Bubble time period estimation of air-gun clusters

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Background

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The main effect is improving the primary to bubble ratio and the bubble time period of the two (or more) clustered air guns, and is determined by the separation distance and size of the guns (Strandenes and Vaage, 1992).

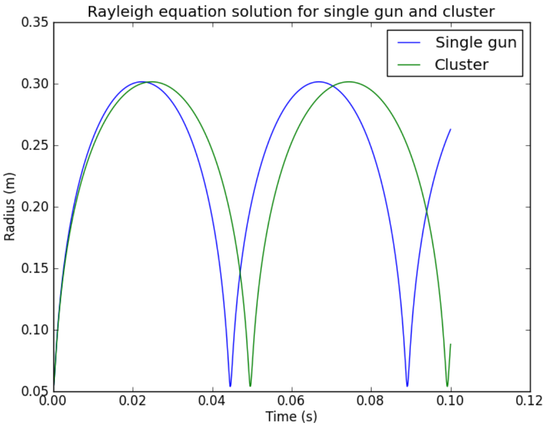
Observation

The simplest model of an oscillating air gun, the Rayleigh equation (Rayleigh, 1917):

$$\ddot{R} = \frac{P - P_\infty}{\rho R} - \frac{3}{2}\frac{\dot{R}^2}{R},$$

retains its general shape when correcting for a cluster case, but is scaled in period.

Observations



The idea

- Estimate a characteristic time factor for the single bubble Rayleigh equation.
- Repeat the process for the cluster case.
- Estimate the bubble time period increase for a real air-gun cluster from these two values.

Implicit assumption

While the Rayleigh equation is not the best model for a single gun, we hope that the relative change will still be a good estimator. This implicitly assumes that:

- Clustered air-gun interaction is dominated by fluid flow as opposed to sound pressure proagation.

The method

Strandenes and Vaage (1992) determined that cluster dynamics were determined by separation distance and equilibrium radius, R_{EQ}, which is the radius where the pressure inside the bubble equals the hydrostatic pressure (and temperature, which we will not require, as it is not certain that exact state will be attained). Assuming adiabatic expansion, this means:

$$R_{\text{EQ}} = R_0 \left(\frac{P_0}{P_\infty} \right)^{\frac{1}{3\gamma}}$$

We will:

- Estimate the bubble wall velocity when $R = R_{\text{EQ}}$, by use of an energy balance.
- Calculate a characteristic time $T = \frac{R_{\text{EQ}}}{U_{\text{EQ}}}$
- Estimate relative bubble time period: $\frac{T_{\text{Cluster}}}{T_{\text{Single}}}$

Single bubble - Energy balance

At the initial state we will have only potential energy (as the bubble wall has not begun moving). This energy can be defined to be equal to the work done to get the bubble from its steady state, $P = P_\infty$, to the initial state. We again assume adiabatic expansion and get

$$\begin{aligned} E_p &= -\int_{V_{\mathrm{EQ}}}^{V} \Delta P \mathrm{d}V \\ &= -\int_{V_{\mathrm{EQ}}}^{V} P_\infty \left(\frac{V_{\mathrm{EQ}}}{V}\right)^{\gamma} - P_\infty \mathrm{d}V \\ &= VP_\infty(1 - \beta\frac{\beta^{-\frac{\gamma-1}{\gamma}}\gamma - 1}{\gamma - 1}), \end{aligned}$$

where $\beta = \frac{P_0}{P_\infty}$ and γ is the adiabatic constant.

