# Strategies for elastic full waveform inversion

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## Introduction

- A crucial step in the full waveform inversion (FWI) method is the modeling of waves propagating in the subsurface. Important wave phenomena like, for instance, shear waves are not included in the acoustic wave equation. Therefore, it is important to use the elastic wave equation, which include more accurate physics, to model wave propagation in a medium. [Tarantola, 1984, Mora, 1987, Pratt, 1999, Aki and Richards, 2002].
- From a computational point of view we are now able to estimate P- and S- wave velocity and density models using FWI. From a mathematical point of view, on the other hand, inverting for three isotropic elastic parameters is difficult due to the complexity of the nonlinear and ill-posed inverse problem, in combination with varying amount of information of the wave phenomena in the observed data [Virieux and Operto, 2009].
- Ocean-bottom cables (OBC) have become common in reservoir monitoring of oil and gas production, and also for $CO_2$ capture and storage experiments. Compared to conventional streamer data, OBC data contain more information, particularly about shear waves.
- We investigate different inversion strategies for estimating the three isotropic elastic parameter models using FWI.
- We study the difference between conventional streamer and OBC datasets in terms of recovering all three isotropic elastic parameters using a two-dimensional synthetic model.

## Full waveform inversion

**Goal:** *Find a parameter model from which it is possible to create synthetic data that is close to some measured data*

Define $S(\mathbf{m})$ as the measure between synthetic and measured data. The least-squares objective functional is given as

$$S(\mathbf{m}) = \frac{1}{2} \sum_{\text{shots}} \sum_{\text{receivers}} \int_0^T \Big( d_{\text{obs}}(\mathbf{x}, t) - d_{\text{mod}}(\mathbf{x}, t; \mathbf{m}) \Big)^2 \, \mathrm{d}t.$$

FWI is then the problem

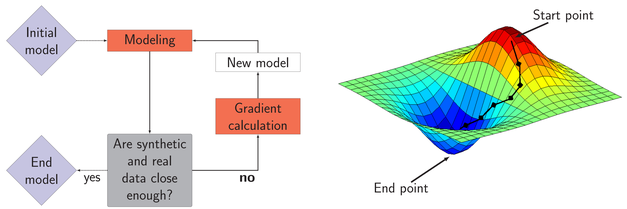
$$\arg\min_{\mathbf{m}} S(\mathbf{m}).$$

Solved using an iterative method

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k \mathbf{H}_k^{-1} \mathbf{g}_k,$$

where $\mathbf{m}_k$ is the model at iteration $k$, $\mathbf{g}_k$ is the gradient of $S(\mathbf{m})$ at iteration $k$, $\mathbf{H}_k$ is the Hessian of $S(\mathbf{m})$ at iteration $k$, and $\alpha_k$ is the step length at iteration $k$.

*Schematic overview*




## Inversion strategies

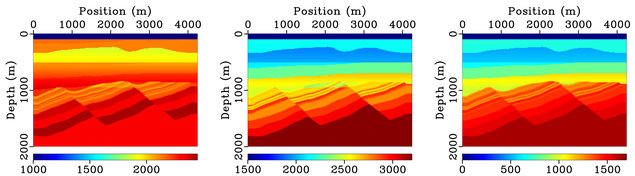
- 10 different inversion strategies
- First class: FWI is used to invert for all three parameters
- Second class: Sequentially inverting for the three parameters using FWI
- Third class: Empirical relations are used to update some parameters
- Fourth class: Leaving the density unchanged and inverting only for the velocity models.
- Empirical relations used in update [Gardner et al., 1974, Castagna et al., 1985]

$$\rho = 310 V_p^{0.25}, \quad \text{and} \quad V_s = 0.862 V_p - 1172.$$

| Strategy | Dataset | Wave field | | | Sequence | | |
|---|---|---|---|---|---|---|---|
| | | $\rho$ | $V_p$ | $V_s$ | $\rho$ | $V_p$ | $V_s$ |
| 1a | Streamer | $P$ | $P$ | $P$ | 1 | 1 | 1 |
| 1b | OBC | $P/V_z/V_x$ | $P/V_z/V_x$ | $P/V_z/V_x$ | 1 | 1 | 1 |
| 2a | Streamer | $P$ | $P$ | $P$ | 3 | 1 | 2 |
| 2b | OBC | $P/V_z/V_x$ | $P/V_z/V_x$ | $P/V_z/V_x$ | 3 | 1 | 2 |
| 3a | Streamer | - | $P$ | - | G | 1 | C |
| 3b | OBC | - | $P/V_z/V_x$ | - | G | 1 | C |
| 3c | OBC | - | $P/V_z$ | $V_x$ | G | 1 | 2 |
| 3d | OBC | - | $P$ | $V_z/V_x$ | G | 1 | 2 |
| 4a | Streamer | - | $P$ | - | - | 1 | - |
| 4b | OBC | - | $P$ | $V_z/V_x$ | - | 1 | 2 |

