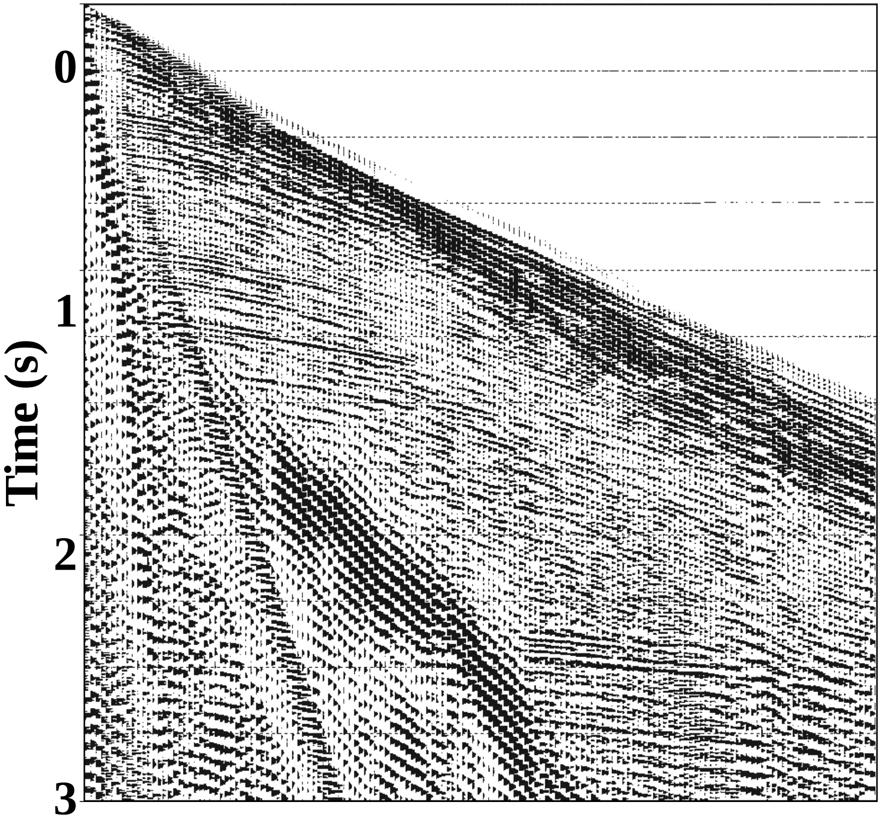
Single-station SVD-based polarization filtering: theoretical and synthetic data investigations

Olena Tiapkina, Martin Landrø and Yuriy Tyapkin

NTNU
Trondheim, Norway

Outline

- Introduction
- Description of polarization filter
- Theoretical investigation
- Stochastic simulation of synthetic data
- Conclusions



Polarization properties of seismic waves

Linearly polarized reflected waves

$$S_x(\omega) = k_x A(\omega) e^{i\varphi(\omega)}$$

$$S_y(\omega) = k_y A(\omega) e^{i\varphi(\omega)}$$

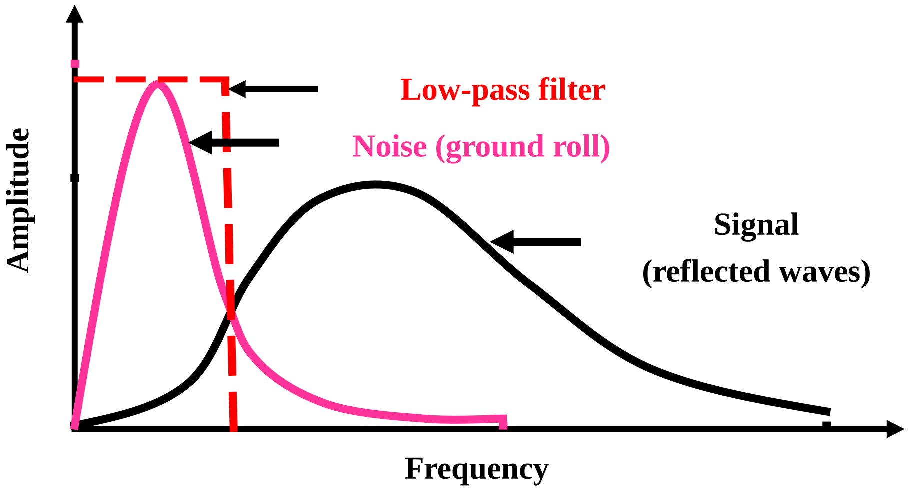
$$S_z(\omega) = k_z A(\omega) e^{i\varphi(\omega)}$$

