New weak-contrast approximation in VTI media

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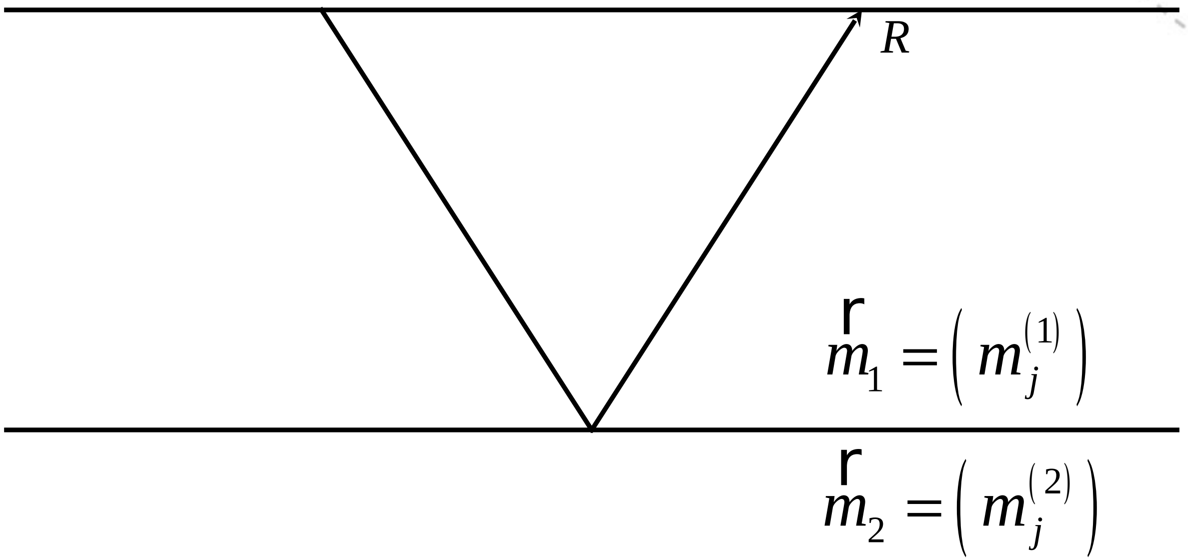
Motivation



Objective: to investigate the weak-contrast approximation of reflection coefficients using the new parametrization

Outline

- Exact solution
- Acoustic approximation
- Weak-contrast approximation
  - Standard parametrization
  - New parametrization
- Numerical results
- Conclusions

Exact solution

$$\mathbf{AR} = \mathbf{b}$$

where **A** is the 4x4 boundary equation matrix dependent on the parameters above and below the interface:

- directional cosines of the qP- and qSV-wave
- vertical slownesses of the qP- and qSV-wave
- horizontal slowness
- stiffness coefficients

$$\mathbf{R} = (R_{PP}, R_{PS}, T_{PP}, T_{PS})^T \quad \mathbf{b} = (-a_{11}, -a_{21}, a_{31}, a_{41})^T$$

Daley and Hron (1977) and Graebner (199

Acoustic approximation

qP- and qSV-wave slowness surface

$$q_P^2 = \frac{1}{\alpha_0^2}\left[1-\left(1+2\delta+S(p)\right)p^2\alpha_0^2\right]$$

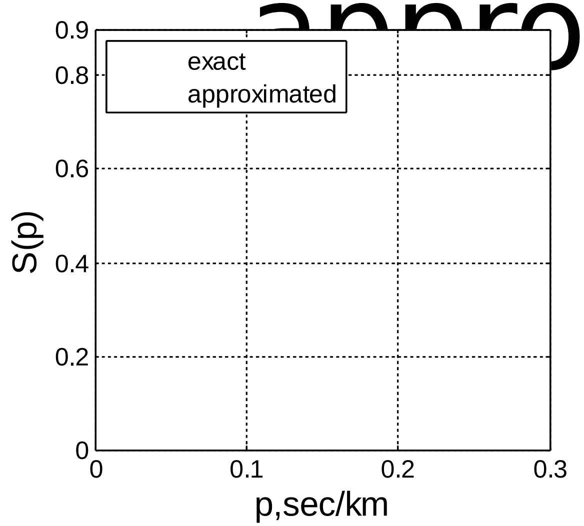
$$q_S^2 = \frac{1}{\beta_0^2}\left[1-\left(1+2\sigma-S(p)\right)p^2\beta_0^2\right]$$

