# Illumination analysis of wave-equation imaging with "curvelets"

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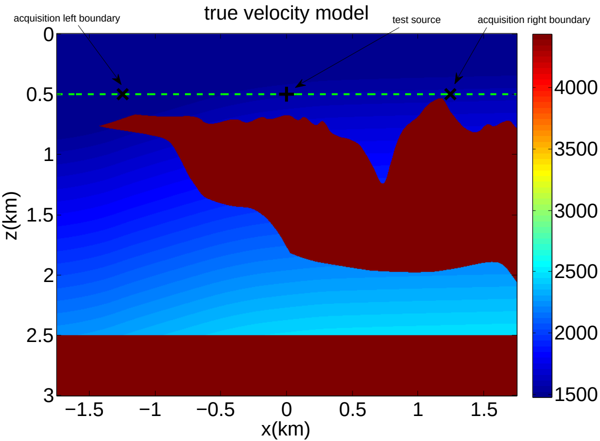
# Overview

- background and previous work
- why wave packets ("curvelets")?
  localization in both space and time
  localized plane wave
- single scattering
- diffraction formulation and partial reconstruction
  illumination correction
  normal operator correction
  via **inverse diagonal approximation** $\longrightarrow$ **partial reconstruction**
- numerical examples
- conclusion

# I. Background and previous work

- limited acquisition aperture VS. complex geological structure
- subsalt imaging: target-oriented illumination analysis and correction

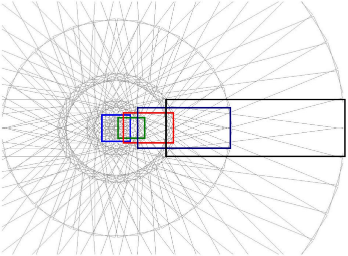


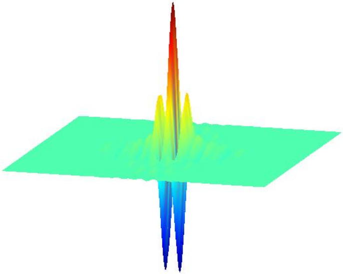
- Muerdter 2001: Raytracing based subsalt illumination
- Rickett 2003: Illumination-based normalization
- Kühl 2003: Least sqaure wave-equation migration
- Wu 2006: Directional illumination based on beamlets
- Xie 2006: Wave equation based illumination analysis
- Malcolm 2007: Illumination with internal multiples
- Alai 2008: Illumination towards shadow zones
- Symes 2008: Approximate linearized inversion
- de Hoop 2009: Partial reconstruction with "curvelets"
- Cao 2009: Frequency domain directional illumination

# II. Why wave packets ("curvelets")?

$$\hat{\varphi}_\gamma(\xi) = \rho_k^{-1/2} \hat{\chi}_{\nu,k}(\xi) \exp[-\mathrm{i}\langle x_j, \xi\rangle], \quad \gamma = (x_j, \nu, k)$$

(de Hoop, Smith, Duchkov, Anderson, Wendt)





- properties of curvelet transform:

  analysis $C$ (forward CT): $\quad d_\gamma = (Cd)_\gamma$

  synthesis $C^T$ (inverse CT): $\quad C^T\{d_\gamma\} = \sum_\gamma d_\gamma \varphi_\gamma$

  recovery: $\quad C^T C = I$

  projection: $\quad \Pi = CC^T \neq I$, with $\quad \Pi_{\gamma'\gamma} = \langle \varphi_{\gamma'}, \varphi_\gamma \rangle$

- operator matrix representation: (de Hoop *et al.* 2009)

  $Fm$

  $C^T C F C^T C m$

  $C^T[CFC^T]Cm$ where $\quad [F]_{\gamma'\gamma} = (CFC^T)_{\gamma'\gamma} = \langle \varphi_{\gamma'}, F\varphi_\gamma \rangle$