Elliptically polarized ground roll

$$R_x(\omega) = q_x B(\omega) e^{i\psi(\omega)}$$

$$R_y(\omega) = q_y B(\omega) e^{i\psi(\omega)}$$

$$R_z(\omega) = q_z B(\omega) e^{i\left[\psi(\omega)+\frac{\pi}{2}\right]}$$

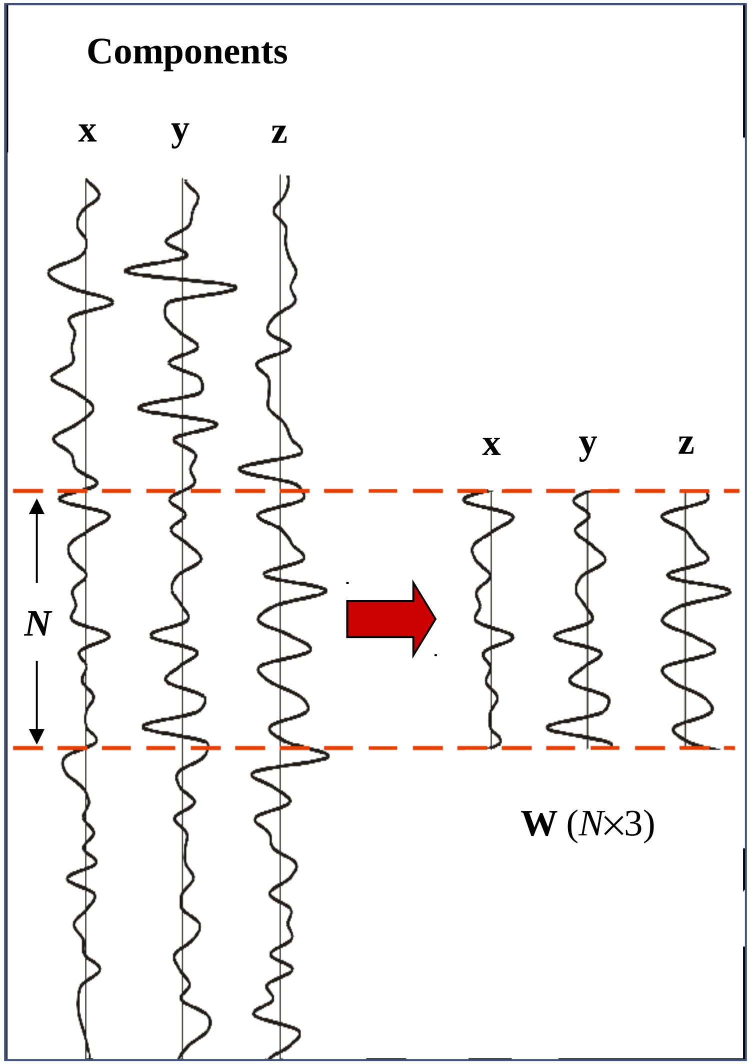


Ground roll has relatively high energy

Ground roll has relatively low apparent velocity

Matrix forming in a sliding window

Jackson, G.M. et.al, 1991



$$\mathbf{W} = (\mathbf{w}_x \ \mathbf{w}_y \ \mathbf{w}_z)$$

$$Dim\,(\mathbf{W}) = N \times 3$$

$$\mathbf{W} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \sum_{i=1}^{3} \sigma_i \mathbf{u}_i \mathbf{v}_i^T$$

without random noise:

Elliptical polarization (ground roll):

$rank(\mathbf{W}) = 2$

$$\mathbf{W} = \mathbf{E}_1 + \mathbf{E}_2$$

Linear polarization (signal):