Acoustic approximation (Alkhalifah, 1998; Stovas and Roganov, 2009

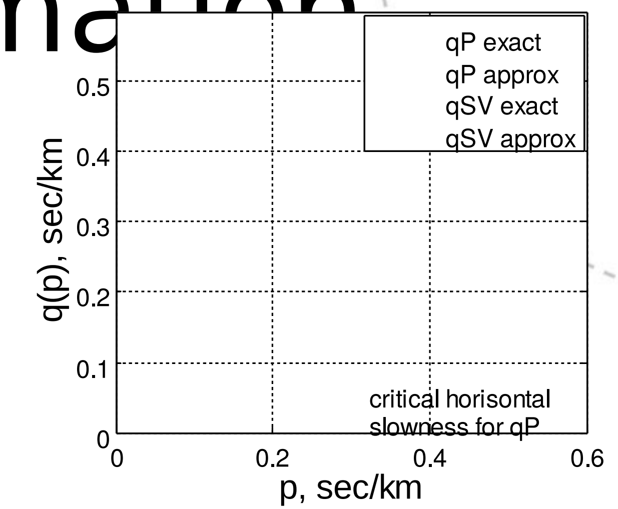
$$q_P^2 = \frac{1}{\alpha_0^2}\frac{1-(1+2\eta)p^2v_{nmo}^2}{1-2\eta p^2 v_{nmo}^2}$$

$$\hat{S}(p) \approx \frac{1}{\alpha_0^2}\frac{2\eta p^2 v_{nmo}^4}{1-2\eta p^2 v_{nmo}^2}$$

