Introduction

Full-waveform inversion of seismic data based on least-squares model fitting is an old technique (Tarantola, 1984) but has recently successfully been used to estimate velocity models from field data. (Sirgue et al. , 2009). A limitation with this technique is that the initial model must be sufficiently close to the true model to prevent cycle-skipping (Virieux and Operto, 2009). A common approach to solve this problem is to rely on velocity models estimated using ray tomography (Sirgue et al. 2009) in combination with depth migration. This usually yields a smooth velocity model with kinematic properties similar enough to the true model to prevent the cycle-skipping problem.

We propose an alternative approach based on wave theory only. By performing model fitting in the image space based on differential semblance (Biondi and Sava, 1999, Shen and Symes, 2008, Weibull and Arntsen, 2011) a low resolution velocity model with good kinematic properties can be obtained. Cycle-skipping can be avoided by using this model as a starting model for full-waveform inversion in the data space. We show synthetic examples where an object function based on differential semblance is used to estimate an initial model and a least-squares object function is then minimized to refine the initial model.

Full-waveform inversion in the data space

Full-waveform inversion in the acoustic approximation is based on minimising the error e between observed seismic data p^{obs} and simulated seismic data p with respect to the unknown seismic velocity c using the least-squares object function

$$e_l = \sum_{r,s}\sum_{t}\left[p^{obs}(x_r,t) - p(x_r,t)\right]^2 , \qquad (1)$$

where x_r is the receiver position, t is the time and s is the source index. The velocity can be estimated using a number of iterative non-linear estimation methods, where the simplest alternative is the steepest descent method. The main problem with this approach is the narrow basin of attraction of the least-squares error function given by equation (1) . In practice this implies that the initial model must be close enough to the true model to avoid significant cycle-skipping. Reducing the frequency content of the input data reduces the problem, but to ensure a correct solution, the initial model must be kinematically close to the true model.

Full-waveform inversion in the image space

Velocity-models derived from ray-tomography combined with depth migration seems to yield good initial models for full-waveform inversion using the least-squares error given by equation (1). It then seems natural to use an object function defined in the migrated image space to estimate velocity models. A good choice is the differential semblance object function

$$e_s = \sum_{x}\sum_{h} h^2\left[\frac{\partial R(x,h)}{\partial z}\right]^2 , \qquad (2)$$

where R is the depth migrated image and h is the subsurface offset. The velocity is updated with a gradient based optimization where the update in iteration n is given by

$$\Delta c_n = -\sum_{s,t}\frac{2}{c_{n-1}^3}\partial_t^2 p_n(x,t)\xi_n(x,t) - \sum_{s,t}\frac{2}{c_{n-1}^3}\partial_t^2 u_n(x,t) q_n(x,t) .$$

Here ξ_n is a time-reversed wavefield driven by the source $\sum_{h,s} h^2 \frac{\partial_z R(x+h,h)}{\partial_z^2} u(x+2h,t,s)$, u_n is the backward extrapolated data and q_n is a wavefield due to the source

$$\sum_{h,s} h^2 \frac{\partial_z R(x-h,h)}{\partial_z^2} p(x-2h,t,s).$$

The velocity-estimation method outlined above is essentially the wave-equivalent of ray tomography methods based on flattening common image point gathers.

Hybrid inversion in the image and data space

To overcome the cycle-skipping problem we suggest to combine inversion in the data space and in the image space. Ideally we want to use an error function equal to $e = w_s e_s + w_l e_l$, where w_l and w_s are appropriate weights. This is difficult in practice due to the different resolution properties of the two error functions; a more pragmatic approach is to first minimize e_s to obtain a kinematically correct low-resolution velocity model, and then minimize e_l to obtain a velocity model with high-resolution. This approach makes it possible to start with a simple initial model far from the true model.

Figure 1 shows the initial one-dimensional velocity model of a synthetic numerical example of full-waveform inversion in the data space minimizing the least squares error e_l. The estimated model is shown in Figure 2 and by comparing with the true model shown in Figure 3 it is clear that the result is quite far from the true model. Figure 4 shows the estimated velocity model obtained by minimizing the differential semblance error function e_s given by equation (2). The resolution of this model is low, but is kinematically close to the true model. Figure 5 shows the result of minimizing the least squares error function e_l with the model shown in figure 4 as the initial model. The result is close the true model and significantly better than the model obtained by using the simple one-dimensional velocity function.