$rank(\mathbf{W}) = 1$

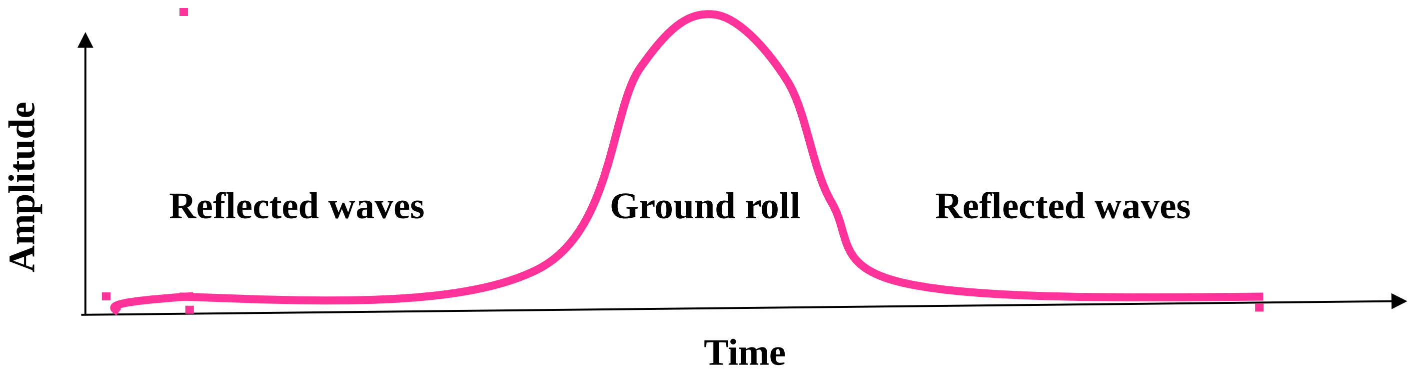
$$\mathbf{W} = \mathbf{E}_1$$

$\mathbf{E}_1$, $\mathbf{E}_2$ and $\mathbf{E}_3$	eigenimages or principal components
$\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$	left singular vectors
$\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$	right singular vectors
σ_1, σ_2 and σ_3	singular values ($\sigma_1 \geq \sigma_2 \geq \sigma_3$)

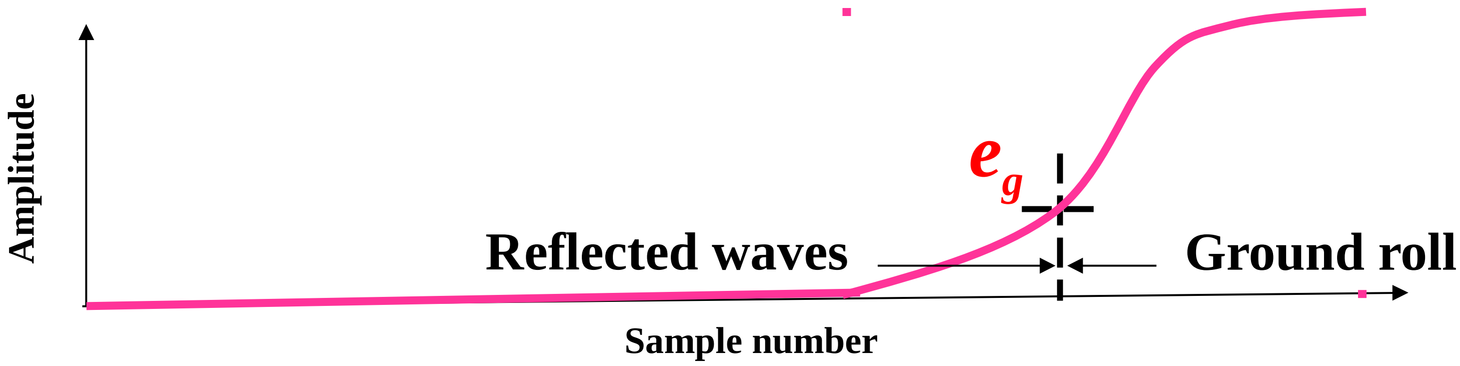
Attribute *e*

(by Jin and Ronen, 2005)

