Strategies for elastic full waveform inversion

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SUMMARY

Ocean-bottom cables (OBC) have become common in reservoir monitoring of oil and gas production, and also for CO_2 capture and storage experiments. Compared to conventional streamer data, OBC data contain more information, particularly about shear waves. This information may be used to estimate the P- and S-wave velocity and density models using the full waveform inversion method. In this study, we investigate different inversion strategies for estimating the three elastic parameter models. We study the difference between conventional streamer and OBC datasets in terms of recovering all three isotropic elastic parameters. We find that the streamer dataset is sufficient for recovering only the P-wave model, while the OBC dataset enables us to recover both the P-wave and S-wave models. There are, however, difficult to invert for the density model using both datasets. An update, however, was achieved by using an empirical relation in the iterations. The best inversion results were obtained using a sequential based approach, where each parameter was inverted for on a one-by-one basis.

INTRODUCTION

A crucial step in parameter estimation problems like the full waveform inversion (FWI) method is the modeling of waves propagating in the subsurface (Tarantola, 1984; Mora, 1987; Pratt, 1999). If the elastic wave equation is used, more physics are included in the propagation, than is the case with the acoustic wave equation (Aki and Richards, 2002). In data fitting methods like FWI, where we try to match synthetic and real datasets, it is important to include as much physics as possible.

From a computational point of view, we are now able to estimate P- and S-wave velocity and density models, using FWI. From a mathematical point of view, on the other hand, inverting for three elastic parameters is difficult due to the complexity of the nonlinear and ill-posed inverse problem, in combination with varying amount of information of the wave phenomena in the observed data.

Ocean-bottom cables (OBC) have become common in oil and gas production, and surveillance of reservoirs used for CO_2 storage. A conventional streamer is able to record P-waves, whereas OBCs are able to record S-waves as well. In elastic FWI, the extra information included in OBC data should give some benefits compared to conventional streamer data. An important question is what benefits the extra information give, and thus if there are any differences in the inversion results for OBC and streamer data acquired over the same model.

The result of the extra parameters included in elastic FWI, is a more complicated inverse problem compared to acoustic FWI. A natural question is then: How should elastic FWI be performed to get three inverted parameter models that are reliable?

In this study, we address the two questions stated above. We use a synthetic elastic model where we simulate both marine streamer and OBC surveys, and use the two datasets to investigate the success of FWI using different inversion strategies. We find that a sequential based approach, where one inverts for one parameter in each inversion, is the best option. Both datasets are able to recover the P-wave model, but only the OBC dataset is able to recover the S-wave model. With this setup, the only updates we achieved for the density were obtained using an empirical relation.
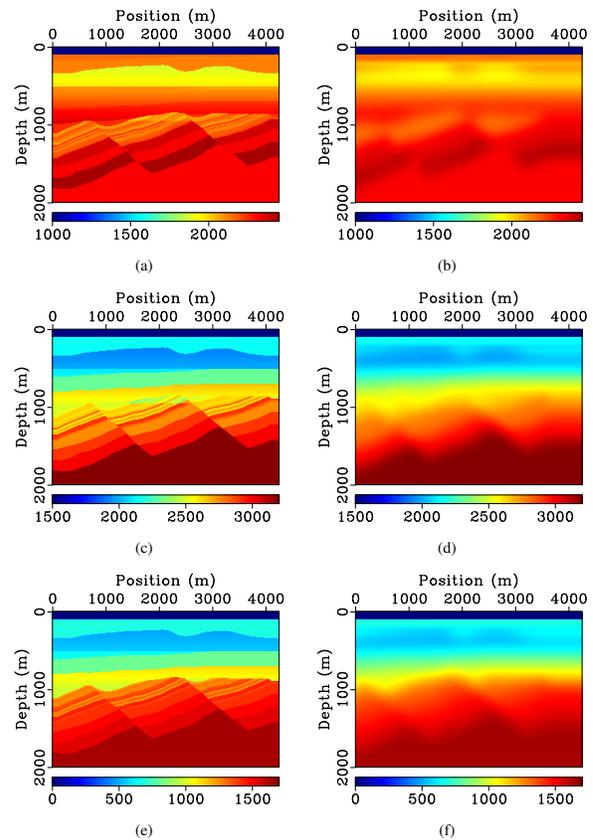
Figure 1: The true (left column) and initial (right column) synthetic elastic model: (a) and (b): ρ (kg/m^3); (c) and (d): V_p (m/s); (e) and (f) V_s (m/s).

INVERSION STRATEGIES

The theory that underlies FWI has been derived several times using different formulations, so we refer to Virieux and Operto (2009) (and the references given therein) for an introduction to the method.