# III. Single scattering

- extension operators

$$E_1: \ (c_0^{-3}\delta c)(z,x) \ \mapsto \ h(z,\bar{x},x) = \delta(x-\bar{x})2(c_0^{-3}\delta c)(z,\tfrac{\bar{x}+x}{2})$$

$$E_2: \ h(z,\bar{x},x) \ \mapsto \ R(z,x,\bar{x},t) = \delta(t)h(z,\bar{x},x)$$

- single scattering operator (Born approximation)

$$F: \ \delta c \mapsto LE_2E_1 2c_0^{-3}\delta c$$

with ($L$: **DSR** propagator)

$$LR(s,r,t) = \int_{\mathbb{R}_+}\left\{\int_{\mathbb{R}^{n-1}}\int_{\mathbb{R}^{n-1}}\int_{\mathbb{R}}\left(\int_0^{t-t_0} G(0,r,t-t_0-\bar{t}_0,z,x)\, G(0,s,\bar{t}_0,z,\bar{x})\,\mathrm{d}\bar{t}_0\right) R(z,x,\bar{x},t_0)\,\mathrm{d}\bar{x}\,\mathrm{d}x\,\mathrm{d}t_0\right\}\mathrm{d}z$$

# IV. Diffraction formulation & partial reconstruction

- normal operator & normal equation
  ($\delta c \in \mathbb{R}^n, d \in \mathbb{R}^{2n-1}, 1^{\mathcal{S}}$:acquisition aperture)

$$F^* 1^{\mathcal{S}} F \delta c = F^* 1^{\mathcal{S}} d, \qquad N = F^* 1^{\mathcal{S}} F$$

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with **thin-slab** propagation ($L = L_0 L_{m-1}$)

$$(E_1^* E_2^* L_{m-1}^* L_0^*) 1^{\mathcal{S}} (L_0 L_{m-1} E_2 E_1) \delta c = (E_1^* E_2^* L_{m-1}^* L_0^*) 1^{\mathcal{S}} d$$

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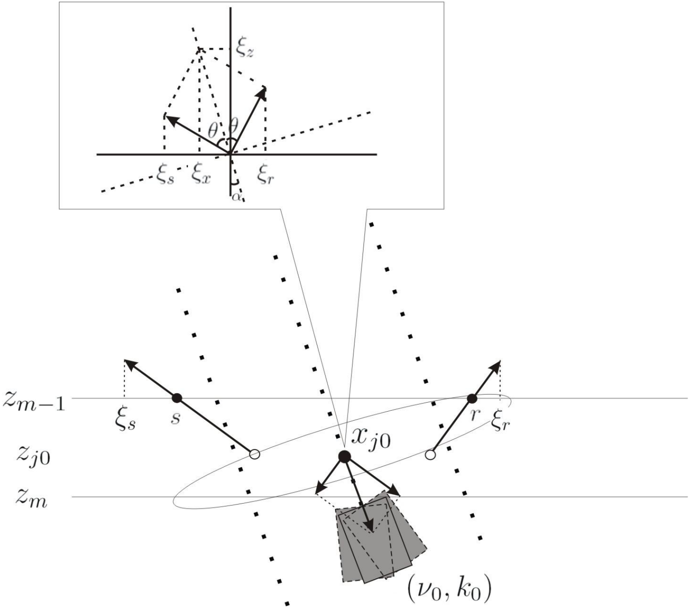
$$(E_1^* \underbrace{E_2^* L_{m-1}^*}_{K_{m-1}^*})[L_0^* 1^{\mathcal{S}} L_0](\underbrace{L_{m-1} E_2}_{K_{m-1}} E_1) \delta c = (E_1^* \underbrace{E_2^* L_{m-1}^*}_{K_{m-1}^*})[L_0^* 1^{\mathcal{S}} d]$$

# illustration: thin-slab propagation and a wave packet



# STEP 1. partial "redatuming"

- illumination correction via inverse diagonal approximation

**illumination operator**: $L_0^* 1^{\mathcal{S}} L_0$

**illumination matrix** correspondingly: $[A^{\mathcal{S}}(z_{\bar{m}-1}, z_{m-1})]_{\bar{\gamma}'\gamma'}$

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Inverse diagonal approximation: $\mathcal{O}(2^{-k/2})$ (de Hoop *et al.* 2009)

$$\widetilde{D}_{\gamma'}(z_{m-1})^{-1} = [A^{\mathcal{S}}(z_{m-1}, z_{m-1})]^{-1}_{\gamma'\gamma'} \Pi_{\gamma'\gamma'}$$

