressure:

$$\Delta v_{Qj} = A(\Delta\sigma - \Delta p_f) + B\Delta p_f$$

$$\Rightarrow$$

$$S_{Qj} = A; \qquad n_{Qj} = 1 - \frac{B}{A}$$

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