Inversion strategies for EFWI

The inversion for the three isotropic elastic parameters may be carried out in several ways. One option is to use FWI to estimate all three parameters at the same time. Another option is to estimate each parameter sequentially, that is, only one parameter is inverted for in each inversion run. A final option is to invert for one of the parameters and link the other parameter updates using some empirical relations.

We have created in total 10 different inversion strategies (see Table 1 for a summary) where we in each strategy try to obtain reliable inverted models for the three elastic parameters. The major differences between the strategies are how the three parameters are updated during the inversion. To investigate what type of extra information included in the particle velocity recordings for the OBC data, we have created some strategies where we use different wave field recordings in the estimation of the three parameters. For the strategies based on empirical updates of the parameter models, we use the following two common relations (Gardner et al., 1974; Castagna et al., 1985)

$$\rho = 310 V_p^{0.25}, \quad \text{and} \quad V_s = 0.862 V_p - 1172, \qquad (1)$$

for the updates in each iteration. Here, ρ is the density, V_p is the P-wave velocity, and V_s is the S-wave velocity.

RESULTS

To test the different inversion strategies we use a realistic model based on the Gullfaks field in the Norwegian North sea (Figure 1). The interesting part in the model is the reservoir at the crest of the rotated fault blocks (approx 1000 m depth and position 2000 m). We emphasize that the model do not follow the empirical relations given in equation 1, so the inversion strategies based on empirical relations introduce errors in the models.

To model the elastic waves we use a higher-order finite difference staggered-grid implementation (Virieux, 1986; Holberg, 1987) of the isotropic elastic wave equations (Aki and Richards, 2002). The streamer and OBC datasets consist of 300 shots, with 20 m shot interval. Both the streamer cable and the OBC consist of 600 receivers separated by 10 m. To avoid numerical aliasing a Ricker wavelet with center frequency 5.0 Hz is used as source signature, and the model sampling is $\Delta x = \Delta z = 10$ m. The model sampling is kept constant in both the modeling and the inversion. The initial models are created by smoothing the true models using a simple triangle smoothing operator (Figure 1).

Inverting for all three elastic parameters at the same time (strategies 1a and 1b) was not successful. The inversions ran quickly into local minima, and the final models were close to the initial models.

All other strategies, except strategy 3b which failed completely, were able to recover the V_p model within the accepted resolution (Figure 2). The images obtained using OBC data include more details compared to the images made by using streamer data. The amount of artifacts are also smaller on the OBC images. The influence of updating ρ using equation 1 in each iteration seems to be small on the final V_p images.

Inverting for V_s turned out to be complicated using streamer data, and the only V_s model we obtained was through the empirical relation. Using the OBC dataset, on the other hand, we were able to obtain reliable V_s models (Figure 3). For strategy 4b, the inversion failed in recovering V_s. We observe that strategy 3a has introduced a false structure in the reservoir, due to wrong empirical relation in this area of the model.

None of the proposed strategies were able to invert for ρ and at the same time give a reliable model (Figure 4). Thus, the best models were obtained using the empirical relation.
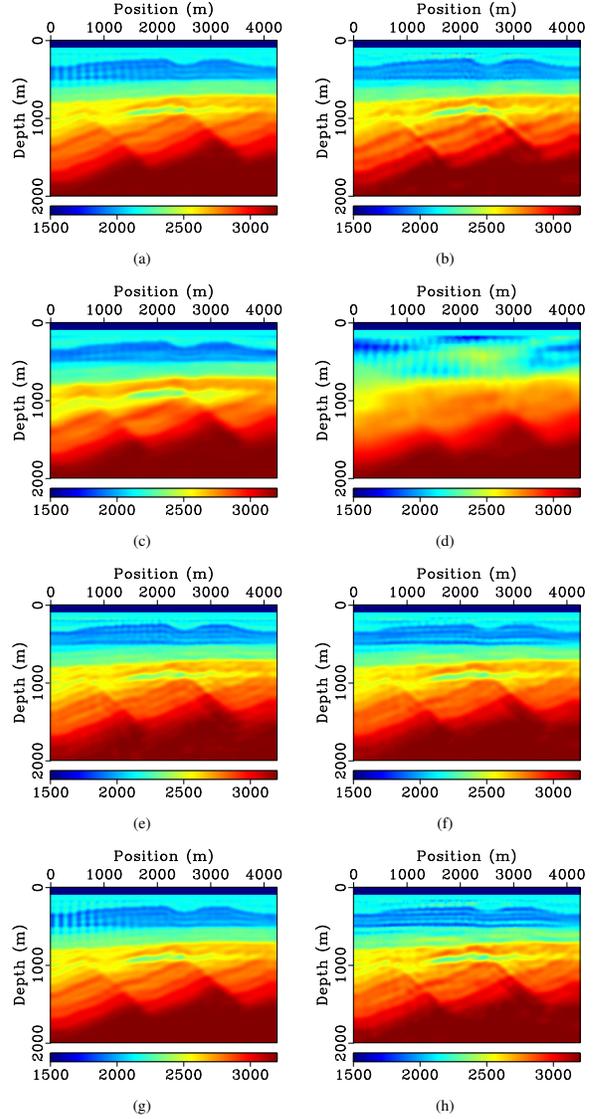
Figure 2: Inversion results for V_p for different inversion strategies (see Table 1): (a) strategy 2a; (b) strategy 2b; (c) strategy 3a; (d) strategy 3b; (e) strategy 3c; (f) strategy 3d; (g) strategy 4a; (h) strategy 4b.

