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Implications of the Born approximation for MVA

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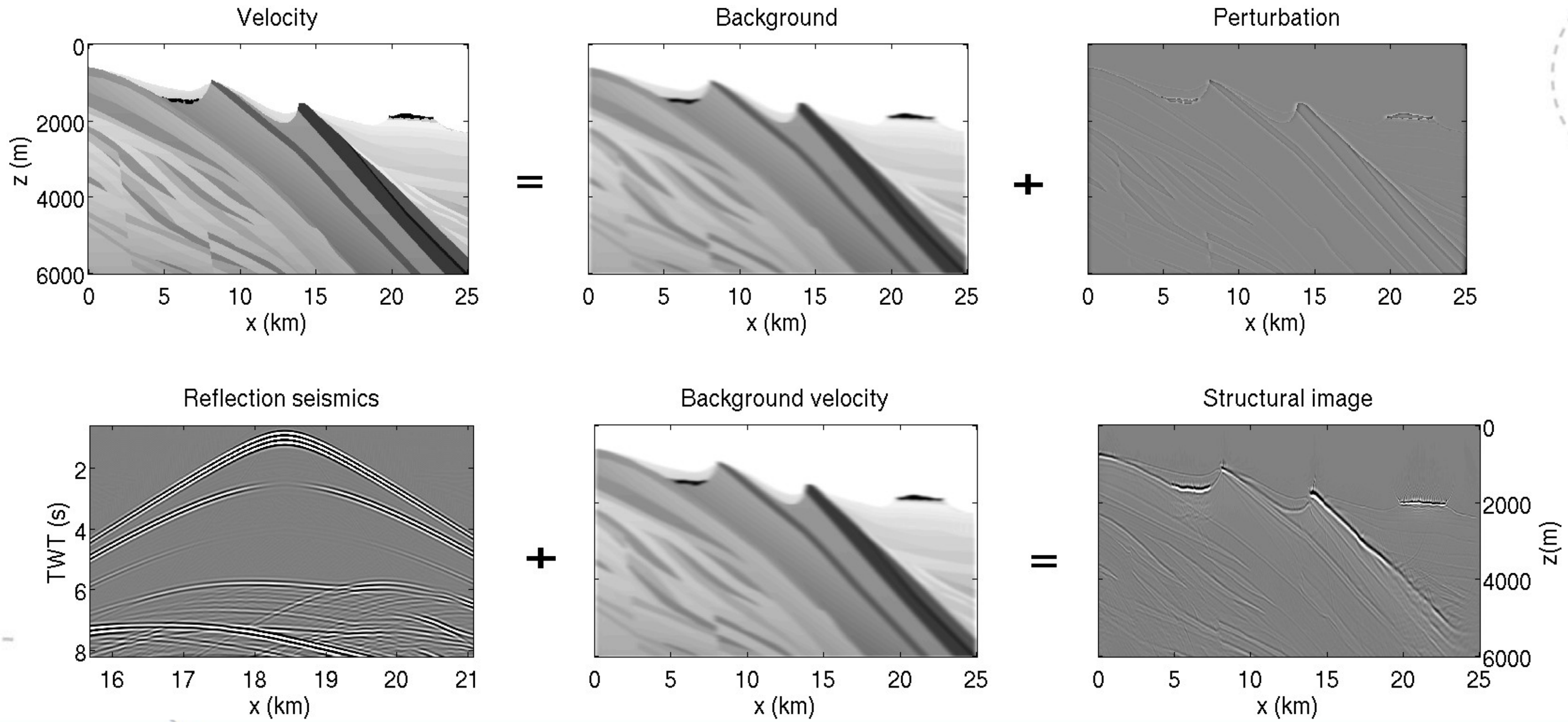
Outline

- Introduction
- Wave equation migration and velocity analysis
- The error of the Born approximation and MVA
- Conclusions

Introduction

- The problem of estimating velocities for prestack depth migration
- Prestack depth migration relie upon a *linearized* model of acoustic scattering (single scattering, Born approximation)
- Using prestack depth migration to estimate the velocities (MVA)

