

## Estimating the Directional Permeability of Reservoir Sandstones Using Image Analysis

*P.A. Lock, X.D. Jing, and R.W. Zimmerman*

Department of Earth Science and Engineering, Imperial College, London SW7 2BP, UK, p.lock@ic.ac.uk, x.jing@ic.ac.uk, r.zimmerman@ic.ac.uk

### Introduction

The goal of this project is to develop a method for predicting the permeability of a rock sample using only a small number of SEM images, without any computationally intensive or time-consuming procedures. The pore structure is idealised as consisting of a cubic network of pore tubes, with the tubes having an arbitrary distribution of cross-sectional areas and shapes. The hydraulic radius approximation is used to compute the individual conductivities of the pores, based on their observed areas and perimeters. The effective medium theory of Kirkpatrick (1973) is then used to estimate the effective pore conductivity.

The methodology has been applied to several reservoir sandstones from the UK continental shelf, having permeabilities in the range of 10-600 mD. It is found that for those samples exhibiting obvious anisotropy, the method yields accurate estimates of the permeabilities, as well as a qualitative indication of anisotropy.

### Calculational Procedure

The first step in the analysis is the estimation of the apparent area and perimeter of each individual pore that can be recognized in the image. When using the intrinsically isotropic Kirkpatrick equation, data only from that face of the cubic sample that is normal to the flow is used as input. The application of a stereological correction factor yields the actual area and perimeter values, from which the pore conductivity can be determined. A hydraulic constriction factor is then used to account for variations in pore radius along the pore length. The resulting set of conductances is truncated to eliminate the contribution of non-conducting features, and is then input into Kirkpatrick's EMT to produce an effective pore conductance,  $G_{eff}$ . The permeability is then calculated, after using another stereological correction to convert the apparent number density of pores in the field of the image to the actual density perpendicular to the flow direction. The complete procedure can be summarized as follows:

1. Take BSEI photographs of polished sections
2. Digitize pore images
3. Apply gray-level thresholding procedure to identify "pores"
4. Compute perimeter and area of each pore with image analyzer
5. Apply stereological correction and hydraulic constriction factor to estimate the hydraulic conductivity  $G_i$  of individual pores
6. Employ areal thresholding procedure to truncate data set
7. Use Kirkpatrick's equation to obtain  $G_{eff}$
8. Compute the areal density of pores inside the image
9. Assuming a cubic lattice, calculate  $k$

## Results

The predictions were within a factor of two of the measured values in 16 of the 18 cases. There seems to be a slight trend of overprediction in the low permeability range, and underprediction in the high permeability range, although this bias may be due to the small number of samples. The average percentage error is 23%, or 47% if absolute values of the error are considered.

The method, although based on an inherently isotropic model, gives qualitative indications of anisotropy, provided that the measured anisotropy is sufficiently substantial to exceed the “error bars” of the measurements. If we assume that each permeability measurement has an error of about 30%, two orthogonal permeability measurements should differ by at least 60% in order for the core to be considered unambiguously anisotropic. By this criterion, the only unambiguously anisotropic core is 4, which has a measured anisotropy of about {6:5:2}, and a predicted anisotropy of {5:3:2}. In this case, not only were each of the three principal permeabilities predicted to 25%, but the ordering of the permeabilities was also predicted quite accurately.

Table 1 lists the predicted permeabilities of the eighteen samples, compared with the values measured in the laboratory. The notation gives a core identifier and thin section number, where the “X,Y,Z” labeling refers to the direction that lies perpendicular to the slice of the thin section.

It was thought that the use of an inherently *anisotropic* model such as that of Bernasconi (1974) would lead to an improvement in the predictions. When using the anisotropic Bernasconi formulation, data from three orthogonal faces are used, and the three permeabilities are computed in a coupled manner. Our results, however, showed that the permeability estimations of the Bernasconi model were only slightly different from that of the Kirkpatrick equation; moreover, they did not lead to systematic improvements of the predictions.

Table 1. Compilation of results for the eighteen UKCS cores.

