Nordic Energy Research Programme Norwegian U. of Science and Technology

Application of Three Dimensional Failure Criteria on High-Porosity Chalk

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Abstract

At low confining pressures high porosity chalk fails in shear like other granular materials, but as the stress level increases, pore collapse becomes increasingly important. High porosity chalk may therefore be described by a Mohr-Coulomb failure surface with an end cap. The Mohr-Coulomb failure surface in the principal stress space appears as a six edged cone, as the criterion is independent of the intermediate principal stress. When the intermediate stress is taken into account, the material becomes apparently stronger and the six sided cone develops into a more rounded shape with a convex triangular cross section. It is a general requirement for failure surfaces to have a convex shape, and an extension of the Mohr-Coulomb criterion must comply to this condition.

Two of the most common failure surfaces reported in the literature involving all three principal stresses, are the extended Drucker-Prager and the Lade criterion. The simulation program ABAQUS, based on the finite element method, is using a formulation deduced from the Drucker-Prager criterion but with a restriction to values of friction angles less than 22 degrees if convexity is to be assured. Van Eekelen however proposed an extension of the Drucker-Prager criterion allowing friction angles up to 46 degree without destroying the convex shape of the surface.

Laboratory data for high porosity chalk showing friction angles ranging from 20 to 40 degrees is used to evaluate the formulations. The criteria are adjusted to match the compressive tests, and the results are then compared to extensional test data.

In addition, a new “end-cap” formulation, giving one continuous formulation is proposed to take care of the pore-collapse failure.

Lade’s criterion demonstrates limitations for higher friction angles. By matching a data set with a friction angle of 38 degrees the surface is showing unphysical behaviour by deviating from the conical shape. Matching a data set with a friction angle of 28 degrees from water-saturated samples gives a poor agreement with the corresponding failure values from extensional tests.

The extended Drucker-Prager formulation in ABAQUS, has a built in limitation, making the criterion unconsistent for friction angles higher than 22 degrees. The agreement with extensional data is good despite the limitation, even for friction angles higher than 22 degrees.

The extended Drucker-Prager criterion based on van Eekelen’s formulation gives a good agreement with extensional failure values and the surfaces remain convex within our range of friction angles.

The shear failure region and the pore collapse region are usually described by two separated mathematical formulations, frequently giving rise to numerical problems during simulation. The end-cap formulation proposed is showing good agreement with laboratory observations.

There is a need to develop models that can describe behaviour of both intact materials as above, and materials under failed conditions. Hence further work will be a study of how the models can be adjusted to a failure history.

Introduction

A rock composed of chalk is built up by coccoliths depositions. The microstructure of the chalk is rather complex and randomly built, giving rise to strong statistical variations in mechanical properties. Samples taken from intact chalk appear as a weakly cemented granular material and failure in shear with low confining pressure is described as brittle. In addition to shear failure the sample may fail in pore collapse. If the material is brought into high confining pressure and pore collapse is triggered, the failure mode is more ductile and the material appears more like a powdery friction material without cohesion.

When the internal bonds are broken and the chalk is saturated with a fluid, it could also be described as a suspension. Given the right condition the chalk will flow and the fluidized chalk properties are showing shear thinning with increasing shear rate and at a certain level of shear rate, called critical shear rate it will show shear thickening.

When modeling rock strength, the condition of failure in the principal stress space is required. Several formulations to predict failure and describe yield are reported in the literature for rocks and soil. Experimental failure surfaces for frictional materials as rock and soil, shown in the principal stress space have been observed to be shaped as cones with cross sections in the deviatoric planes which are triangular, monotonically curved surfaces with smoothly rounded corners. Still, the most prevalent failure criteria for isotropic rocks and soil, is the Mohr-Coulomb criterion, predicting a linear increase in strength with increasing hydrostatic stresses.

In this project, laboratory data from triaxial tests are used to adjust common three-dimensional yield criteria to describe the failure surface for chalk. The yield conditions or the flow and hardening rules are not included in this study, it means that the behaviour of the surface when yielding is not taken into account, but the shape of the surface according to experimental results have been the main purpose of this study. The chosen criteria are the ones proposed by Lade and Drucker Prager. Two different versions of the Drucker Prager is applied, one available in ABAQUS (a Finite Element program) and one that was optimized by van Eekelen (1980).

A three dimensional criterion is taking the intermediate stress into account, but in a conventional triaxial test two of the principal stresses are equal. The effect of the intermediate stress is therefore omitted in this study and the main purpose of this study is to use both data from compression and extension tests and evaluate how well the criterion will match the failure points in the triaxial-plane.

Lade's failure criteria

A general three-dimensional failure criterion for rocks and soil has been proposed by P. Lade. This criterion is formulated in terms of the first and the third stress invariant of the stress tensor, it involves only three independent material parameters. The parameters interact with one another but each parameter corresponds to one of three failure characteristics of rock behaviour.