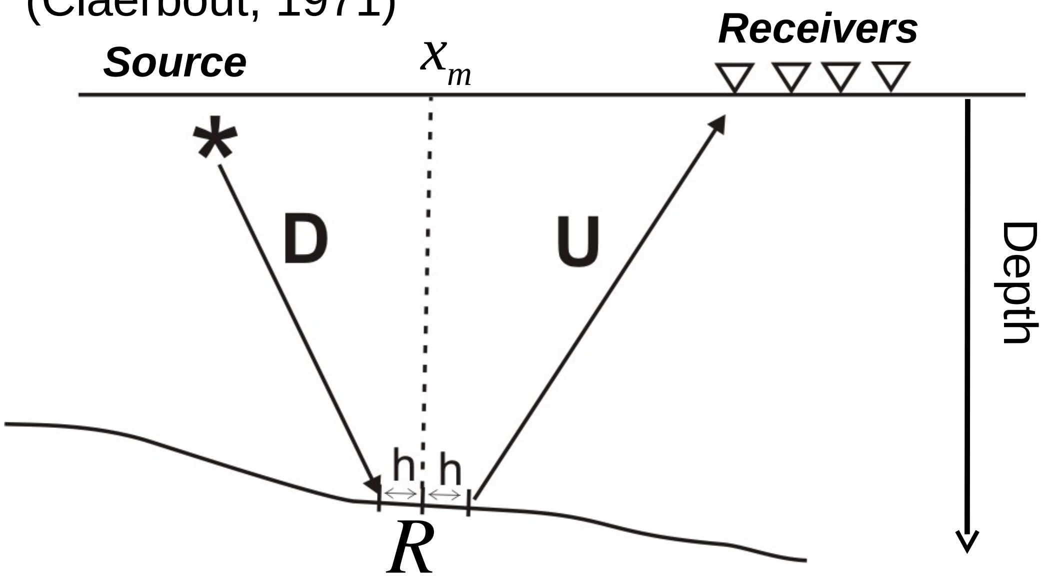


Goals

- Determine how the linearization error (Born approximation error) affects the result of MVA.

Wave equation migration and velocity analysis (1)

- Wave equation migration = Downward extrapolation + crosscorrelation (Claerbout, 1971)



D = Downgoing wave

U = Upgoing wave

R = Reflectivity

$\omega = 2\pi f$ = frequency

h = half offset

p_h = horizontal slowness

Crosscorrelation (imaging condition)

$$R(\mathbf{x}_m, h, z) = \sum_{shots} \sum_{\omega} U(\mathbf{x}_m + h, \omega, z) D^*(\mathbf{x}_m - h, \omega, z)$$ (Rickett and Sava, 2002)

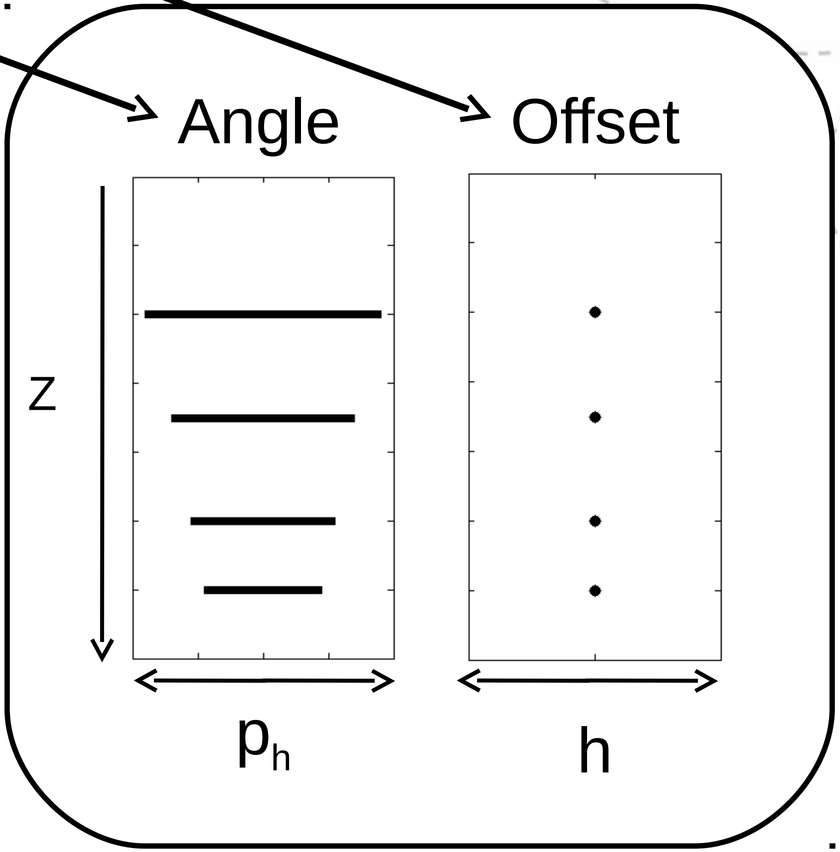
$$R(\mathbf{x}_m, p_h, z) = \sum_{shots} \sum_{\omega} R(\mathbf{x}_m, p_h, \omega, z)$$ (deBruin *et al*., 1990)

Wave equation migration and velocity analysis (2)

- CIPs at correct slowness:

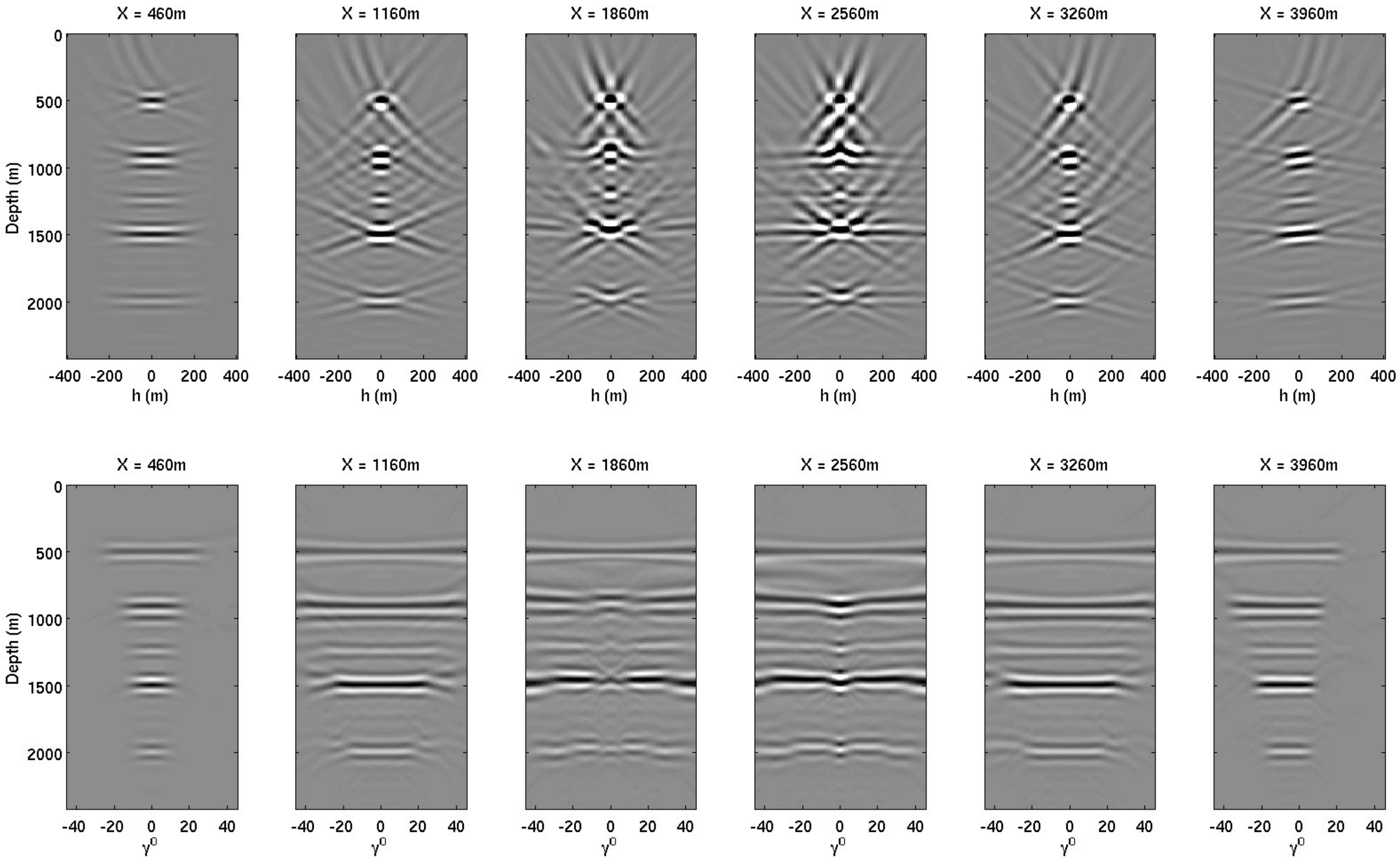
$$R(\mathbf{x}_m, h, z) = R(\mathbf{x}_m, h, z)\delta h$$

$$R_z(\mathbf{x}_m, p_h, z) = R_z(\mathbf{x}_m, z)$$



Common image point gathers (CIPs)

offset



Offset gathers

Angle gathers

angle

Wave equation migration and velocity analysis (3)

- Objective functions

  Target image fitting (TIF; Sava and Biondi, 2004):

$$J(s)=\frac{1}{2}\left\|\delta R\right\|^2; \quad \textit{Where } \|\times\| \textit{ denotes Hilbert norm;}$$
$$\delta R \textit{ represents an image perturbation.}$$

  Differential semblance optimization (DSO; Symes and Carazzone, 1991):

$$J(s)=\frac{1}{2}\left\|h\delta R(\mathbf{x}_m,h,z)\right\|^2.$$

- The objective functions can be minimized iteratively using Newton methods (Nocedal & Wright, 1999)

$$\underbrace{s_{k+1}}_{\textbf{Slowness at iteration k+1}} = s_k - \underbrace{\alpha\nabla_s J(s)_k}_{\textbf{Slowness update}} \qquad \textit{Where } \alpha \textit{ is a step length.}$$

Wave equation migration and velocity analysis (4)

- The gradient of the objective functions with respect to the velocity:

TIF: $$\nabla_s J = \Re e\left\{\left(\frac{\partial \delta R}{\partial s}\right)^* \delta R\right\}$$

DSO: $$\nabla_s J = \Re e\left\{\left(h\frac{\partial \delta R}{\partial s}\right)^* h\delta R\right\}$$

- Image perturbation and wavefield perturbations

$$\delta R(\mathbf{x}_m, h) = U(\mathbf{x}_m - h)\delta D(\mathbf{x}_m + h)^* + \delta U(\mathbf{x}_m - h)D(\mathbf{x}_m + h)^*$$