$$e = (\sigma_1 - \sigma_3)(\sigma_2 - \sigma_3)$$

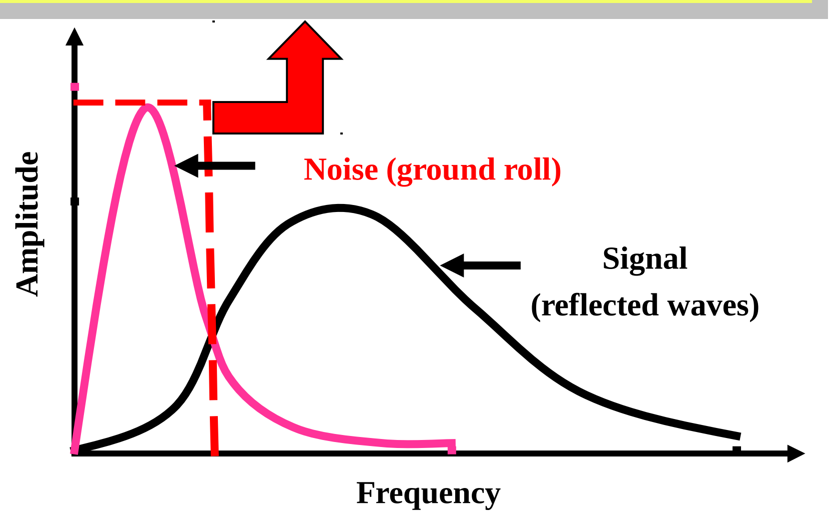


Attribute *e* arranged in non-descending order



Filtering

$$\mathbf{F} = \begin{cases} \mathbf{W} & \text{if } e < e_g \\ \mathbf{W} - (\mathbf{E}_1 + \mathbf{E}_2) & \text{if } e \geq e_g \end{cases}$$



F result of filtering

W original 3C data

$\mathbf{E}_1$ and $\mathbf{E}_2$ first two eigenimages of low-pass filtered original data

… How much signal energy remains in the third SVD term E3?

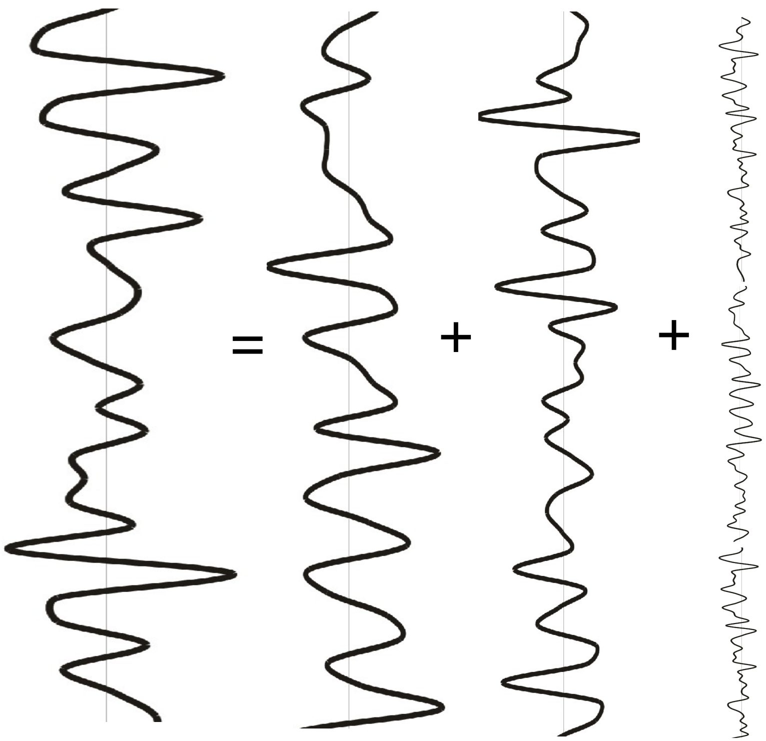
Mathematical model of the record

$$\mathbf{W} = (\mathbf{w}_x \ \mathbf{w}_y \ \mathbf{w}_z)$$

$$\mathbf{w}_i = a_i \mathbf{D}_i \mathbf{g} + b_i \mathbf{s} + \mathbf{n}_i$$

with

$$\mathbf{D}_i = \begin{cases} \mathbf{I}, & i = x, y \\ \mathbf{H}, & i = z \end{cases}$$