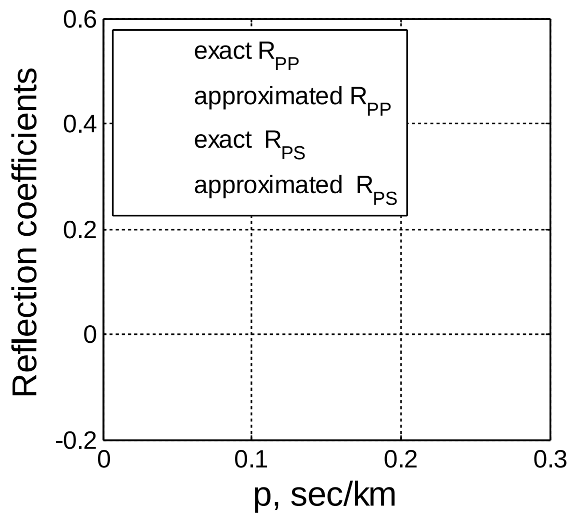
Accuracy of acoustic approximation



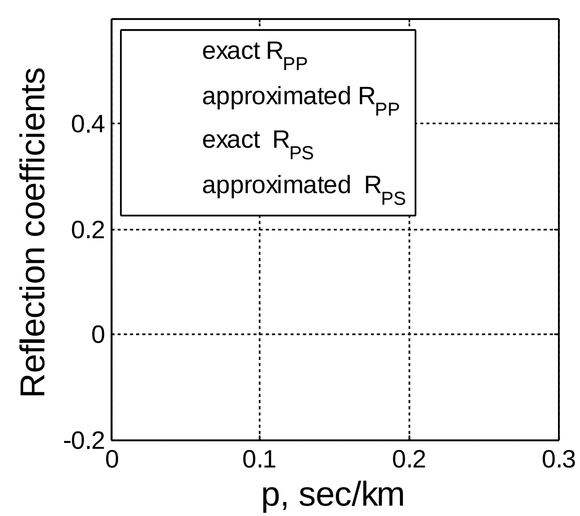
S-function approximation



Vertical slowness approximation

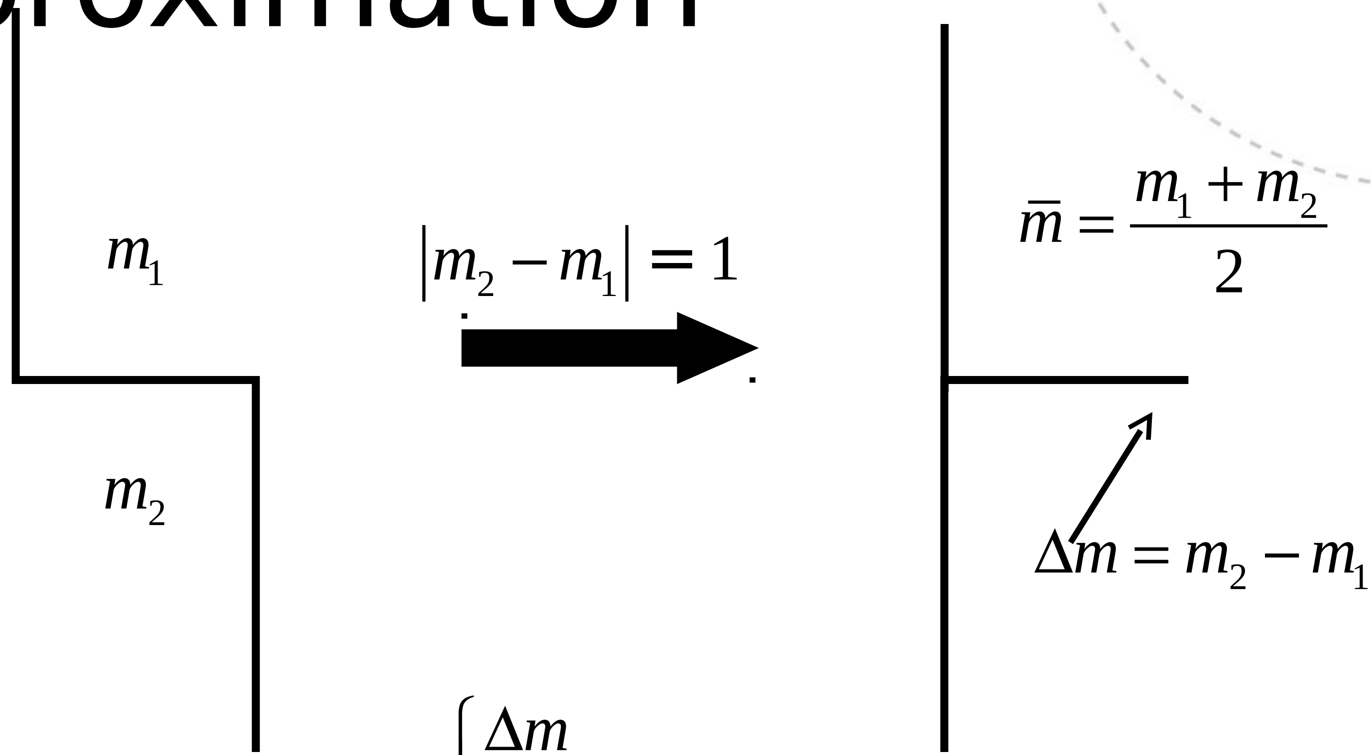


Model with strong anellipticity



Model with weak anellipticity

Weak-contrast approximation



$$dm = \begin{cases} \dfrac{\Delta m}{\bar{m}} \\ \Delta m \end{cases} \quad \text{contrast}$$

Standard parametrization in VTI media

P-wave vertical velocity $\alpha_0 = \sqrt{\dfrac{c_{33}}{\rho}}$

S-wave vertical velocity $\beta_0 = \sqrt{\dfrac{c_{44}}{\rho}}$

Anisotropic parameters

$$\begin{cases} \varepsilon = \dfrac{c_{11} - c_{33}}{2c_{33}} \\ \delta = \dfrac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})} \end{cases}$$

Density ρ

Thomsen, 1986

New parametrization

Standard parametrization

P-wave vertical velocity α_0

S-wave vertical velocity β_0

Anisotropic parameters $\left\{ \begin{array}{l} \varepsilon \\ \delta \end{array} \right.$