- Under the Born approximation:

$$\delta U = L\delta s$$ ***L: Forward Born operator***

$$\delta D = L\delta s$$

Error of the Born approximation

- In constant background medium the error of the Born approximation is given by:

$$Error = O(s_0^2) = \underbrace{U(s_0+\delta s) - U(s_0)}_{\delta U(exact)} - \underbrace{DU(s_0)\delta s}_{\delta U(linearized)}$$

$$Error(\beta,\rho,\theta_0) = \left| 1 - \left(1 + i\frac{\beta\rho}{\cos\theta_0}\right) \mathbf{exp}\left(-i\beta\left(\cos\theta_0 - \sqrt{(1+\rho)^2 - sin^2\theta_0}\right)\right)\right|$$

$$\beta = \omega s_0 \Delta z = \frac{\Delta z}{\lambda_0}$$

$$\rho = \frac{\delta s}{s_0}$$

$$\theta_0$$

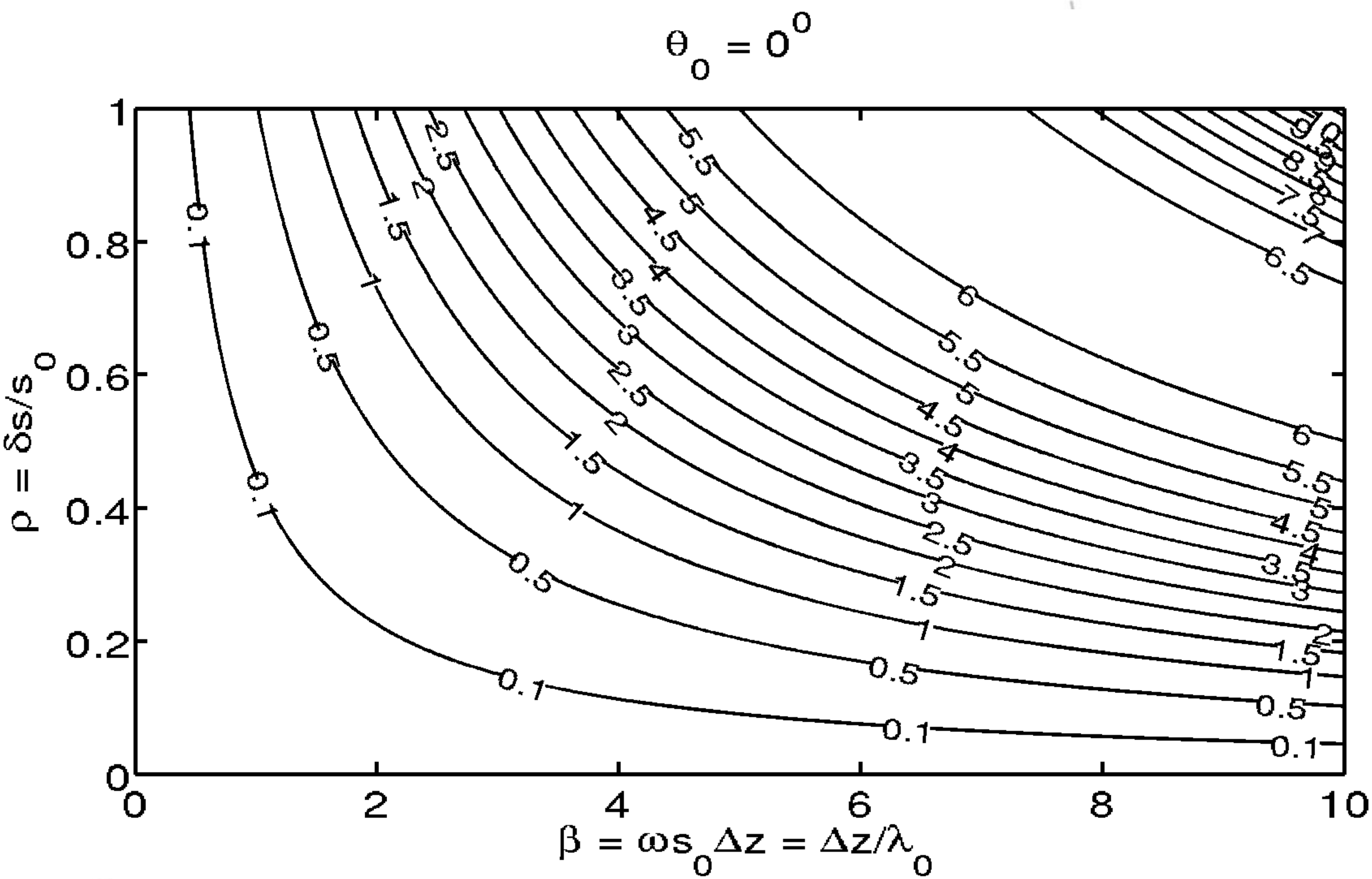
δs : **slowness perturbation**

Δz : **extent of perturbation**

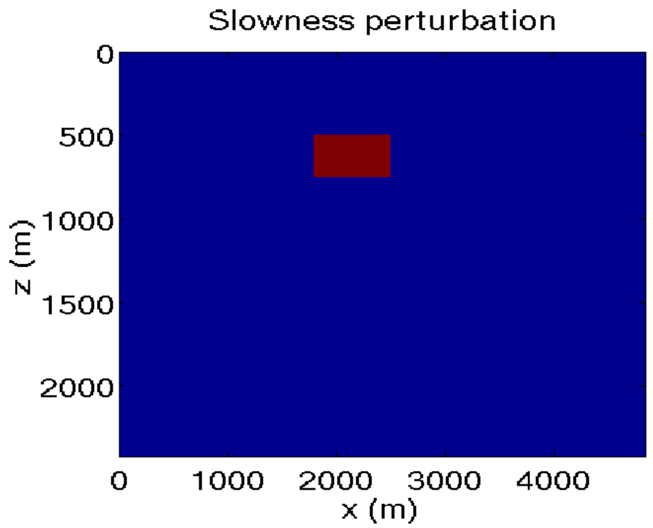
s_0 : **background slowness**

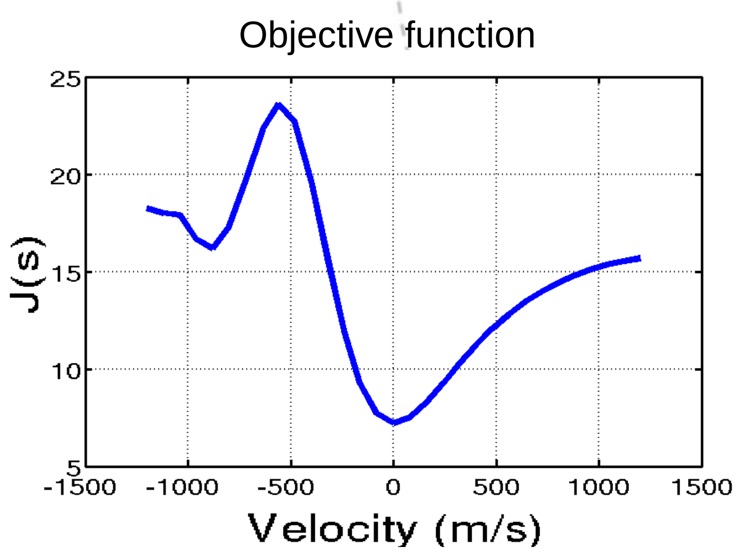
θ_0 : **Take off angle in background medium**