I identity operator

H discrete Hilbert transform

a_i and b_i amplitudes of ground roll and signal

g and **s** “forms” of ground roll and signal

$$\|\mathbf{g}\| = \|\mathbf{Hg}\| = \|\mathbf{s}\| = 1$$

$$\|\mathbf{n}_i\| = c^2$$

Cross-correlation matrix

$$\mathbf{R} = \mathbf{W}^T\mathbf{W} = \begin{bmatrix} a_x^2 + b_x^2 + c^2 & a_x a_y + b_x b_y & b_x b_z \\ a_x a_y + b_x b_y & a_y^2 + b_y^2 + c^2 & b_y b_z \\ b_x b_z & b_y b_z & a_z^2 + b_z^2 + c^2 \end{bmatrix}$$

Characteristic polynomial $\left|\mathbf{R} - \lambda\mathbf{I}\right| = 0$

$$\lambda_0 = \lambda - c^2$$

$$\lambda_0^3 + q_2\lambda_0^2 + q_1\lambda_0 + q_0 = 0$$

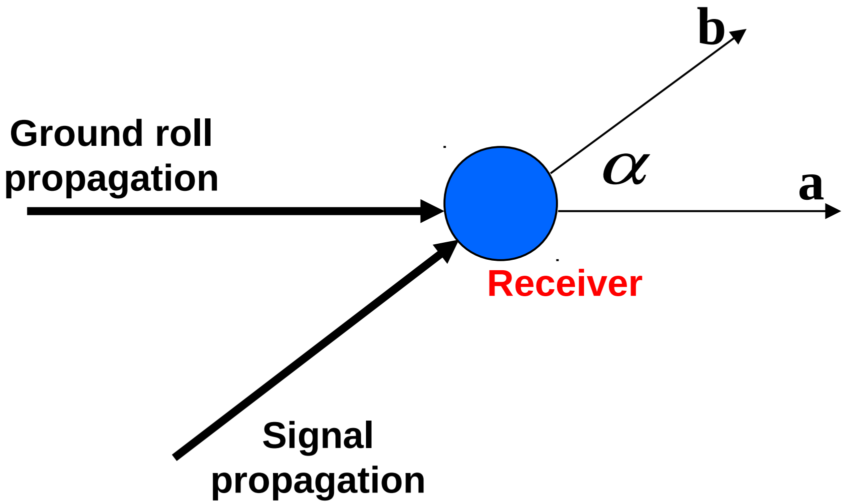
q_0, q_1, q_2 are functions of ground roll and signal amplitudes

Cardano's formula:

$$\lambda_3 = \sigma_3^2 = 2\sqrt{-Q}\cos\left[\left(\theta + 2\pi\right)/3\right] + A/3 + c^2$$

A, Q, θ are functions of ground roll and signal amplitudes

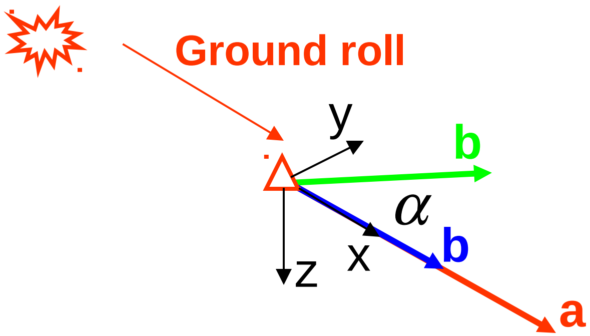
$$\mathbf{a} = \{a_x, a_y\} \quad \mathbf{b} = \{b_x, b_y\}$$



If $\alpha = 0$, then $\lambda_3 = c^2$

In this case filter subtracts not only ground roll, but also signal.

$\mathbf{a} = \{a_x, a_y\}$ $\mathbf{b} = \{bx, by\}$



Signal

Signal

If ground roll is much stronger than signal, only an appreciable part of the horizontal signal component perpendicular to vector a remains after polarization filtering.