For soils the expression is as follows,

$$f(s_1, s_2, s_3) = \left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{P_a}\right)^m - h_1 \tag{1}$$

where,

$$I_1 = s_{11} + s_{22} + s_{33} \tag{2}$$

$$I_3 = \det(s\,) = s_{11}(s_{22}s_{33} - s_{23}^{2}) - s_{12}(s_{12}s_{33} - s_{13}s_{23}) + s_{13}(s_{12}s_{23} - s_{13}s_{22}) \tag{3}$$

I_1 and I_3 are the first and the third the stress invariants and P_a is the atmospheric pressure expressed in the same units as the stresses. The value of h_1 and m in the equation is determined by plotting (I_1^3/ I_3-27) vs (P_a/I_1) at failure in a log-log diagram and locating the best fitting straight line. The intercept of this line with $(P_a/I_1)=1$ is the value of h_1, and m is the slope of the line. In the principal stress space, the failure surface is shaped like an asymmetric bullet with the tip pointed at the origin of the stress axis. In order to use the criteria expressed by equation (1) for materials with cohesion and tensile strength, a translation of the surface along the hydrostatic axis is performed. This is done by adding a constant stress *a?P*$a?P_a$ to the normal stresses before substituting in the equation (1),

$$\bar{s}_x = s_x + a \cdot P_a \tag{4a}$$

$$\bar{s}_y = s_y + a \cdot P_a \tag{4b}$$

$$\bar{s}_z = s_z + a \cdot P_a \tag{4c}$$

where a is a dimensionless parameter and P_a is the atmospheric pressure in the same units as the stresses. The value of $a?P_a$ reflects the tensile strength of the rock. Lade investigated the criterion with respect to range and flexibility and used the following range for m=0.5-2.0, $h_1=10^4$-10^8 and a=10-200 and found that the criterion remains convex. The mathematical expression in equation (1) indicates that some value of m exists for which the failure surface becomes concave with respect to the hydrostatic axis.

Drucker-Pager's failure criterion (The ABAQUS formulation)

In the Finite Element software ABAQUS, a formulation of the Drucker-Prager criterion intended for geological materials is included. The formulation is written in terms of the standard three stress invariants with the linear expression written as,

$$f(s_1, s_2, s_3) = t - p\tan(b) - d \;\; ; \tag{5}$$

where the deviatoric stress measure is,

$$t = \frac{1}{2} q \left(1 + \frac{1}{K} + \frac{\left(1 - \frac{1}{K}\right) r^3}{q^3} \right) \tag{6}$$

b(q) is the slope of the linear yield surface in the p-t stress plane and is commonly referred to as the friction angle of the material, *d* is the cohesion of the material, *K(q)* is the ratio of the yield or failure in the triaxial extension to the yield stress in triaxial compression. θ is the Lode angle in this formulation.

The standard invariants are the equivalent pressure stress,

$$p = -\frac{1}{3} trace(s\,)\,; \tag{7}$$

the von Mises equivalent stress,

$$q = \sqrt{\frac{2}{3}(S:S)}\,; \tag{8}$$

and the third invariant of deviatoric stress,

$$r = \left(\frac{9}{2} S \cdot S : S\right)^{\frac{1}{3}}, \tag{9}$$

where S is the stress deviator, defined as $S = s + pI$ or $s_i = s_i - s_m$, i=1, 2 and 3

Adjustment of the criterion is done by plotting failure values into a p-q diagram. Then by fitting the best straight line through the yield or failure points from compression tests will provide *b* and *d* for the model. To define K, the results from extension tests are needed. This is illustrated in Figure 1.

When K=1, t=q, which implies that the yield surface is the von Mises circle in the deviatoric plane. To ensure that the surface remains convex requires 0.778≤K≤1.0. The result for K shows that this implies ϕ≤22°. Many real materials have larger Mohr-Coulomb friction angles than this value. If ϕ is significantly larger than 22°, the model may provide a poor match between compression and extension failure points. The deviatoric plane is illustrated in Figure 2.

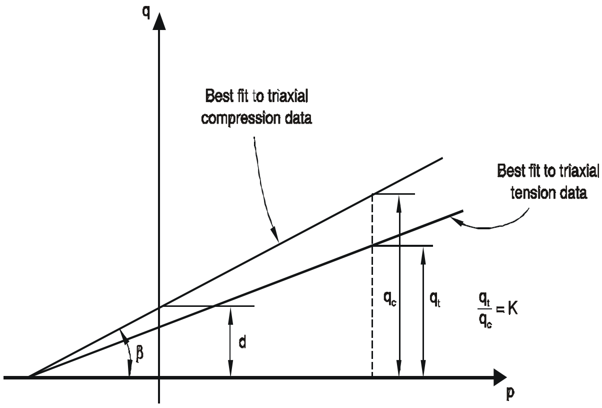

Figure 1. The linear model in a p-q plot.