Error of the Born approximation



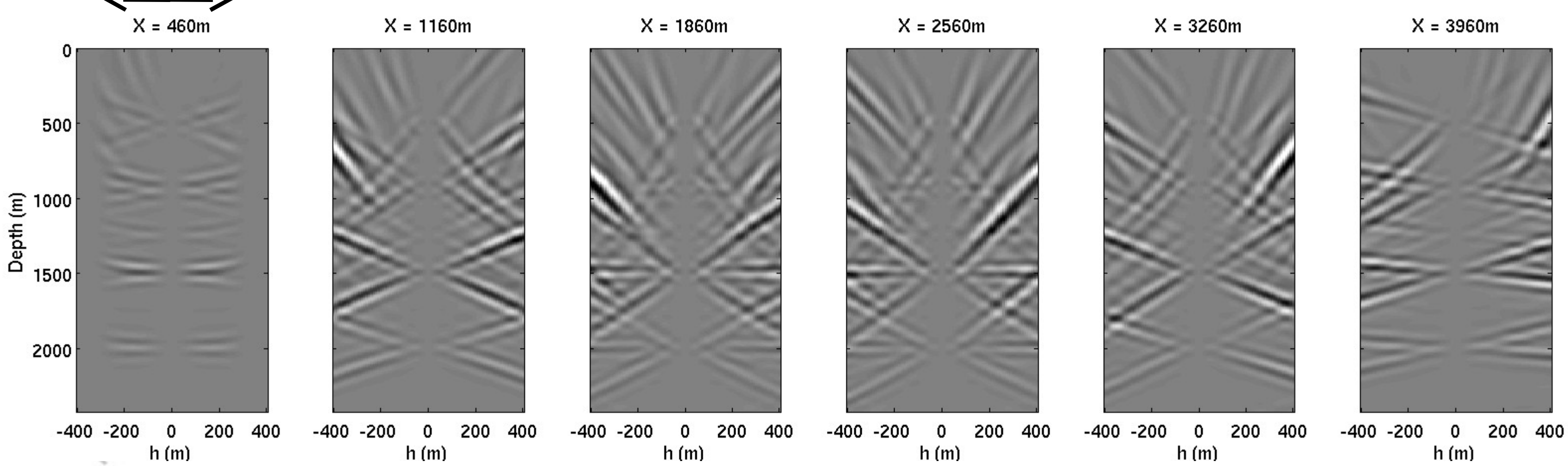
Example 1

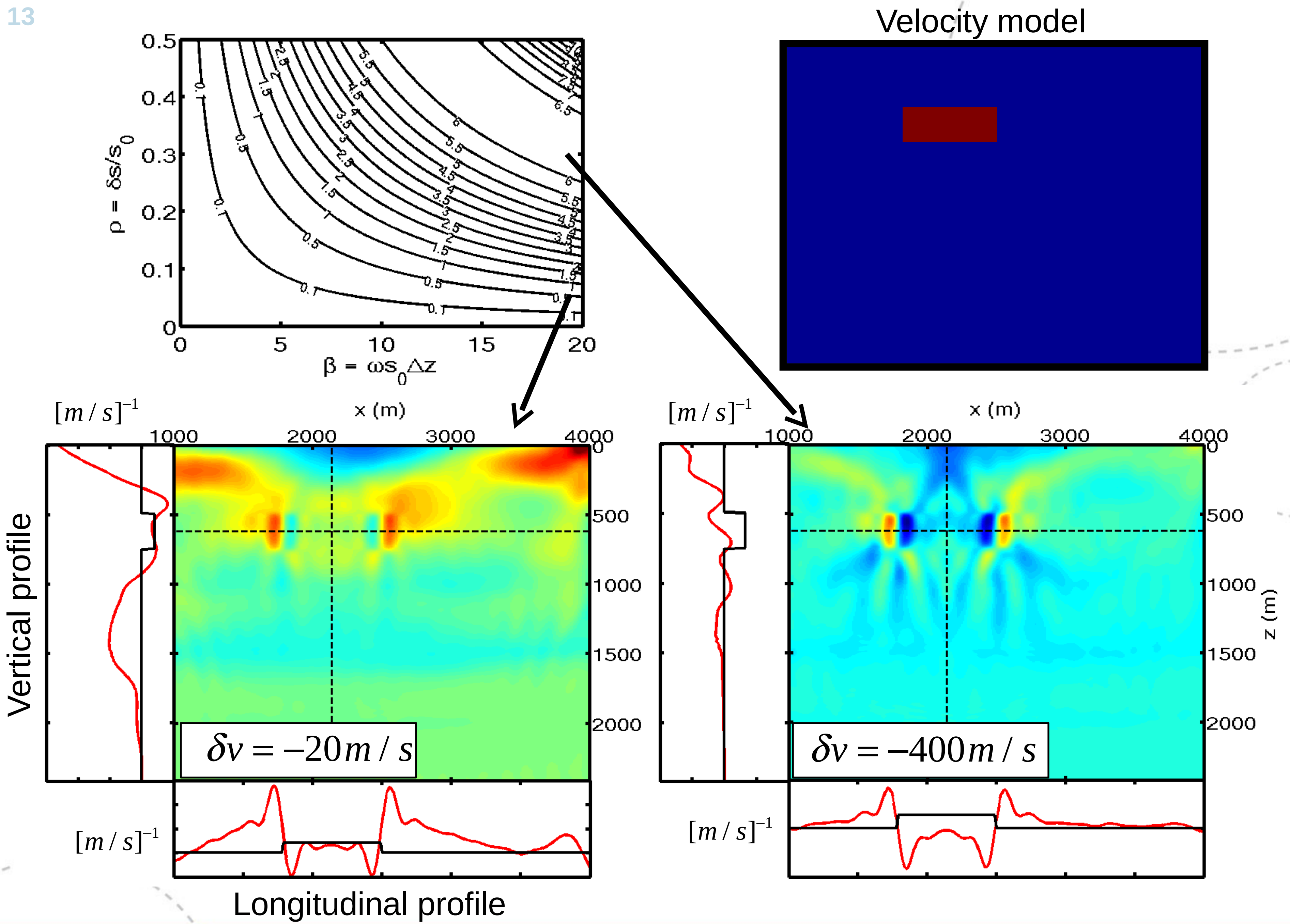




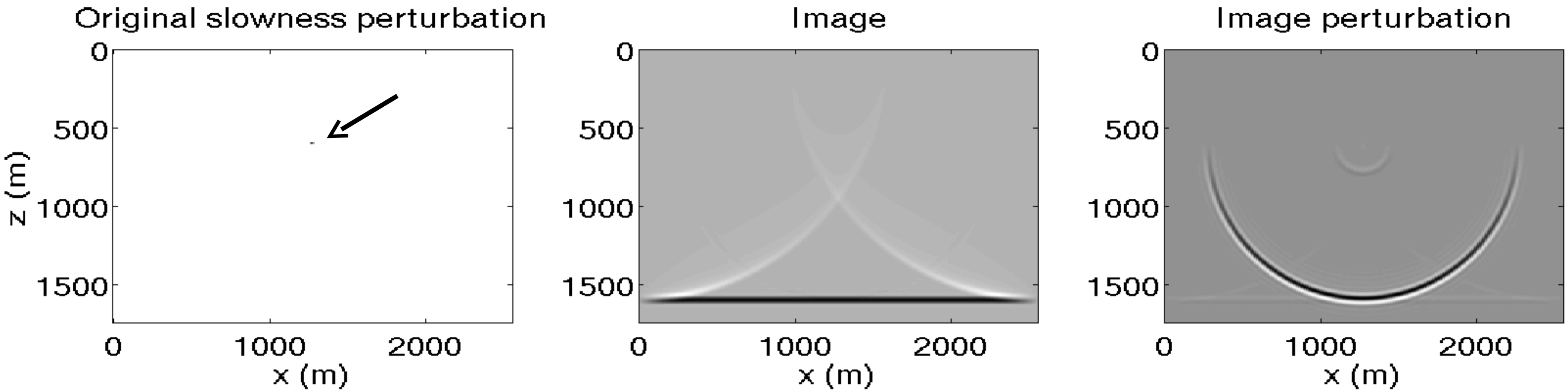
offset

$$h^2 \delta R(\mathbf{x}_m, h, z)$$