Stochastic simulation of synthetic data

$$\mathbf{w}_i = a_i \mathbf{D}_i \mathbf{g} + b_i \mathbf{s}$$

"Forms" **g** *and* **s** *are independent stochastic processes*

Random noise is negligible

Signal has three components

Ground roll has x and z components with fixed ratio of their energies: $\frac{a_z^2}{a_x^2} = 4$

We studied the performance of polarization filtering depending on

(1) ground roll-to-signal energy ratio $e = \left(a_x^2 + a_z^2\right) / \left(b_x^2 + b_y^2 + b_z^2\right)$

(2) vertical-to-horizontal signal component energy ratio $p = b_z^2 / \left(b_x^2 + b_y^2\right)$

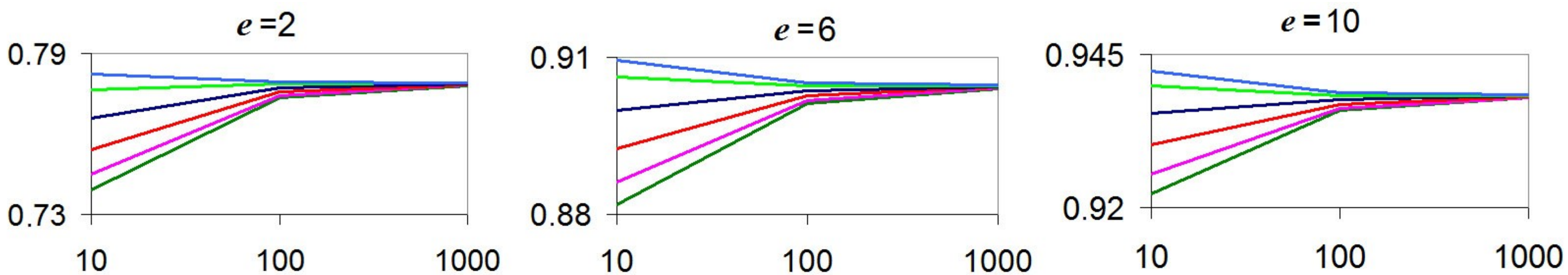
(3) angle α *between vectors* $\mathbf{a}$ *and* $\mathbf{b}$

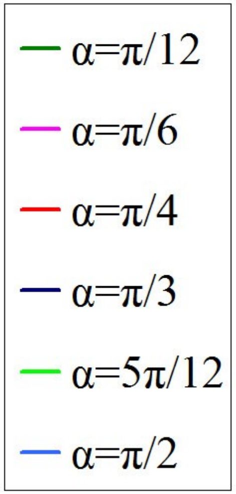
We consider how much signal energy remains on y component

Since P and S waves behave differently, we consider them separately.

P waves

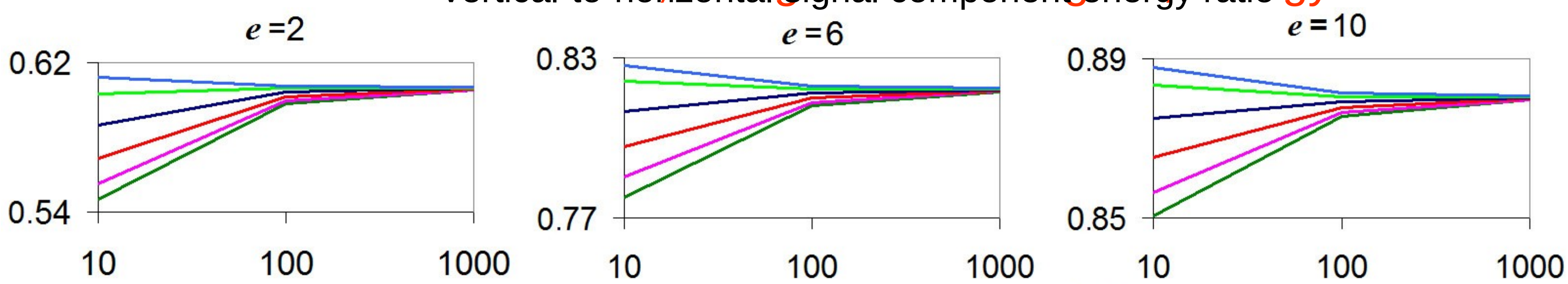
the correlation coefficient with the “pure” signal