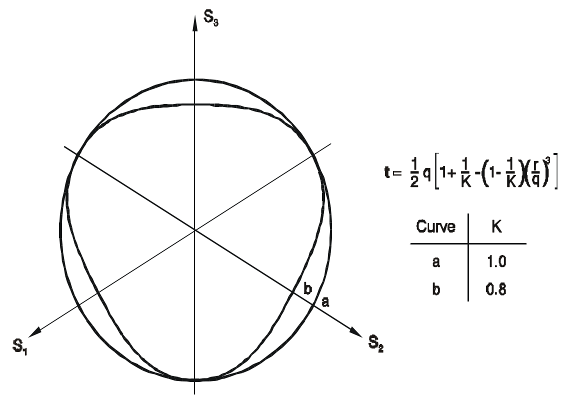

Figure 2. Typical failure surface of the linear model in the deviatoric plane.

Drucker-Prager's failure criterion (van Eekelen formulation)

A special class of yield surfaces is considered by van Eekelen (1980), given by

$$J = (p + a)a(1 - b \sin(3u))^n \tag{10}$$

where u is the Lode angle, a is the attraction and α and β is model parameters. The purpose of his work was to optimize the function with respect to convexity and agreement with experimental data. It was found that the function is optimal regarded convexity with n = -0.229.

The formulation is written in terms of the three stress invariants p, J and ν:

$$p = \frac{1}{3} trace\, s \tag{11}$$

$$J = \left[\frac{1}{2} trace S^2 \right]^{\frac{1}{2}} \tag{12}$$

$$u = -\frac{1}{3} \arcsin \left[\frac{3\sqrt{3}}{2} \frac{\det S}{J^3} \right] \tag{13}$$

where S = $s_i = s_i - s_m$, i=1, 2 and 3

$$a = \sin(j_0^{'}) \tag{14}$$

$$b = 0.85\sqrt{a} \tag{15}$$

f_o' is the angle of shearing resistance at u =0

To provide f_o', an effective angle of shearing resistance can be obtained by:

$$f^{'}(u) = \arcsin \frac{\cos(u)}{\frac{1}{g(u)} - \frac{1}{\sqrt{3}}\sin(u)} \tag{16}$$

where

$$g(v) = \frac{\sin(f_c^{'})}{\cos(u) + \frac{1}{\sqrt{3}}\sin(u)\sin(f_c^{'})} \tag{17}$$

and f_o' is the Mohr-Coulomb angle of shearing resistance, provided by compression tests.

End cap formulation

In ABAQUS, the "end-cap" is provided by a separate function. It has been reported that this could lead to numerical problems during simulations. A function to close the yield envelope or define the "end-cap" is therefore proposed and applied. This is done by a function being one in the shear failure region, but will approach zero when entering the pore collapse region. When hydrostatic yield is reached, the function is zero. The formulation is a function of the first stress invariant and is as follows:

$$f(p) = (1 - e^{-a(\frac{Po - p^{'}}{Po})^m}) \tag{18}$$

where

$$p = \frac{s_1 + s_2 + s_3}{3} \tag{19}$$

were α and *m* is adjustable parameters and P_o is the value of hydrostatic yield.

No attempt is done to formulate flow and hardening rules with this expression, which is needed before it could be used in numerical simulations.

Triaxial test data

To adjust the criterion, triaxial test data from compression and extension tests are used. The chalk has been saturated with water and methanol to provide different failure strength. All data is taken from Trond Eie's thesis (19xx). The chalk that was saturated with methanol is called A, and the chalk saturated with water is called D.

The triaxial test data form chalk A is presented in Table 1 and test data from chalk D is presented in Table 2. The Mohr-Coulomb strength parameters for compression is used to adjust the Drucker-Prager failure failure criterion; the data is presented in Table 3.

Table 1. Values at failure for compression and extension tests for chalk saturated with methanol.

Compression tests, chalk A		Extension tests, chalk A	
s_1 (MPa)	s_2=s_3 (MPa)	s_1=s_2 (MPa)	s_3 (MPa)
10.69	0.3	10	2.08
12.39	0.5	10	1.13
13.10	0.8	11	3.53
11.25	1.0	12	4.43
14.87	1.2		
14.82	1.5		

Table 2. Values at failure for compression and extension tests for chalk saturated with water.

Compression tests, chalk D		Extension tests, chalk D	
s_1 (MPa)	$s_2=s_3$ (MPa)	$s_1=s_2$ (MPa)	s_3 (MPa)
5.35	0.99	6.20	0.49
6.19	1.41	7.40	1.23
7.65	2.12	8.00	2.06
		8.50	2.19

Table 3. Strength data from compression tests.

Chalk	Angle of shearing resistance (φ)	Failure angle (β)	Cohesion (S_o)	Compression strength (C_o)
A	38.38	64.19	2.20	9.10
D	28.99	59.49	0.98	3.33

Adjustment of the Lade criterion

The Lade criterion is adjusted by determing the value of $?_1$ and *m* in the equation. This is done by plotting (I_1^3/I_3-27) vs. (P_a/I_1) at failure in a log-log diagram and locating the best fitting straight line. The intersection of this line with (P_a/I_1)=1 is the value of $?_1$, and m is the slope of the line.