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applying this inverse diagonal on both sides of the normal equation (reaching the top of the thin slab):

$$\widetilde{D}_{\gamma'}(z_{m-1})^{-1}[L_0^* 1^{\mathcal{S}} L_0] d(z_{m_0-1}) = \widetilde{D}_{\gamma'}(z_{m-1})^{-1}[L_0^* 1^{\mathcal{S}} d]$$

# STEP 2: micro-diffraction tomography within the thin slab

- thin-slab normal operator:

$$\Xi_{m-1} = K_{m-1}^{*} 1^{\mathcal{S}(z_{m-1})} K_{m-1}$$

in which $K_{m-1} = L_{m-1} E_2$

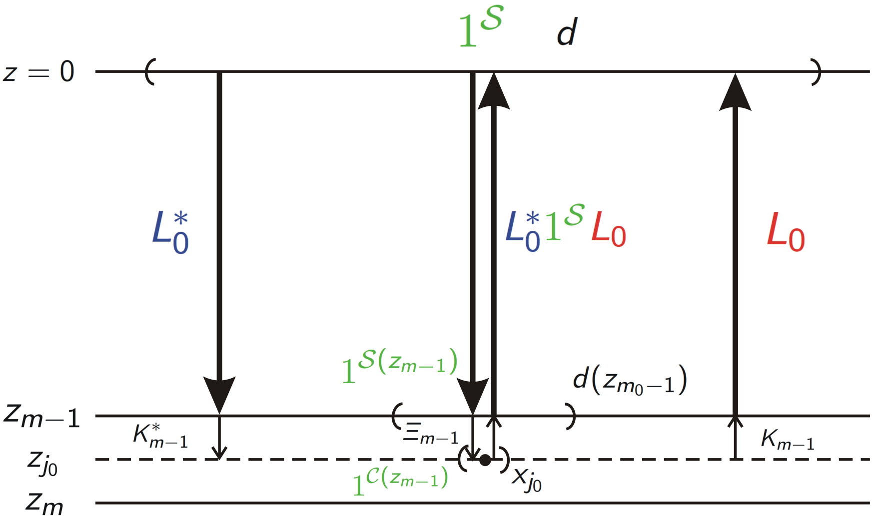
- inverse diagonal approximation of $\Xi_{m-1}$

$$(\widetilde{D}_{\Xi})_{\gamma_0}(z_{m-1})^{-1} = [\Xi_{m-1}]^{-1}_{\gamma_0\gamma_0} \Pi_{\gamma_0\gamma_0}$$