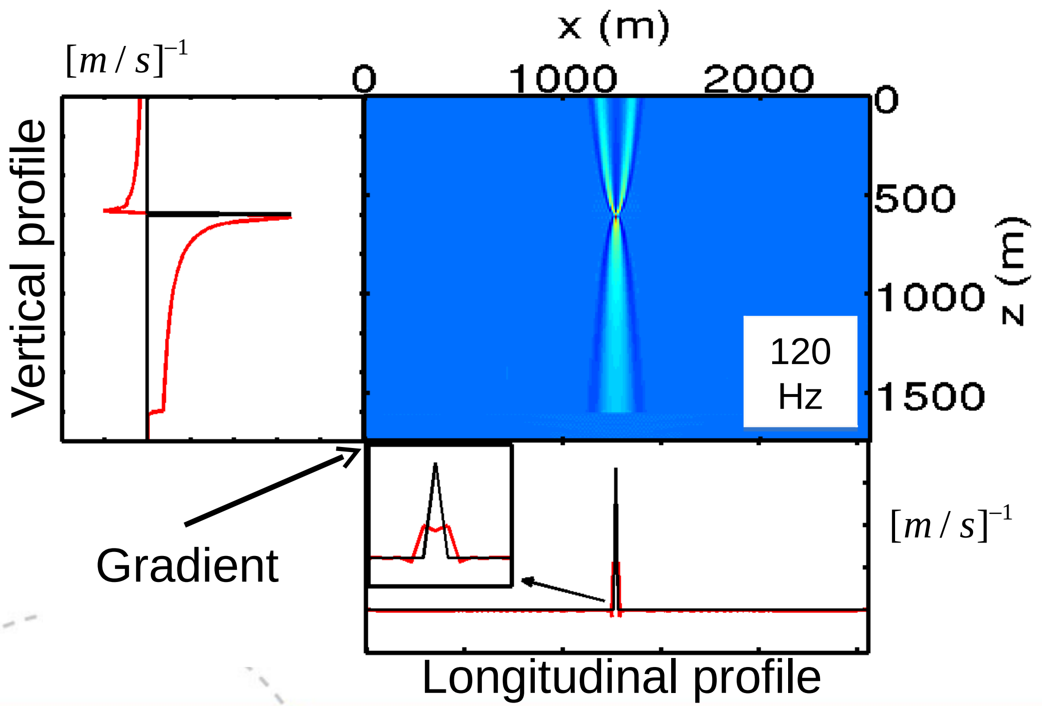



Velocity model
$\rho = \delta s/s_0$
$\beta = \omega s_0 \Delta z$
x (m)
z (m)
$[m/s]^{-1}$
Vertical profile
$\delta v = -20 m/s$
$\delta v = -400 m/s$