Chalk A (methanol saturated chalk)

These values is plotted for chalk A in Figure 3. Based on this plot $?_1$ is 5?10^{12} and *m* is 4.7. By looking at the log-log plot, it is quite apparent that extrapolation of the line until intersecting with (P_a/I_1)=1 will add a significant uncertainty to the $?_1$ and *m* values. When Lade investigated the flexibility of the criteria, the value of *m* was within the range of 0.5-2.0. It means that we could expect some odd behaviour of the failure surface.

The surface is projected in to the triaxial-plane together with the failure values for compression and extension for chalk A in Figure 5. It is quite obvious that the surface is showing strange behaviour and the match with experimental data is rather poor.

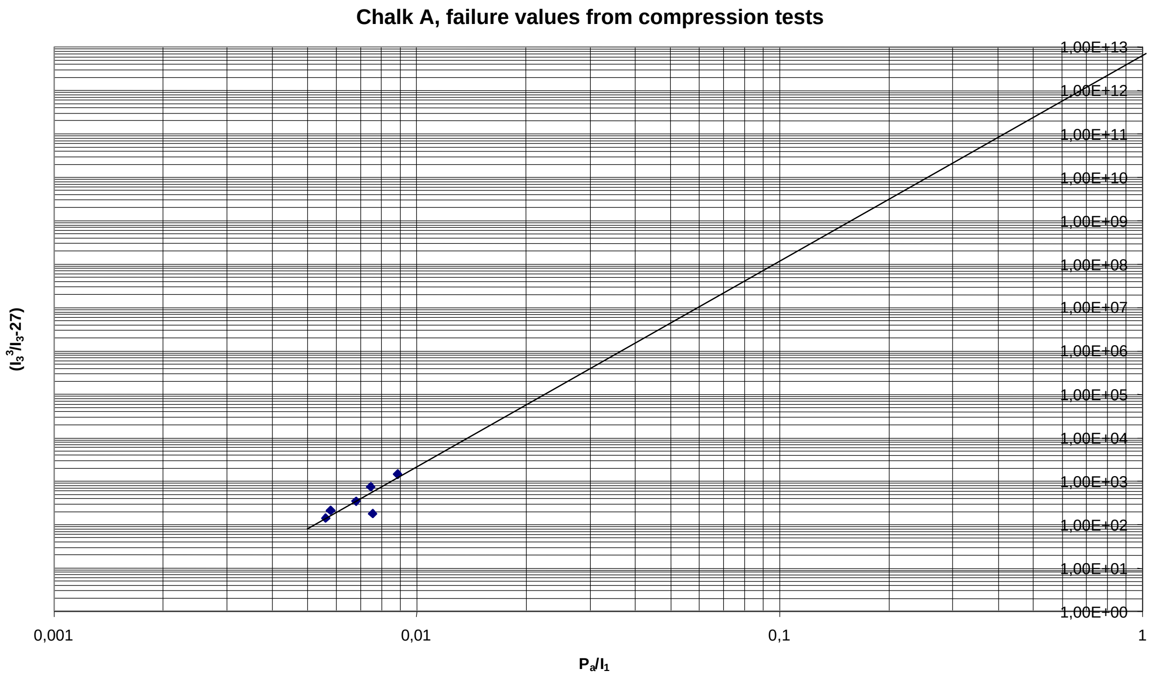

Figure 3. Log-log plot of (I_1^3/I_3-27) vs. (P_a/I_1) at failure for chalk saturated with methanol.

Chalk D (water saturated chalk)
The determination of the parameters $?_1$ and *m* at failure for chalk D is based on the log-log plot in Figure 4. $?_1$ is 2.5?10^4 and *m* is 1.46.

The surface for chalk D is also projected in to the triaxial-plane together with the failure values for compression and extension, see Figure 6. The surface do match the data-point from compression tests, but the match with extensional data is poor, and in this case we are within the range where Lade found the criteria to be appropriate. Compared with the surface for chalk A the surface for chalk D is showing a more normal behaviour.

Adjustment of the Drucker-Prager criterion (The ABAQUS formulation)
An approach to match Mohr-Coulomb and Drucker-Prager is provided in the ABAQUS manuals. This is done by letting the two models define the same failure definition in triaxial compression and extension. The outcome is a relation between *f* , the Mohr-Coulomb shearing resistance, and *ß*, the friction angle in the Drucker-Prager criterion, and a relation between K, the ratio between compression and extension and *f* , the Mohr-Coulomb shearing resistance. The expressions is as follows,