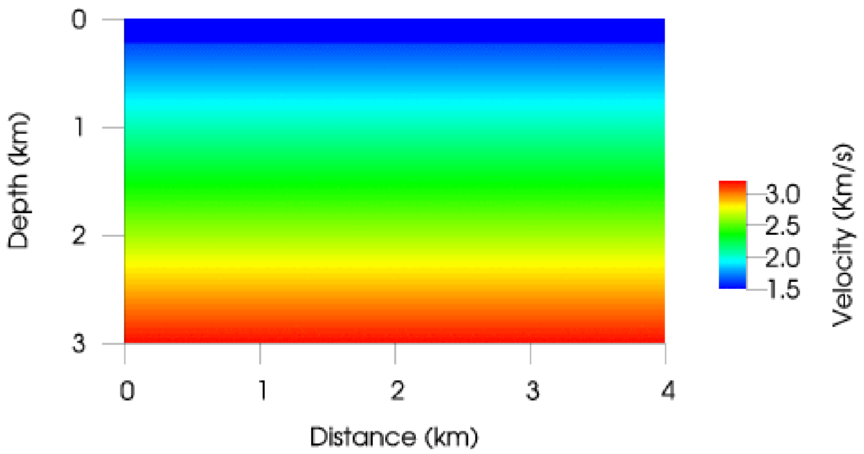

Figure 1 *One-dimensional initial model with linear velocity increase.*

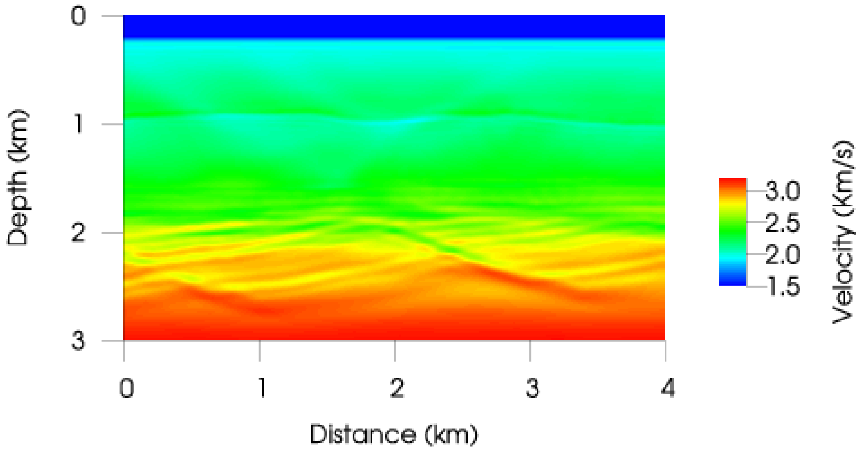

Figure 2 *Velocity model estimated with full-waveform inversion in the data space using the model in Figure 1 as the initial velocity field. Note the discrepancy with the true model shown in Figure 3.*