Density ρ

P-wave time processing parametrization

α_0	*P-wave vertical velocity*
$v_{nmo}^2 = \alpha_0^2 (1 + 2\delta)$	*qP-wave normal moveout velocity squared*
$\eta = (\varepsilon - \delta) / (1 + 2\delta)$	*Anellipticity factor*
$\gamma_0^2 = \beta_0^2 / \alpha_0^2$	*S- to P-wave velocity ratio squared*
$Z = \rho\alpha_0$	*P-wave impedance*

P-wave time processing

Interpretation

Background medium vs. parametrization

$$\hat{\beta}_0 = \hat{\alpha}_0\sqrt{\hat{\gamma}_0^2} = \bar{\beta}_0\left(\frac{1}{1-\frac{(d\alpha_0)^2}{4}}\sqrt{\left(1+\frac{(d\alpha_0)^2}{4}\right)\left(1+\frac{(d\beta_0)^2}{4}\right)} - d\alpha_0 d\beta_0\right)$$

$\hat{\beta}_0 = \bar{\beta}_0$ when $d\beta_0 = d\alpha_0$ or when $d\beta_0 = 3d\alpha_0\left(1-\frac{d\alpha_0^2}{12}\right)\Big/\left(1+\frac{d\alpha_0^2}{4}\right)$

$$\hat{\delta} = \frac{1}{2}\left(\frac{\hat{v}_{nmo}^2}{\hat{\alpha}_0^2}-1\right) = \bar{\delta}\left(1+\frac{(d\alpha_0)^2}{4}\right)+\frac{(d\alpha_0)^2}{8}+\frac{d\alpha_0 d\delta}{2}$$

$$\delta_1 = \delta_2 = \bar{\delta} \Longrightarrow \hat{\delta} = \bar{\delta}\left(1+\frac{(d\alpha_0)^2}{4}\right)+\frac{(d\alpha_0)^2}{8}$$

$$\delta_1 = \delta_2 = 0 \Longrightarrow \hat{\delta} = \frac{(d\alpha_0)^2}{8}$$ - Induced anisotro

Reflection coefficients

$$R_{PP} = R_0 + R_2 \sin^2\theta + R_4 \sin^4\theta$$

$$R_{PS} = R_1 \sin\theta + R_3 \sin^3\theta$$

Standard parametrization

$$R_0 = \frac{1}{2}\frac{\Delta Z}{Z}$$

$$R_i = R_i\left(\frac{\Delta\alpha_0}{\alpha_0}, \frac{\Delta\beta_0}{\beta_0}, \Delta\varepsilon, \Delta\delta, \frac{\Delta Z}{Z}\right)$$

$$i = 1, 2, 3, 4$$

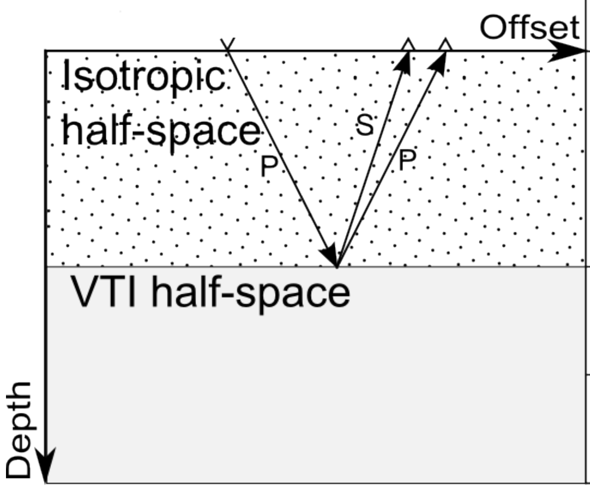
Proposed parametrization

$$R_0 = \frac{1}{2}\frac{\Delta Z}{Z}$$

$$R_i = R_i\left(\frac{\Delta\alpha_0}{\alpha_0}, \frac{\Delta\gamma_0^2}{\gamma_0^2}, \frac{\Delta v_{nmo}^2}{v_{nmo}^2}, \Delta\eta, \frac{\Delta Z}{Z}\right)$$