$$\tan b = \frac{6 \sin f}{3 - \sin f} \tag{20}$$

and

$$K = \frac{3 - \sin f}{3 + \sin f} \tag{21}$$

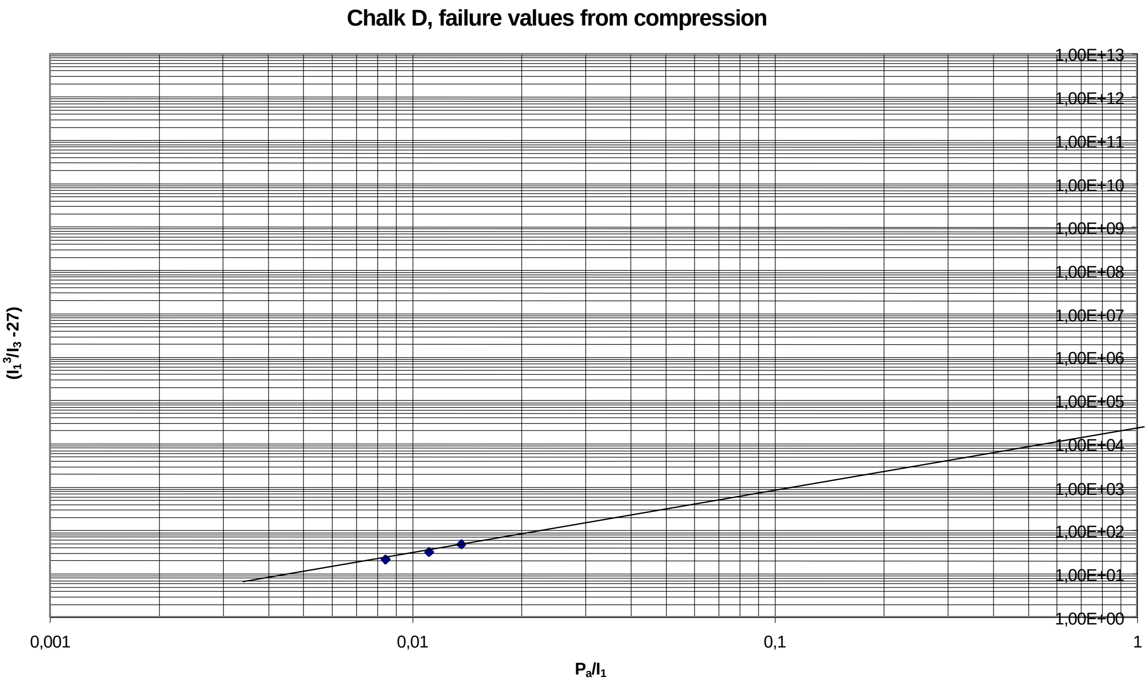

Figure 4. Log-log plot of (I_1^3/I_3-27) vs. (P_a/I_1) at failure for chalk saturated with water.

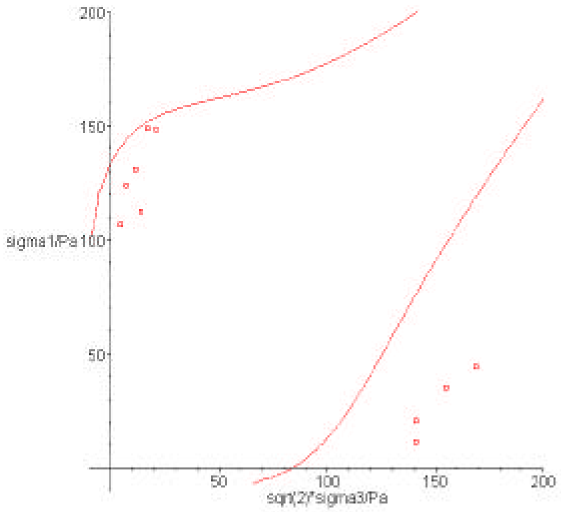

Figure 5. The Lade criterion adjusted to methanol saturated chalk.

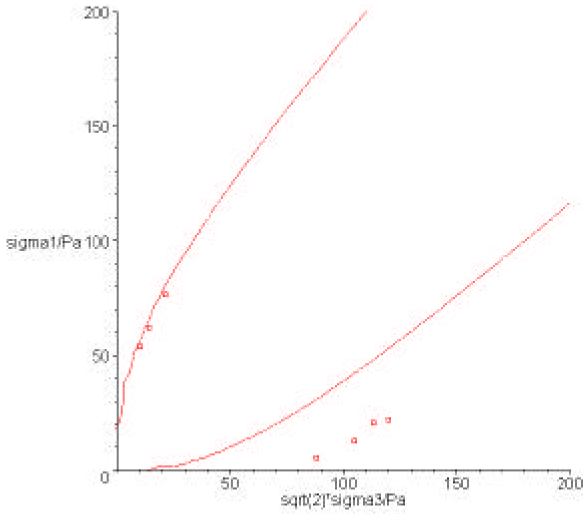

Figure 6. The Lade criterion adjusted to water saturated chalk.

Chalk A (methanol saturated chalk)

Using equation (20), the Drucker-Prager friction angle *ß* is 57.48° for chalk A . The cohesion *d* is 4.34 and the ratio between compression and extension is set to 0.778 according to the limitation with respect to convexity. In Figure 7, the surface is projected in to the triaxial-plan. The axes are (σ_1, *l*), where *l* is the $\sqrt{2}\ \sigma_3$. The surface show poor match with the data-points from extension tests, but this is due to the high confining pressure. The confining pressure used in these tests are around 10 MPa, which is close to hydrostatic yield, it means that the stresses are close to the "end-cap " and could therefore not be described by this linear criterion. The failure mode for these particular extension tests, could be described more like a combination of pore collapse and shear failure. This will be adjusted by the "end-cap" function later.