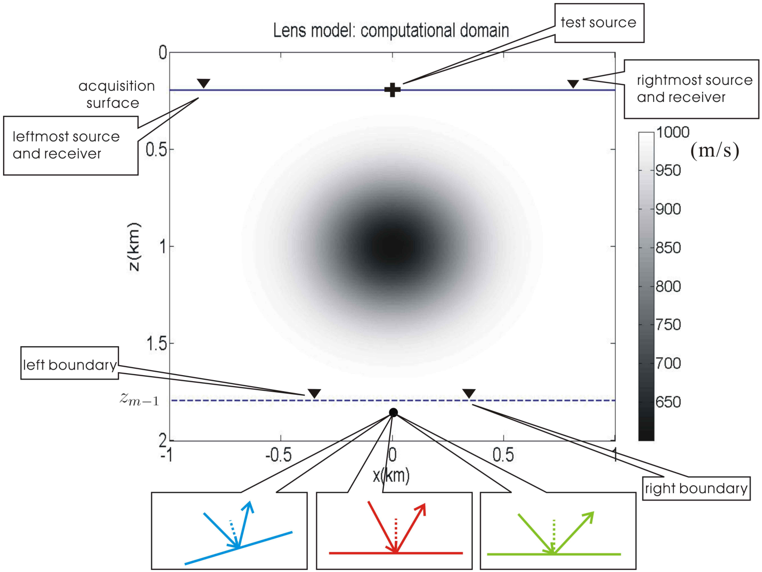
- partial reconstruction

$$\begin{aligned}\widetilde{D}_{\Xi}(z_{m_0-1})^{-1}[K_{m_0-1}^{*}]\Pi^{\mathcal{S}(z_{m_0-1})}[K_{m_0-1}]\Pi^{\mathcal{C}(z_{m_0-1})}(h) \\ = \widetilde{D}_{\Xi}(z_{m_0-1})^{-1}[K_{m_0-1}^{*}]\Pi^{\mathcal{S}(z_{m_0-1})}(d(z_{m_0-1}))\end{aligned}$$

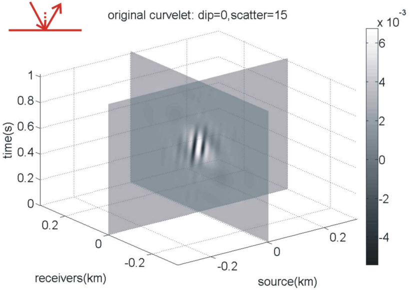
# illustration: "anatomy" of the normal operator