CORE	$G_{eff}$ (m <sup>4</sup> )	No. of pores	No. of images	Total area (m <sup>2</sup> )	$k_{pred}$ (mD)	$k_{measured}$ (mD)	Error (%)
1X	$1.14 \times 10^{-21}$	3495	2	$1.52 \times 10^{-5}$	29	18	59
1Y	$2.42 \times 10^{-21}$	7397	5	$3.80 \times 10^{-5}$	52	18	191
2X	$41.2 \times 10^{-21}$	1843	5	$3.80 \times 10^{-5}$	222	137	62
2Y	$34.4 \times 10^{-21}$	1244	3	$2.28 \times 10^{-5}$	209	119	76
2Z	$57.3 \times 10^{-21}$	1579	5	$3.80 \times 10^{-5}$	265	109	143
3X	$29.3 \times 10^{-21}$	2310	5	$3.80 \times 10^{-5}$	198	202	-2
3Y	$22.6 \times 10^{-21}$	2652	5	$3.80 \times 10^{-5}$	175	196	-11
3Z	$23.5 \times 10^{-21}$	2600	5	$3.80 \times 10^{-5}$	179	222	-19
4X	$9.2 \times 10^{-21}$	3229	5	$3.80 \times 10^{-5}$	87	83	4
4Y	$5.3 \times 10^{-21}$	2876	4	$3.04 \times 10^{-5}$	56	72	-23
4Z	$2.67 \times 10^{-21}$	1785	2	$1.52 \times 10^{-5}$	35	28	25
5X	$39.4 \times 10^{-21}$	1656	4	$3.04 \times 10^{-5}$	239	470	-49
5Z	$51.3 \times 10^{-21}$	1829	5	$3.80 \times 10^{-5}$	275	500	-45
6X	$26.8 \times 10^{-21}$	2394	5	$3.80 \times 10^{-5}$	188	275	-32
6Z	$27.8 \times 10^{-21}$	1854	4	$3.04 \times 10^{-5}$	189	228	-17
7X	$9.09 \times 10^{-21}$	3605	5	$3.80 \times 10^{-5}$	96	109	-12
7Y	$10.6 \times 10^{-21}$	2448	4	$3.04 \times 10^{-5}$	95	109	-12
7Z	$14.4 \times 10^{-21}$	2179	4	$3.04 \times 10^{-5}$	115	69	66

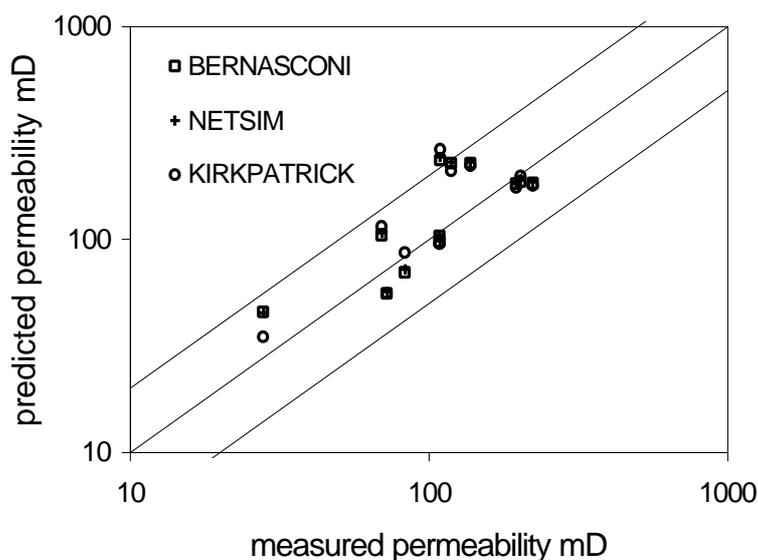


Fig. 1. A comparison of the permeability predictions using Kirkpatrick's isotropic EMT, Bernasconi's anisotropic EMT, and exact network calculations using NETSIM.

The use of the Kirkpatrick equation to estimate the effective conductivity was tested against the network simulation code NETSIM (Jing, 1990), which performs an essentially *exact* calculation by solving the flow equations in the entire network of tubes. This test was thought to be necessary, because previous studies of the validity of the Kirkpatrick equation have all been based on idealized conductivity distributions. Reassuringly, we found that the effective medium predictions agreed to within a few percent with the exact network calculations.

A cross-plot of the predicted permeability against the laboratory measurements is shown in Fig. 1, with lines that indicate errors of a factor of two in either direction.

### Discussion and Conclusions

We have developed a network model that allows predictions of the hydraulic permeability of consolidated sedimentary rocks such as sandstones, based on image analysis of polished sections of a rock core sample. The method sometimes overpredicts, and sometimes underpredicts the measured permeability, with an (absolute value) error that is on average only 47%. To put this in perspective, we point out that, for example, McPhee and Arthur (1994) made a series of gas permeability measurements on a Clashach quarried sandstone with an average permeability of 693 mD, and found an “error” of 32%, based on the standard deviation of all the measurements. Although these values are not directly comparable, the mean (absolute) relative error in our measurements was 47%, and the median (absolute) relative error was 23%. So, it seems that the errors in our predictions are comparable to the error inherent in the laboratory measurements.

Furthermore, the use of more computationally intensive procedures such as the inherently anisotropic effective medium theory of Bernasconi or essentially *exact* network calculations fail to produce any systematic improvement on the predictions obtained using the isotropic effective medium theory of Kirkpatrick.

It should be emphasized that our model is based entirely on measured attributes of the pore space, and contains no adjustable parameters. The question may be raised as to why our estimates are so much more accurate than those made by Koplík *et al.* (1984), using a broadly similar procedure. The main difference seems to be our inclusion of the constriction factor

and the stereological correction factors for pore size and number density, each of which lower the estimated pore conductance by about a factor of 2. Taken together, the absence of these corrections may explain why their estimates were too high by about an order of magnitude.

### **Acknowledgement**

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