Chalk D (water saturated chalk)

Using the same equation (20) on chalk D, the Drucker-Prager friction angle *ß* is 49.05°. The cohesion *d* is 2.04 and the ratio between compression and extension is still set to 0.778 according to the limitation with respect to convexity. In Figure 8, the surface is projected in to the triaxial-plane. The data-points for chalk D gives a good match with the failure surface. The confining pressure for the extension tests are close to 7 MPa, which is within the elastic region.

Adjustment of the "end-cap" function

By introducing an "end-cap" function, it is possible to describe the region where the failure mode is a combination of shear failure and pore collapse. Equation (18) is adjusted to chalk A and D and applied on the surfaces above. The parameters are presented in Table 4.

Figure 9 is showing the surface for chalk A and the surface for chalk D is shown in Figure 10 with the "end-cap" function applied. Now we can se that the failure points from extension tests in Figure 9, is laying on the "end-cap" part of the surface.

The surfaces are illustrated in three dimensions in Figure 11 and 12. The surfaces are closed and shaped as cones with cross sections in the deviatoric plane which are rounded triangular.

Table 4. Parameter used to adjust the “end-cap” function to chalk A and D.

Chalk	a	m	P_o (hydrostatic yield in MPa)
A	5	1	10
D	8	1	8

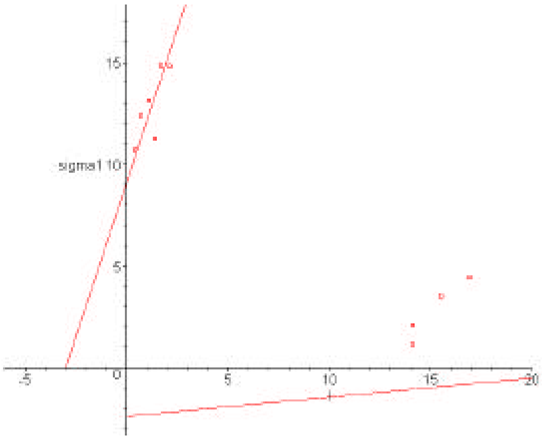

Figure 7. The Drucker-Prager (ABAQUS) criterion adjusted to methanol saturated chalk.

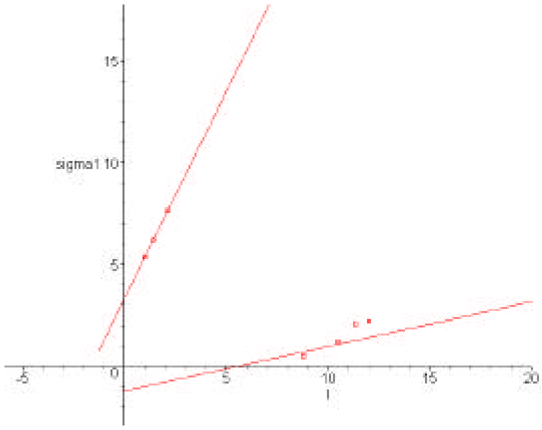

Figure 8. The Drucker-Prager (ABAQUS) criterion adjusted to water saturated chalk.

Figure 9. The surface for chalk A with the “end-cap” function applied.

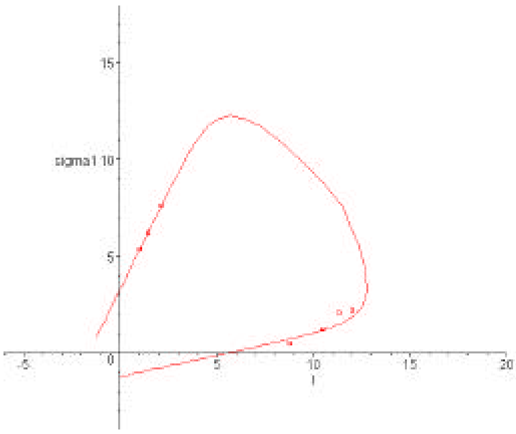

Figure 10. The surface for chalk D with the “end-cap” function applied.

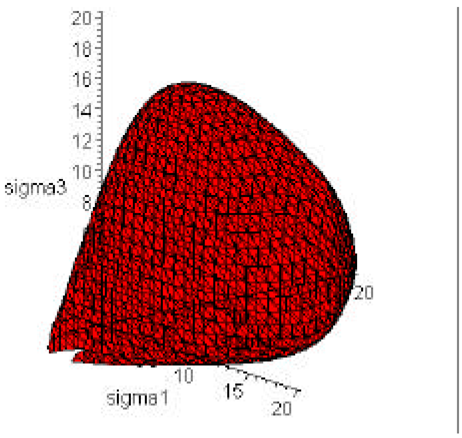

Figure 11. The surface for chalk A in the three-dimensional principal stress space.