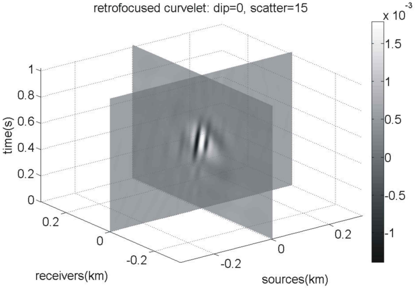


# V. Numerical examples

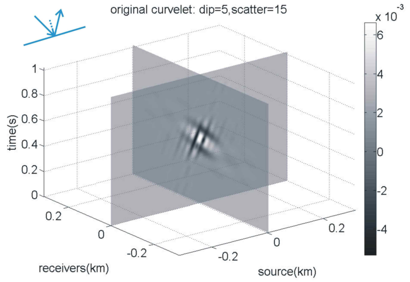


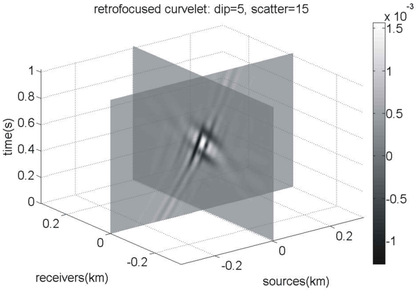
# lens: retrofocusing 1



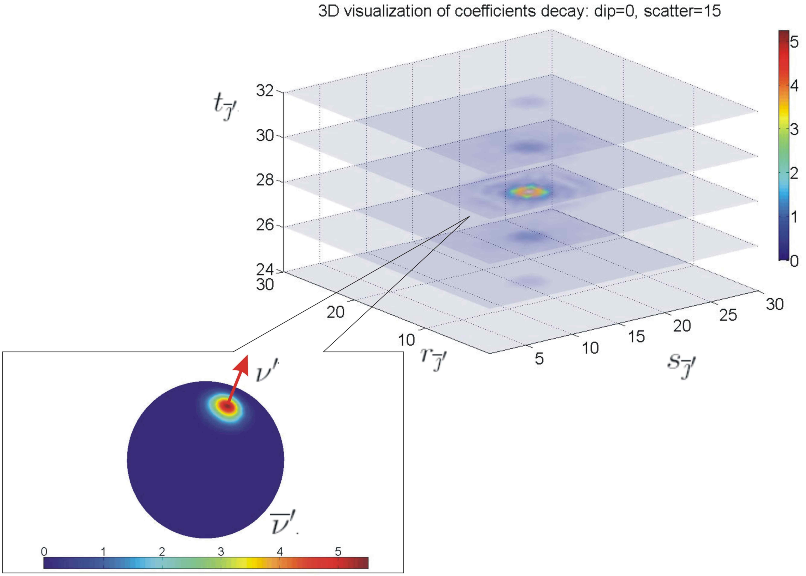



# lens: retrofocusing 2

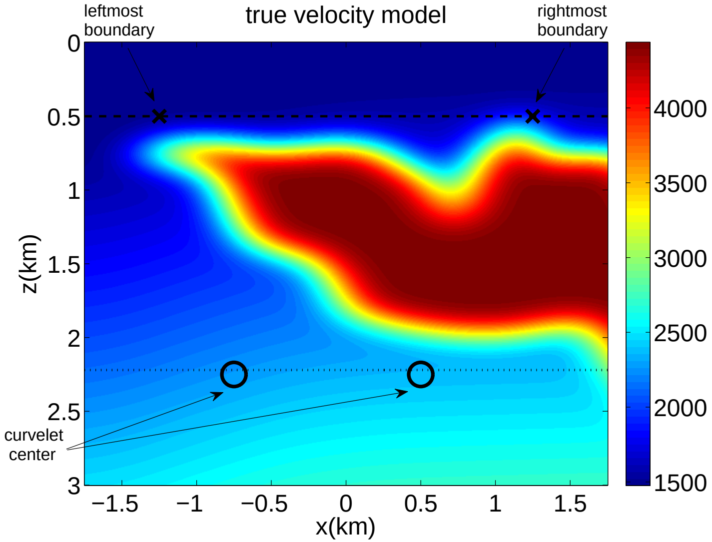


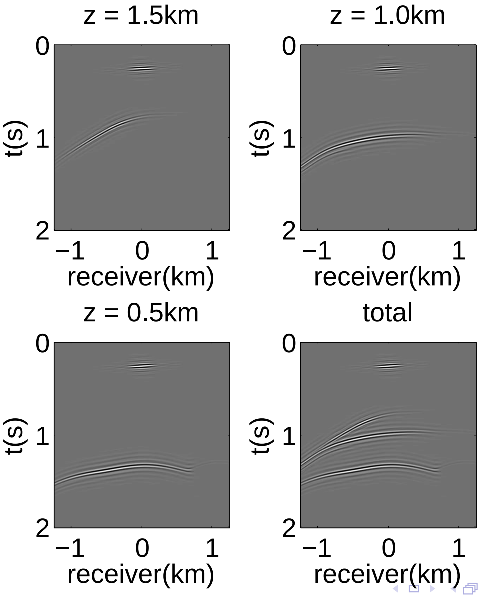
# curvelet coefficients decay exponentially



# example 2: salt

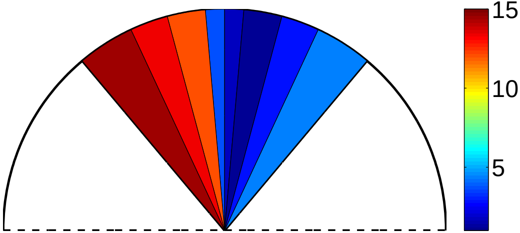


# salt: upward continuation, dip 0°, scattering 15°

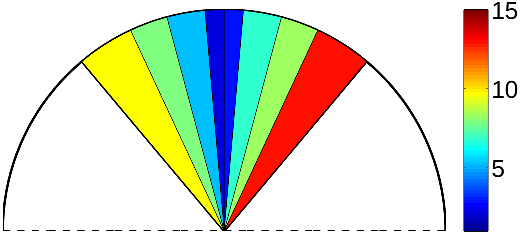


# salt: illumination dip angle response

- source1: $z = 2.25km, x = -0.75km$

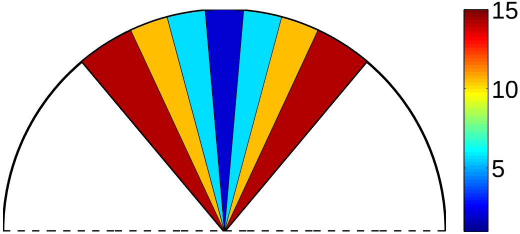



- source2: $z = 2.25km, x = 0.5km$



# salt: illumination scattering angle response

- source: $z = 2.25km, x = 0.5km$



# VI. Conclusions

- wave-equation based illumination analysis and correction

- wave packets ("curvelets"): localization in both space and time

- **inverse diagonal approximation & partial reconstruction**

  illumination correction

  normal operator correction

- artifacts minimization

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