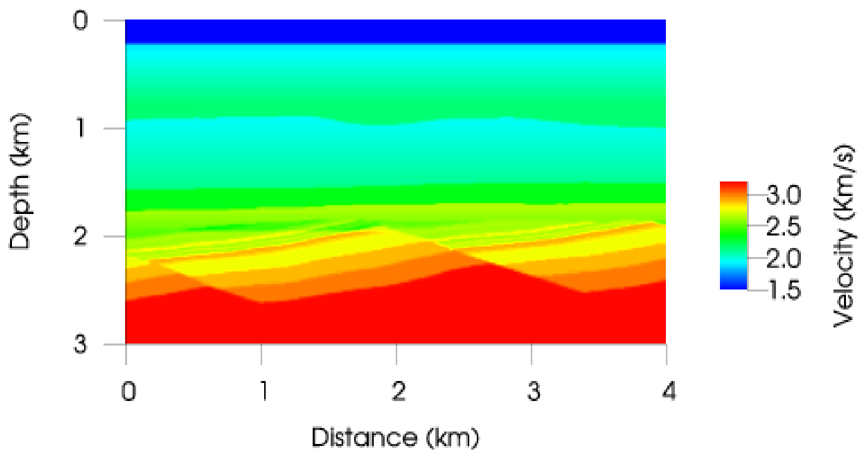

Figure 3*The true velocity model. Compare with Figures 2 and 5.*

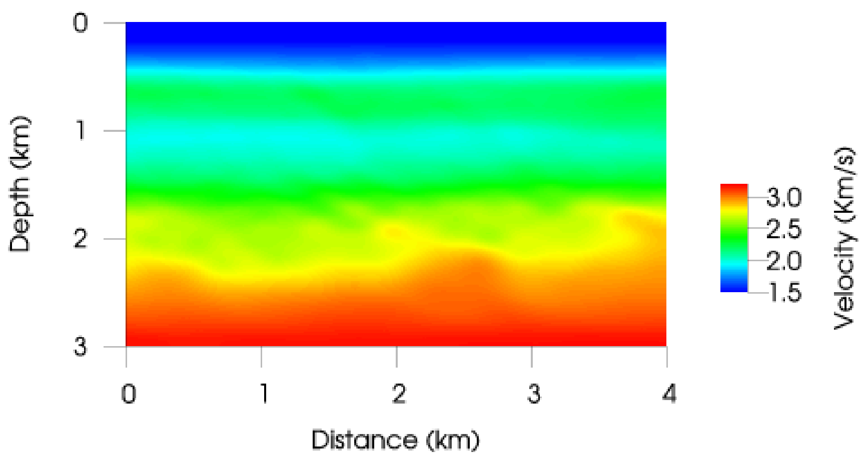

Figure 4 *Velocity model estimated with full-waveform inversion in the image space using the model in Figure 1 as the initial velocity field.*

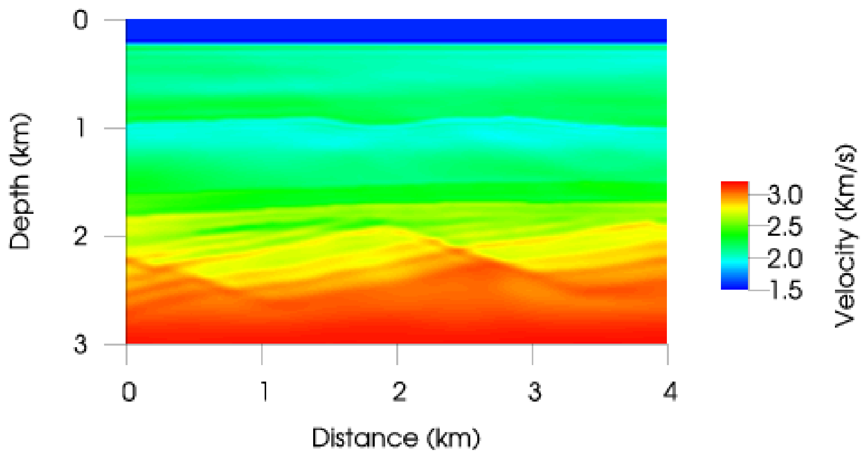

Figure 5 *Velocity model estimated with full-waveform inversion in the data space using the model in Figure 4 as the initial velocity field.*

Conclusions

By combining waveform inversion in the image and data spaces the problem of cycle-skipping can be reduced, and a numerical example shows that the velocity field of a realistic model can be accurately estimated using a simple one-dimensional initial model.

References

Biondi, B., and Sava, P., Wave equation migration velocity analysis, 1999, SEG Expanded Abstracts, 18, 1723-1726.

Sirgue, L., O.I. Barkved, J.P. Van Gestel, O.J. Askim, and J.H. Kommedal, 2009, 3D Waveform inversion on Valhall Wide-azimuth OBC, 71st EAGE Conference and Exhibition.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation, Geophysics, 49, 1259-1266.

Shen,.P, and Symes, W., 2008, Automatic velocity analysis via shot profile migration, Geophysics, 73, VE49-VE59.

Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics, Geophysics, 74, WCC1-WCC26.

Weibull, W., and B. Arntsen, 2011, Automatic Migration velocity Analysis using Reverse Time Migration, 73rd EAGE Conference and Exhibition.