Table 1: *Overview of the inversion strategies ($P$: pressure, $V_z$ : vertical particle velocity, $V_x$ : horizontal particle velocity, G: Gardner relation used in update, C: Castagna relation used in update).*

### Test model



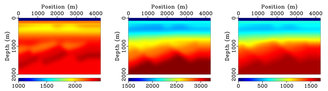

Left: Density ($\rho$), middle: P-wave velocity ($V_p$), right: S-wave velocity ($V_s$).

- Based on the Gullfaks field in the Norwegian North sea
- Target zone is the reservoir on the crest of the rotated fault blocks (approx 1000 m depth and position 2000 m)
- Model do not follow the empirical relations above. Thus, the inversion introduce errors when empirical relations are used in the parameter updates.

## Modeling and acquisiton geometry

- Higher-order finite difference staggered-grid implementation of the isotropic elastic wave equation
- 300 shots with 20 m shot interval
- Streamer cable: 600 receivers separated by 10 m
- OBC cable: 600 receivers separated by 10 m
- Ricker wavelet with center frequency 5.0 Hz.

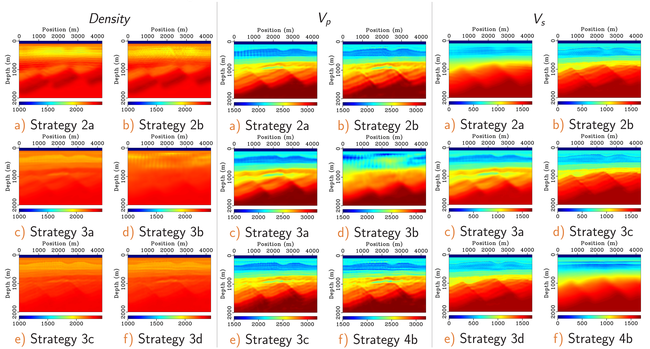
### Initial model




Left: $\rho$, middle: $V_p$, right: $V_s$.

## Results

- Inverting for all three simultaneously failed completely (strategies 1a and 1b)
- All strategies, except strategy 3b which failed completely, were able to recover the $V_p$ model within the accepted resolution.
- Inverting for $V_s$ turned out to be complicated using streamer data, and the only $V_s$ model we obtained was through the empirical relation. Note that the empirical relation introduce a false structure in the reservoir.
- Using the OBC dataset, the inversion gave reliable $V_s$ models.
- None of the proposed strategies were able to invert for $\rho$ and at the same time give a reliable model. Thus, the best models were obtained using the empirical relation.




- The images obtained using OBC data include more details compared to the images made by using streamer data. The amount of artifacts are also smaller on the OBC images.
- The influence of updating $\rho$ using the empirical relation seems to be small on the final $V_p$ images.
- The update of $\rho$ is importing for recovering $V_s$ (see results for strategy 4b).
- FWI using OBC is sensitive to the parameters relative close to the sea floor. Small errors in these parts, may have major impacts on the recorded datasets, and the inversion gets easily stuck in a local minima.

## Discussion

- Empirical relations should be used with extreme care, since spurious structures may be introduced in the models. On the other hand, it may be better to update the involved parameters during the inversion, than leaving them unchanged.
- An extension of the sequential strategies is to run a final inversion, where several parameters are inverted for simultaneously. This could potentially explain correlation effects between the parameters inverted for.
- The different strategies may be modified by introducing a data-weighting matrix in the objective functional. Using this matrix it is possible to focus the inversion on different parts of the data set.

## Summary and remarks

- We have investigated different strategies for inverting the three isotropic parameters using streamer and OBC datasets.
- The inversion for $V_p$ was successful for both datasets, while the inversion for $V_s$ was only successful using the OBC dataset.
- The density ($\rho$) was difficult to invert for using our setup. Hence, the updates for $\rho$ were achieved using an empirical relation in each iteration.
- Inverting for several parameters simultaneously was difficult, and thus it may be better to invert for each parameter sequentially.

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