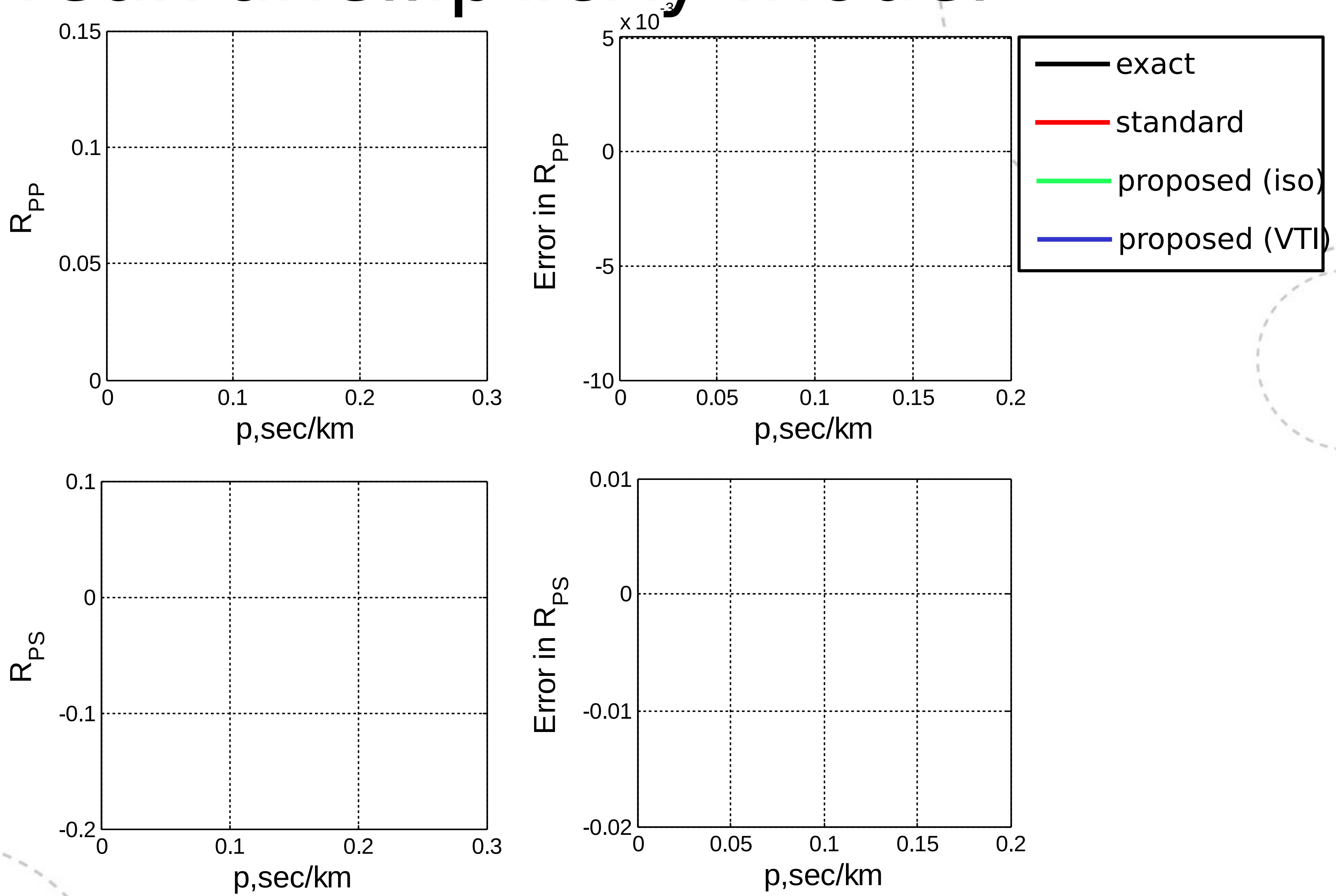
$$i = 1, 2, 3, 4$$

Numerical results

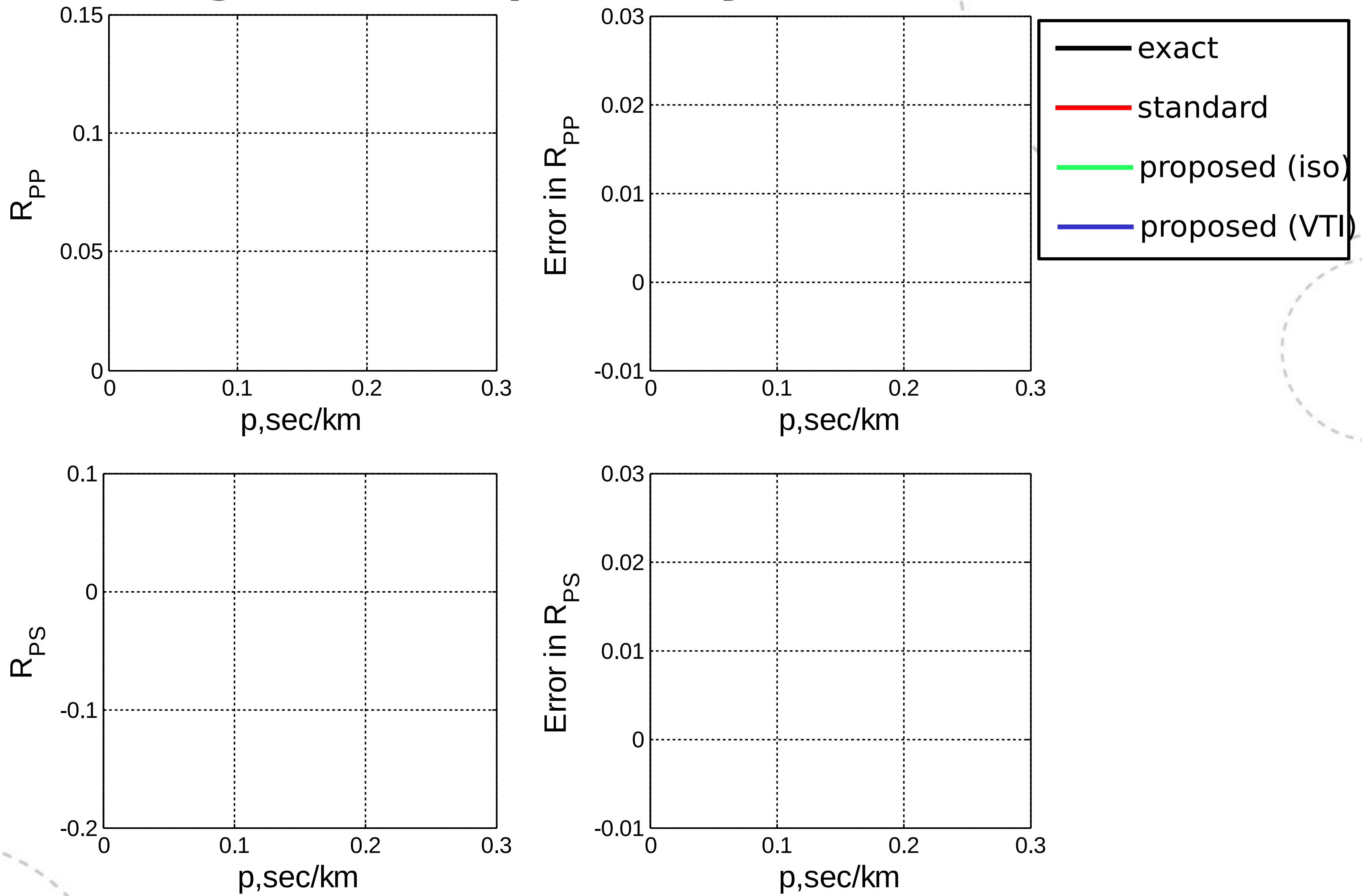


	α_0 $[km/s]$	β_0 $[km/s]$	ε	δ	v_{nmo}^2	η	γ_0^2	ρ $[kg/cm^3]$
	3	1.5	0	0	9	0	0.25	2
model 1 Green River shale	3.33	1.77	0.20	-0.22	6.21	0.74	0.28	2.2
model 2 Taylor sand	3.37	1.83	0.11	-0.04	10.56	0.16	0.30	2.2

Weak anellipticity model



Strong anellipticity model



Conclusions

- We propose new weak-contrast approximation qPqP- and qPqSV reflection coefficients using acoustic approximation
- The approximation is based on new parametrization of a VTI medium using qP-wave processing parameters and interpretation parameters
- We develop new approximation for both isotropic and VTI background
- Comparison with standard weak-contrast approximation shows that the proposed one gives similar or even better results

Acknowledgements

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Thank you
for your attention!
Questions?