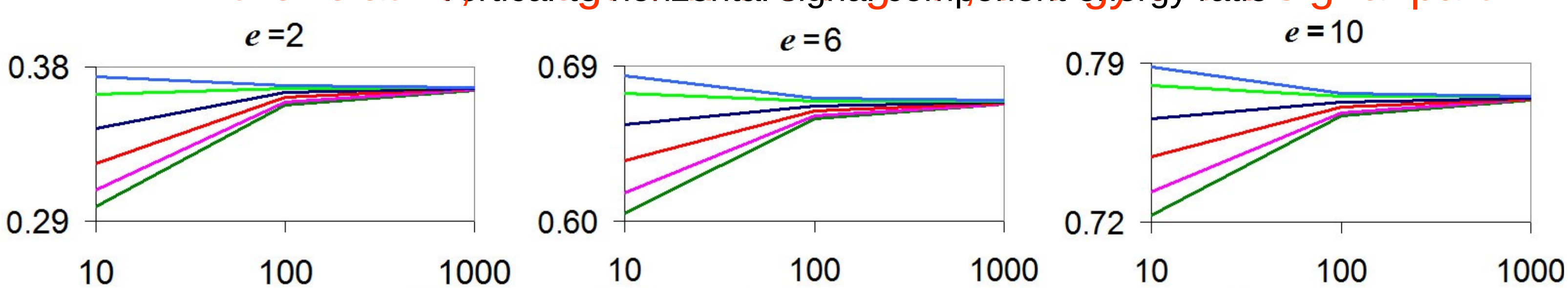
Vertical-to-horizontal signal component energy ratio

the relative, in regard to the original, energy



Vertical-to-horizontal signal component energy ratio

the relative, in regard to the original, energy of the signal part

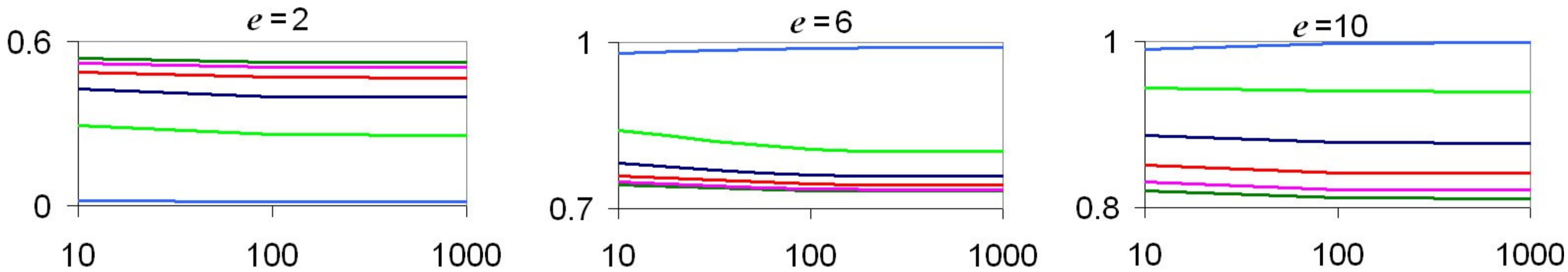


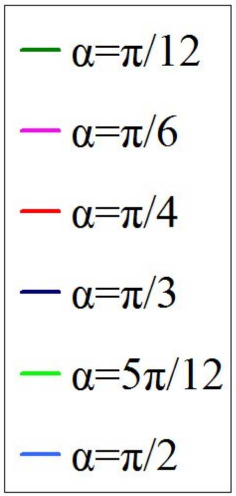
Vertical-to-horizontal signal component energy

Y component characteristics after polarisation filtering depending on ground roll-to-signal energy ratio *e*, vertical-to-horizontal signal component energy ratio *p*, and angle α between vectors **a** and **b**.