Longitudinal profile

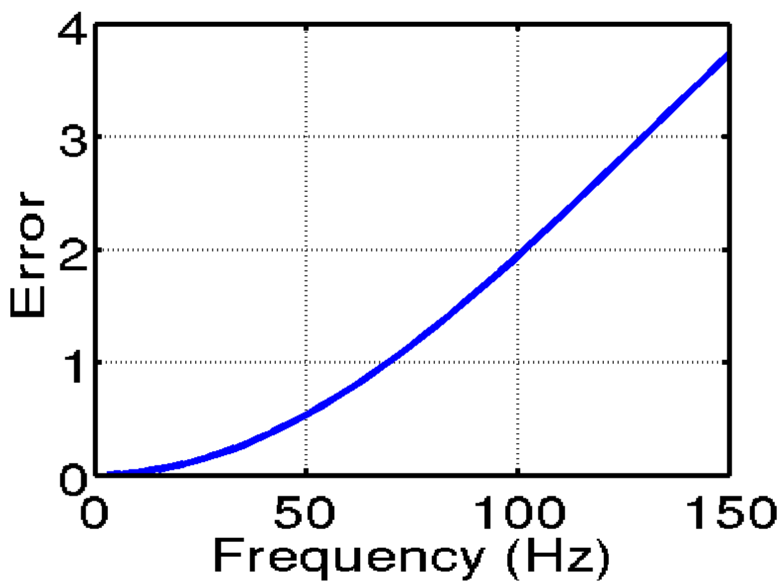
Example 2



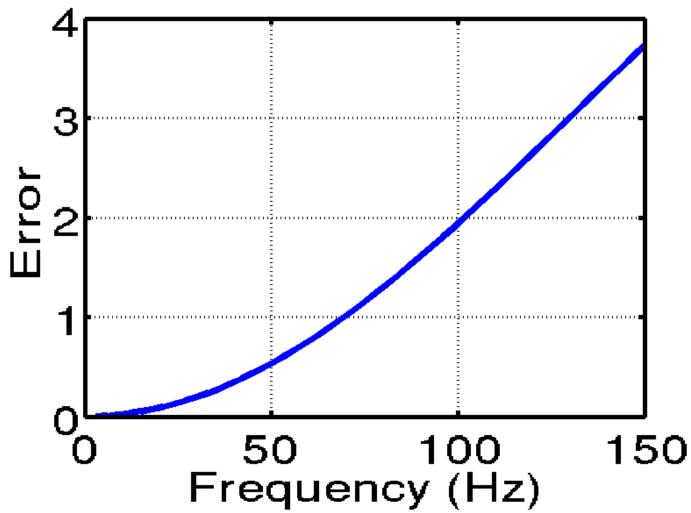
Original slowness perturbation
Image
Image perturbation
z (m)
0
500
1000
1500
0
1000
2000
x (m)