Single bubble - Energy balance

To get the kinetic energy we assume flow as describe by a single monopole potential

$$\phi = \frac{-UR^2}{r}$$

and integrate the kinetic energy over all the fluid to get

$$\begin{aligned} E_k(R, U) &= \int_\Omega \frac{1}{2}\rho u^2 \mathrm{d}V \\ &= 4\pi \int_R^\infty \frac{1}{2}\rho \left|\nabla\phi\right|^2 \mathrm{d}r \\ &= 2\rho\pi U^2 R^3 \end{aligned}$$

Single bubble - Energy balance

The energy balance then yields:

$$2\rho\pi U_{\mathrm{EQ}}^2 R_{\mathrm{EQ}}^3 = \frac{4}{3}\pi R_0^3 P_\infty \left(1 - \beta \frac{\beta^{-\frac{\gamma-1}{\gamma}}\gamma - 1}{\gamma - 1}\right)$$

$$U_{\mathrm{EQ}} = \frac{1}{3}\sqrt{\frac{6P_\infty}{\rho}\frac{\hat{\beta}}{\beta^{\frac{1}{\gamma}}}},$$

where $\hat{\beta} = 1 - \beta\frac{\beta^{-\frac{\gamma-1}{\gamma}}\gamma-1}{\gamma-1}$.

Single bubble - Characteristic time

Our characteristic time for a single bubble is:

$$T_{\mathsf{Single}} = \frac{R_{\mathsf{EQ}}}{U_{\mathsf{EQ}}} = \frac{R_{\mathsf{EQ}}\beta^{\frac{1}{2\gamma}}}{2}\sqrt{\frac{6\rho}{\hat{\beta}P_{\infty}}}.$$

Two bubbles - Energy balance

In the 2-bubble case, the potential energy will remain the same (per bubble) as in the single bubble case. This means that for our relative bubble time period estimate we do not need to assume anything about the bubble expansion.

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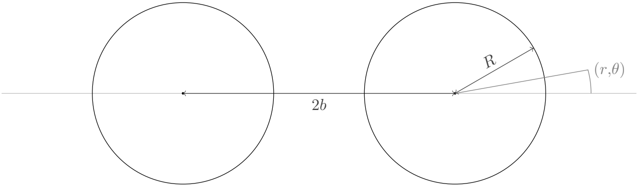
The kinetic energy, however, will be considerably more tricky to evaluate.

Two bubbles - Energy balance

We assume a two monopole potenial, with a distance $2b$ between the two singularities:



$$\phi = -\frac{U\,R^2}{r} - \frac{U\,R^2}{\sqrt{r^2 + 4\,b^2 + 4\,br\cos\left(\theta\right)}}.$$

Two bubbles - Energy balance

To evaluate the energy integral we apply Green's identity to get

$$\begin{aligned} E_k =& \frac{1}{2}\rho \int_{\Omega} |\nabla \phi|^2 \, \mathrm{d}V \\ =& -\frac{1}{2}\rho \int_{\partial\Omega} \frac{\partial \phi}{\partial r} \phi \, \mathrm{d}S - \underbrace{\frac{1}{2}\rho \int_{\Omega} \phi \Delta \phi \, \mathrm{d}V}_{0, \text{ as } \phi \text{ is harmonic}}, \end{aligned}$$

which has the effect of separating the total energy to the energy contribution from each bubble.

Two bubbles - Energy balance

Assuming non-overlapping bubbles, the kinetic energy at equilibrium radius can be evaluated to be (Barker and Landrø, 2012):

$$E_k = -\frac{1}{2}\,\pi\,\rho\,U_{\mathrm{EQ}}^2 R_{\mathrm{EQ}}^3 \frac{4 + 6\,\kappa - 4\,\kappa^2 - 4\,\kappa^3 + \left(1 - \kappa^2\right)\ln\left(\frac{\kappa+1}{\kappa-1}\right)}{\kappa\left(\kappa^2 - 1\right)},$$

where $\kappa = \frac{2b}{R_{\mathrm{EQ}}}$.