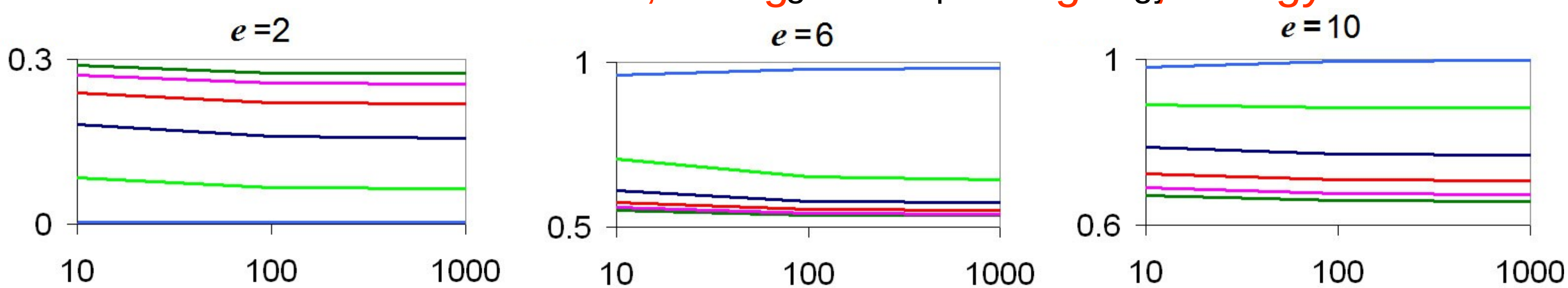
S waves

the correlation coefficient with the “pure” signal

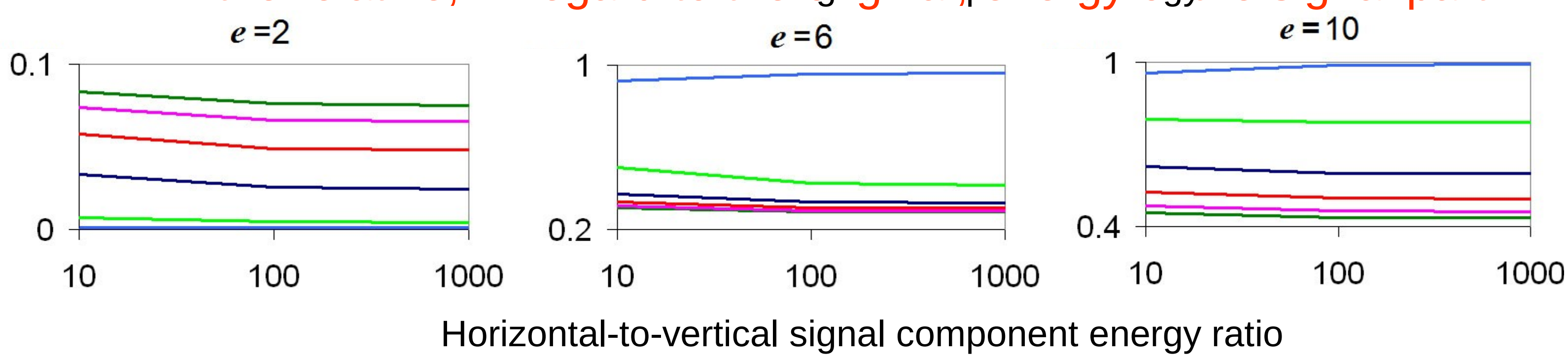




the relative, in regard to the original energy



the relative, in regard to the original, energy of the signal part



Y component characteristics after polarisation filtering depending on ground roll-to-signal energy ratio *e*, horizontal-to-vertical signal component energy ratio *p*, and angle α between vectors **a** and **b**.

Conclusions

- Single-station SVD-based polarization filtering has been investigated theoretically and using stochastic simulation of synthetic data

- After filter application, most of signal energy can be preserved only on horizontal component perpendicular to the plane where ground roll propagates

- For P- and SV-wave data, if α is large and horizontal to vertical components energy ratio of signal is large, then application of the filter is favorable.

- For SH-wave data, application of the filter is favorable if ground roll-to-signal energy ratio is rather high.

- Influence of errors in scaling between data components is planned to be investigated

Acknowledgements

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