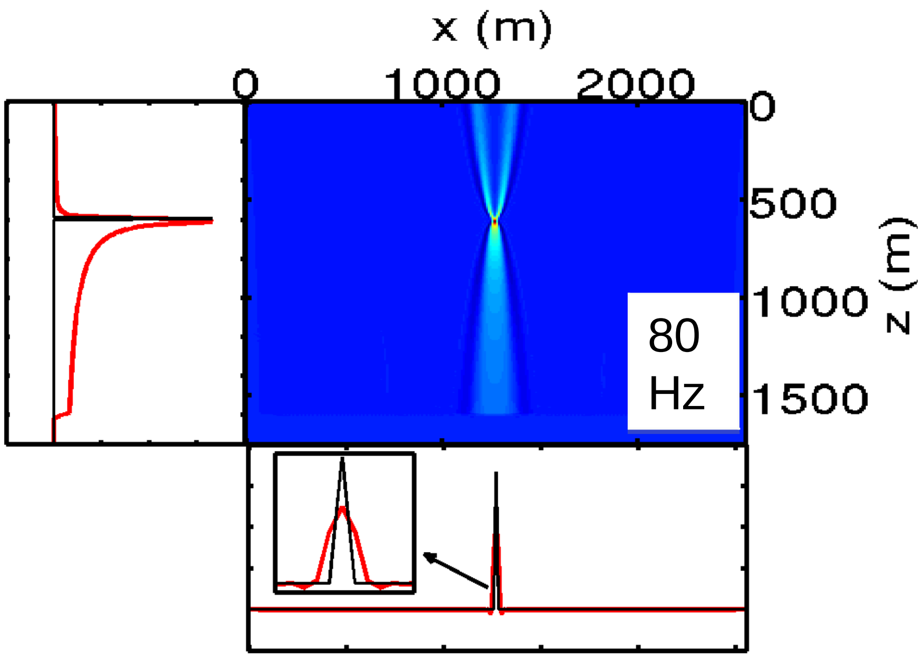


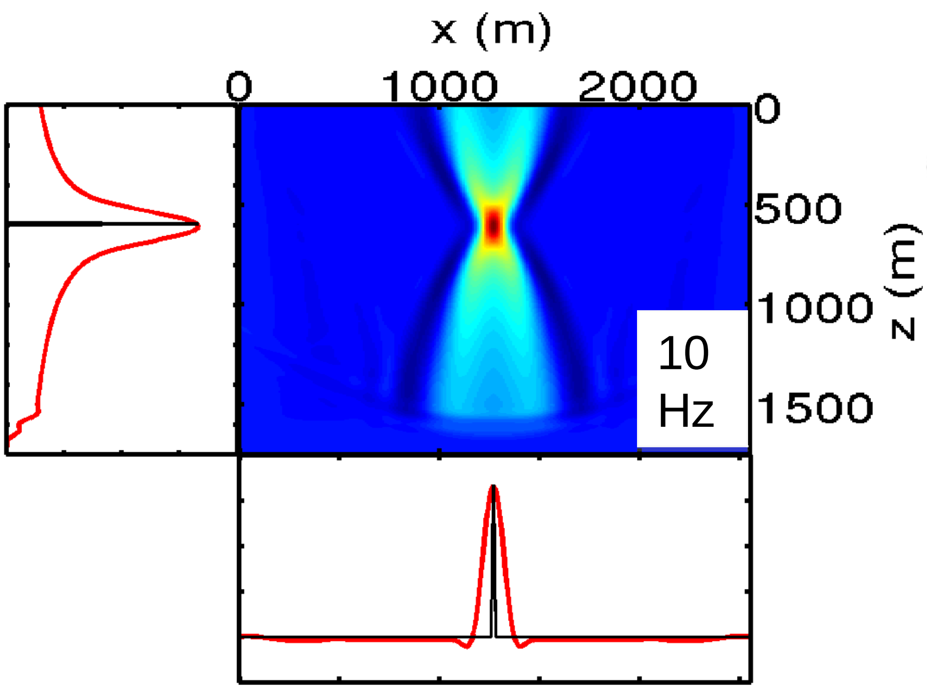
$[m/s]^{-1}$
x (m)
0
1000
2000
0
500
1000
1500
z (m)
120
Hz
Vertical profile
Gradient
$[m/s]^{-1}$
Longitudinal profile



Error
0
1
2
3
4
0
50
100
150
Frequency (Hz)


Error
Frequency (Hz)
0
1
2
3
4
0
50
100
150


x (m)
0
1000
2000
0
500
1000
1500
z (m)
80
Hz


x (m)
0
1000
2000
0
500
1000
1500
z (m)
10
Hz

Conclusions

- Our error analysis shows that MVA could benefit from the use of low frequencies. For practical applications, where background slowness and slowness perturbations are not known, this could mean that very low frequencies (less than 10 Hz) must be used for the method to work.

- We also conclude that the extent of the slowness perturbation has equal impact on the error as the frequency. Therefore it could be useful to limit the depth extent of the model in the first iterations.

- Finally we remind that the initial model must be sufficiently close to the true background model. This means we have a small ratio between the slowness perturbation and the background slowness

Acknowledgements

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