Two bubbles - Characteristic time

Following the single bubble procedure we get

$$T_{\mathrm{Cluster}} = \frac{1}{4}\,\beta^{\frac{1}{2\gamma}}\,R_{\mathrm{EQ}}\sqrt{\frac{6\rho}{\hat{\beta}P_\infty}}\sqrt{\frac{4\,\kappa + 4 - \ln\left(\frac{\kappa-1}{\kappa+1}\right)}{\kappa} + \frac{2}{1-\kappa^2}},$$

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and by dividing by T_{Single} we get the relative time period estimate

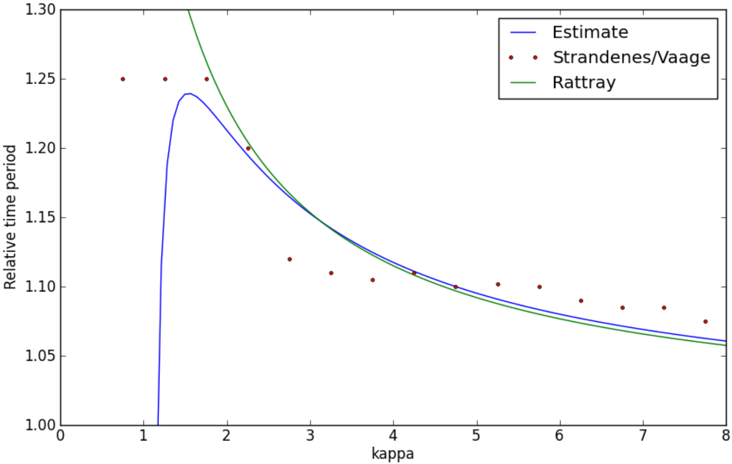
$$\frac{T_{\text{Cluster}}}{T_{\text{Single}}} = \frac{1}{2} \sqrt{\frac{4\,\kappa + 4 - \ln\left(\frac{\kappa-1}{\kappa+1}\right)}{\kappa} + \frac{2}{1-\kappa^2}}, \qquad \kappa \geq 2.$$

Rattrays estimate

For comparison, Rattray (1951) inferred the following relative time period estimate for cavities near a wall, which is (in our context and notation):

$$\frac{T_{\text{Wall}}}{T_0} = 1 + 0.41\frac{1}{\kappa}$$

Real data comparison



Real data comparison

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Naturally, we would like to extend the estimation down to $\kappa = 0$.

From spheres to isosurfaces

To extend the domain of validity we drop the assumption of spheres, and instead let the bubbles be represented by isosurfaces in the potential field.

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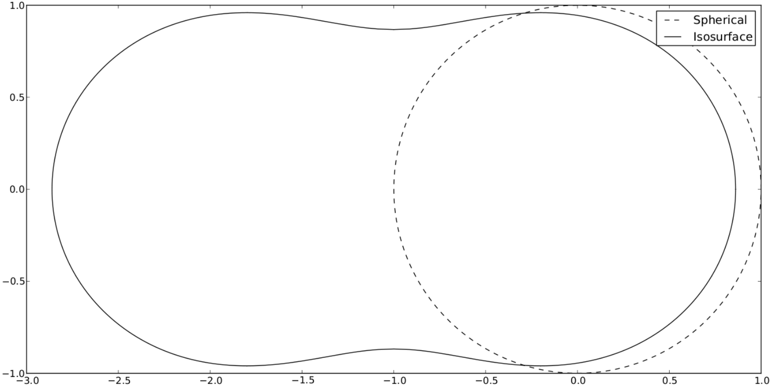
The tradeoff is that we can no longer present an analytical expression.

From spheres to isosurfaces

An example isosurface:

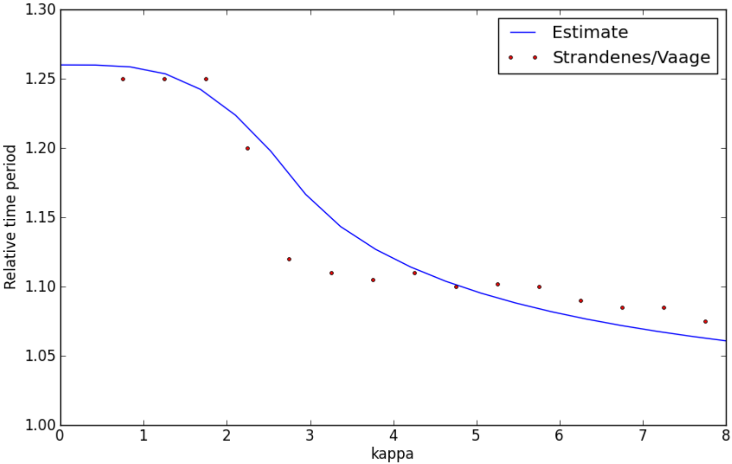


The numerical approach

Since we are using an isosurface in the potential field, the evaluation of the kinetic energy interval will in fact be a lot simpler, as only the iso-constant is needed. This means the method is, for a given κ:

- Determine the constant needed to get an isosurface containing the equilibrium volume.
- Divide this by the constant needed in the single bubble (sperical) case, $-R * U$.
- Take the square root to get the estimate for U
- Get the relative time period estimate as earlier described.

Real data comparison



Clusters with more than 2 guns

Of course, this method can be extended to work for any number of guns as long as the the guns are placed equally on a circle or sphere.

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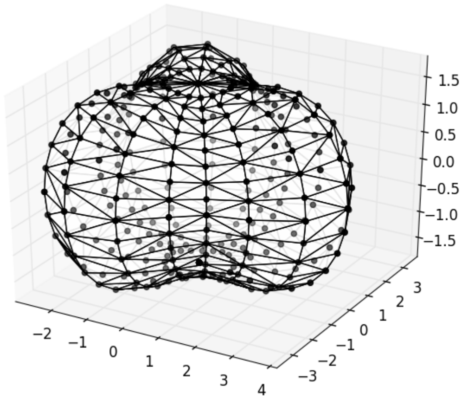
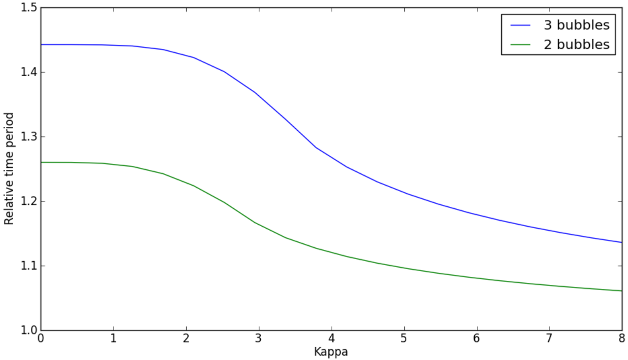
In the following slides we will see the estimated relative bubble time period change for a 3-gun symmetric cluster, where b is the distance to the center. Sadly, we have no data to compare to in this case.

3-gun cluster

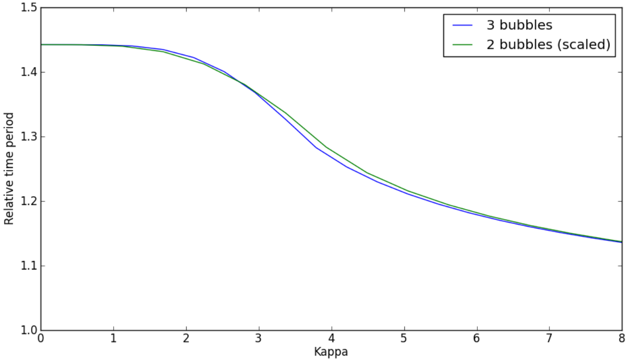
Isosurface example



3-gun cluster



3-gun cluster



Conclusions

- We have found a simple analytic expression for estimating the relative time period change of two-gun clusters vs. single air guns, when the bubbles are non-overlapping.
- By introducing isosurfaces of the potential as an abstract bubble wall, we can estimate the relative bubble time period of coalescing bubbles as well.
- The relatively good fit for smaller values of κ might indicate that the main form of interaction of guns in clusters is the fluid movement.

Acknowledgements

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