Inversion strategies for EFWI

Strategy	Dataset	Wave field			Sequence		
		ρ	V_p	V_s	ρ	V_p	V_s
1a	Streamer	P	P	P	1	1	1
1b	OBC	$P/V_z/V_x$	$P/V_z/V_x$	$P/V_z/V_x$	1	1	1
2a	Streamer	P	P	P	3	1	2
2b	OBC	$P/V_z/V_x$	$P/V_z/V_x$	$P/V_z/V_x$	3	1	2
3a	Streamer	-	P	-	G	1	C
3b	OBC	-	$P/V_z/V_x$	-	G	1	C
3c	OBC	-	P/V_z	V_x	G	1	2
3d	OBC	-	P	V_z/V_x	G	1	2
4a	Streamer	-	P	-	-	1	-
4b	OBC	-	P	V_z/V_x	-	1	2

Table 1: An overview of the different inversion strategies (P: pressure, V_z: vertical particle velocity, V_x: horizontal particle velocity, G: Gardner relation used in update, C: Castagna relation used in update). The first column is the enumeration of the strategies, the second column is the type of dataset, the third column is the wave field type used in the inversion for the different parameter models, and the fourth column gives the order in which the parameters are inverted for.
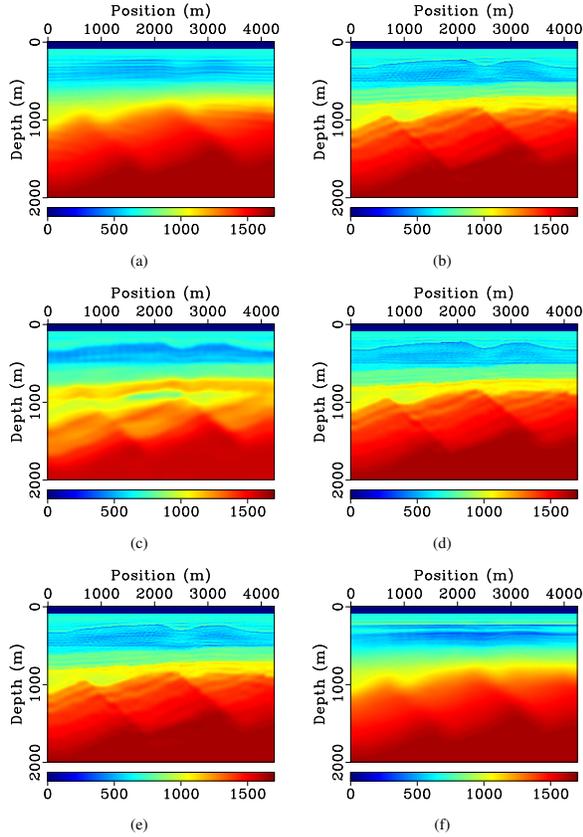
Figure 3: Inversion results for V_s for different inversion strategies (see Table 1): (a) strategy 2a; (b) strategy 2b; (c) strategy 3a; (d) strategy 3c; (e) strategy 3d; (f) strategy 4b.
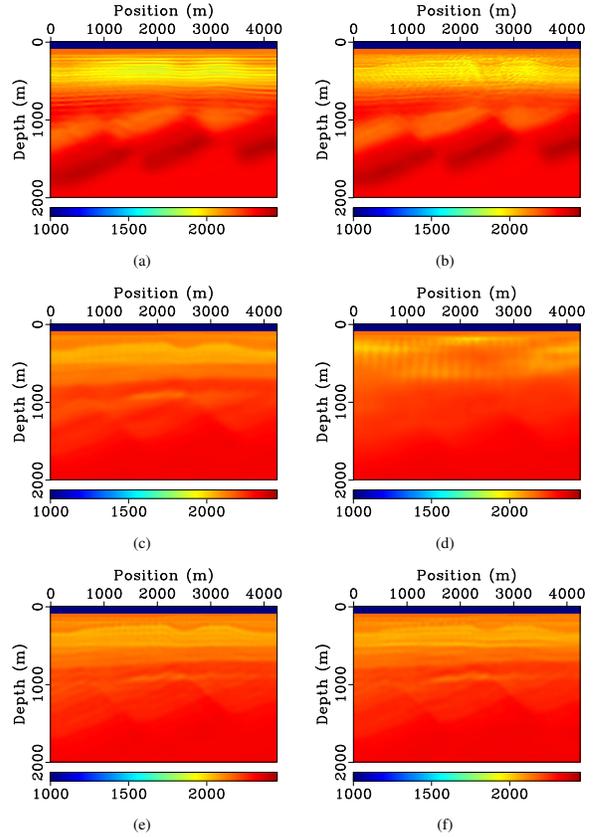
Figure 4: Inversion results for ρ for different inversion strategies (see Table 1): (a) strategy 2a; (b) strategy 2b; (c) strategy 3a; (d) strategy 3b; (e) strategy 3c; (f) strategy 3d.