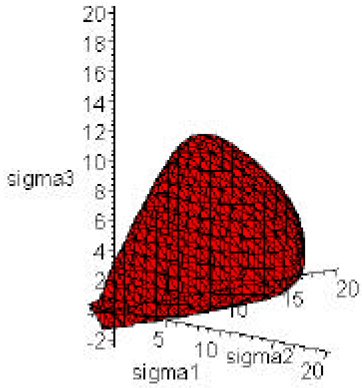

Figure 12. The surface for chalk D in the three-dimensional principal stress space.

Adjustment of the Drucker-Prager criterion (The van Eekelen formulation)

The adjustment of the criterion is based on the Mohr-Coulomb parameters and equation (15) and (16). For chalk A the angle of shearing resistance, f_o’ at *u* =0 is 38.38°, the attraction is 3.8 and the parameter *n* is set to –0.229. For chalk D the angle of shearing resistance, f_o’at ?=0 is 28.98°, the attraction is 2.35 and the parameter *n* is set to –0.229. In Figure 13 the surface for chalk A is projected in to the triaxial-plane, the match is the same as with the ABAQUS formulation and the agreement with extension tests are deviating from the linear criterion because these tests are in the “end-cap” region of the surface. Figure 14 is showing the agreement for chalk D. By a closer look at the line intersecting the points from compression tests, it can be seen that the adjustment is a bit out of accuracy and is causing the surface to be more closed than the adjustment on the ABAQUS formulation.

Adjustment of the “end-cap” function

In Figure 15, equation (18) is applied on the van Eekelen formulation and adjusted with the same parameters as presented in Table 4 and used on the ABAQUS formulation. Applying the “end-cap” function on the surface for chalk A, the result is much the same as with the ABAQUS formulation. The extension tests are laying in the “end-cap” region. It could be discussed which one is preferable. The same “end-cap” parameter used with the ABAQUS formulation is also used on the surface for chalk D. In Figure 16 it is seen that due to the lack of accuracy in the linear adjustment, the mach is not as good with the ABAQUS formulation.

By tuning the surface for chalk D to match the experimental data a better agreement is obtained. The result is shown in Figure 17 where the parameters are; angle of shearing resistance, f_o’=34.38° at ?=0, and attraction = 1.5 and the parameter *n* is still set to –0.229.

The surfaces based on the van Eekelen are illustrated in the three-dimensional principal stress space in Figure 18 and 19. The surfaces are shaped as cones and could constitute a failure criterion for high porous chalk.

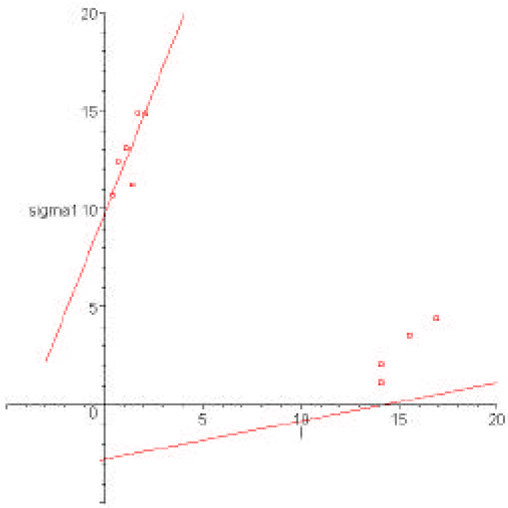

Figure 13. The Drucker-Prager (van Eekelen) criterion adjusted to methanol saturated chalk.

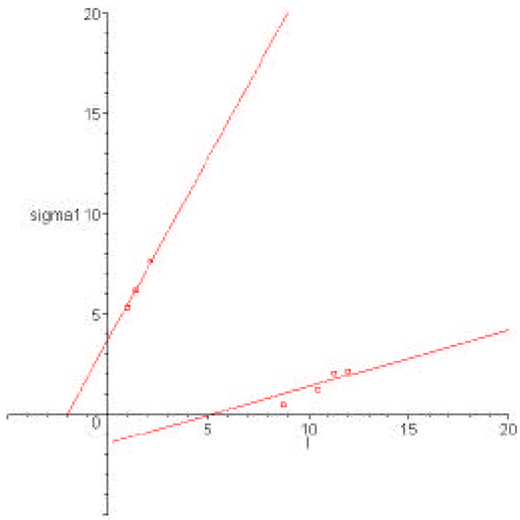

Figure 14. The Drucker-Prager (van Eekelen) criterion adjusted to water saturated chalk.

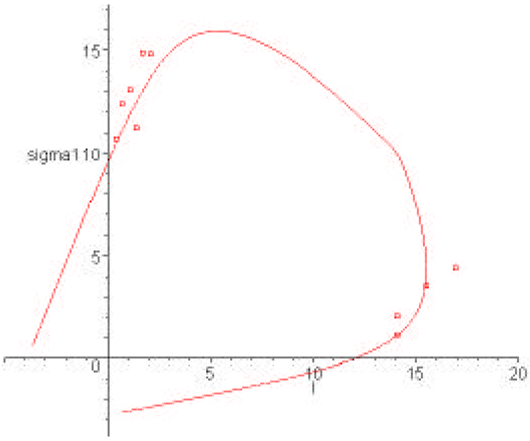

Figure 15. The surface for chalk A with the “end-cap” function applied.