DISCUSSION

Our results show that the best strategy for getting reliable inverted models for the three elastic parameters, is a sequential based strategy where one invert for each parameter on a one-by-one basis. This was not a big surprise, due to the non-linearity and complexity of the problem.

In terms of resolution, the general observation is that the V_p models obtained with the OBC dataset include more details and less artifacts than the streamer models. By comparing the results obtained with strategies 3c, 3d and 4b, it is clear that the updates for V_p are less sensitive to a good estimate of ρ, and that one to some extent may neglect to update ρ in the inversion as long as the initial estimate is good enough.

Our results demonstrate that for the chosen model pressure data recorded by conventional streamers are not enough for recovering the V_s model, and that particle velocity fields are necessary to be able to recover the model. An important observation is that the OBC strategy where ρ was not updated (strategy 4b) failed in recovering V_s. The inversion for V_s seems to be more dependent on a good estimate for ρ than the inversion for V_p. Hence, a good estimate of ρ is important for the success in recovering V_s using OBC data.

The failure of strategy 3b was a big surprise, since the similar strategy for the streamer dataset (strategy 3a) was able to recover the V_p model. Thus, the OBC data is more sensitive to ρ and V_s, particularly the parts relative close to the sea floor. Small errors in these parts, may have major impacts on the recorded datasets, and the inversion is not able to explain these errors by only updating the V_p model.

None of our strategies were able to recover the ρ model using FWI. Mora (1987) discussed the importance of the choice of model parameters, and claimed that inversion for ρ is difficult in cases where the source is dominated by P-waves and the model parameters are ρ, V_p and V_s. Our results substantiate this claim.

An alternative to the challenging problem of estimating ρ is to use empirical relations, either the inverted V_p or V_s models. It is possible to do the same with V_s in cases where inversion fails. Empirical relations should be used with care since they may introduce spurious structures in the models, as can be seen in the inversion results for V_s (Figure 3). These structures may distort the overall results.

There are small differences in the V_p images using different wave fields in the inversion. The model obtained using the pressure and V_z field in the inversion is slightly better than the inversion using only the pressure field (Figures 2(e) and 2(f), respectively). For the V_s images there are no differences in the images using the V_x wave field, or both V_x and V_z wave fields in the estimation.

The computational cost of the proposed strategies vary, and a sequential based inversion strategy is far more costly than the other strategies. In three dimensions, where the runtime of the involved modeling methods are high compared to two dimensions, one may be restricted to perform fewer iterations in the inversion. In these cases, empirical relations may be used to have an update of the parameter models which are not updated by FWI, to restrict the total runtime. An alternative is to let the other parameter models be unchanged during the iterations. We believe, however, it is better to have an update of all parameters, as long as the empirical relations are not too far away from the solution.

An extension of our strategies is to run a final inversion, after V_p and V_s models are obtained, where one invert for the two velocity models at the same time. This could potentially explain correlation effects between the models, and thus sharpen the results. This would, however, increase the total cost of the inversion.

CONCLUSION

We have investigated different strategies for estimating the P- and S-wave velocity and density models using FWI, where we have compared conventional streamer and OBC datasets acquired over the same model. We find that both datasets are able to invert for V_p, while the OBC dataset is necessary for the inversion for V_s. The ρ model is difficult to invert for with our setup using both dataset, and the updates for this model is obtained using an empirical relation. A sequential based inversion strategy, where one invert for each parameter on a one-by-one basis is the best option when an estimate for all three elastic parameters is wanted.

ACKNOWLEDGMENTS

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