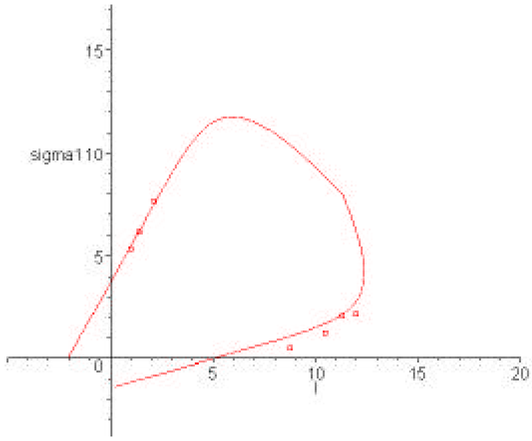

Figure 16. The surface for chalk D with the “end-cap” function applied.

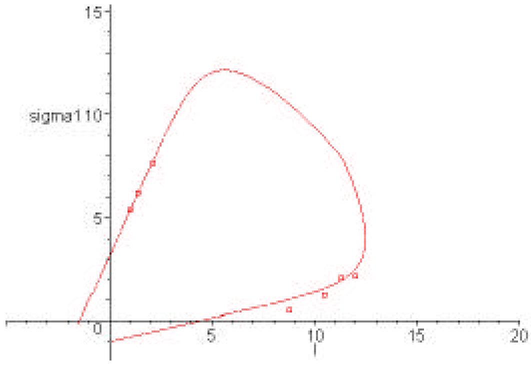

Figure 17. The surface for chalk D, tuned to match the experimental data.

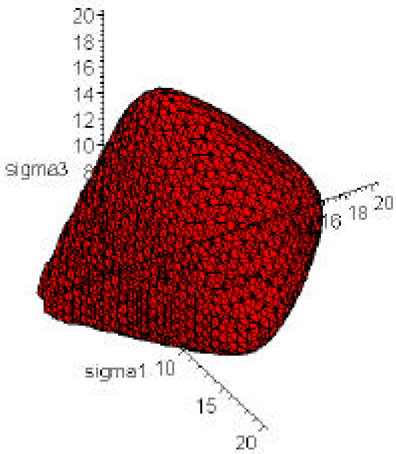

Figure 18. The surface for chalk A in the three-dimensional principal stress space.

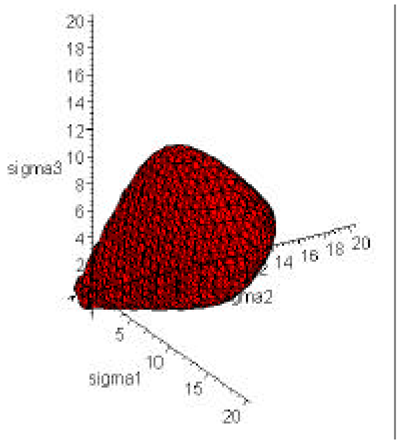

Figure 19. The surface for chalk D in the three-dimensional principal stress space.

Conclusion

- This study shows how well the criterion is adjusted to experimental data in the triaxial-plane and how easily they are adjusted. Adjustment of the Lade criterion is straight forward, but the extrapolation of the line until intersecting with $(P_a/I_1)=1$ will add an uncertainty to the values of $?_1$ and m. It may be possible to tune the criterion to match the compression and extension data, but using the previous described method is in our case providing a poor match with high porosity chalk.
- Adjustment of the Drucker-Prager criterion (ABAQUS formulation) is straight forward and gives good agreement with experimental data despite its limitation regarding convexity.
- The Drucker-Prager based on van Eekelen's formulation gives good agreement with experimental data, but the adjustment is not quite accurate and some tuning is needed.
- The "end-cap" formulation shows good agreement and flexibility.
- Both formulations based on the Drucker-Prager criterion could be used to describe the failure surface of high porosity chalk.

References

Steinar Nordal, 1983. "Elasto-Plastic Behaviour of Soils Analysed by the Finite Element Method," Norges Tekniske Høgskole, University of Trondheim, 454 p.

H.A.M. van Eekelen, 1980: "Isotropic Yield Surfaces in Three Dimensions for Use in Soil Mechanics," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 4, no. 1, pp. 89-101.

M.K. Kim and P.V. Lade, 1984: "Modeling Rock Strength in Three Dimensions," University of California, Los Angeles.

ABAQUS, Users Manuals, ver 5.8.

Rasmus Risnes *et al*., 1992: "Petroleum Related Rock Mechanics," Høgskolen i Stavanger, 314 p.

Philippe Charlez, 1991: "Rock Mechanics," Institute of Physique du Globe de Paris, 328 p.

Trond Victor Eie, 1999: "Ekstensjons- og kompresjonstester innenfor og nær det elastiske området for høyporøs kalk," Høgskolen i Stavanger, 74 p.