PROBLEM 1 (5 POINTS). Consider a vertical tubing through which only undersaturated oil is flowing. Consider that wellhead pressure is kept constant and equal to separator pressure. Assume that at time t1, the rate is $5000 \mathrm{Sm} 3 / \mathrm{d}$. At a later time t2, the rate is $3000 \mathrm{Sm} 3 / \mathrm{d}$. What can you say about flowing bottomhole pressure in t2 and t1?
a) pwf at t1 is greater than at t2
b) pwf at t1 is smaller than at t2
c) pwf at t1 and at t2 are equal

THIS PROBLEM IS CORRECTED AUTOMATICALLY BY INSPERA

PROBLEM 2 (5 POINTS). As a follow-up to problem 2, assume that the IPR of the well can be modeled with the equation $q \mathrm{o}=\mathrm{J} *(\mathrm{pR}-\mathrm{pwf})$. What can you say about reservoir pressure in t 1 and t 2
a) Reservoir pressure in t1 is greater than at t2
b) Reservoir pressure in t 1 is smaller than at t2
c) Reservoir pressure at t1 and at t2 are equal
d) We need more information to determine the relationship between reservoir pressure in times t1 and t2

THIS PROBLEM IS CORRECTED AUTOMATICALLY BY INSPERA

PROBLEM 3 (5 POINTS). Consider a horizontal flowline that transports gas from a compression station to a gas terminal located on the shore. The compressors ensure the pressure at the inlet to the pipeline is always constant. Select the alternatives that make sense:
a) When the rate through the pipeline is $30 \mathrm{E} 06 \mathrm{Sm} 3 / \mathrm{d}$, the pressure at the exit of the pipeline is higher than when the rate through the pipeline is $20 \mathrm{E} 06 \mathrm{Sm3} / \mathrm{d}$
b) When the rate through the pipeline is $30 \mathrm{E} 06 \mathrm{Sm} 3 / \mathrm{d}$, the pressure at the exit of the pipeline is lower than when the rate through the pipeline is $20 \mathrm{EO6} \mathrm{Sm3/d}$
c) We need more information to determine the pressure at the exit of the pipeline when the flow is 30 E06 Sm3/d and 20 E06 Sm3
d) When the valve is closed at the terminal, and there is no flow of gas, the pressure at the exit of the pipeline is equal to the pressure at the inlet of the pipeline

PROBLEM 4 (2 POINTS). For a production system consisting of a single well producing to a separator through a flowline, the curve of available pressure at the wellhead consists of pressures calculated cocurrent from reservoir to wellhead considering pressure drop in the formation and in the tubing for several well rates.
a) False
b) True

THIS PROBLEM IS CORRECTED AUTOMATICALLY BY INSPERA

PROBLEM 5 (3 POINTS). As a follow-up to problem 4, when comparing two curves of available pressure at the wellhead for the same dry gas well, but using two different tubing diameters, Phi1 > Phi2, for a given standard condition rate $\mathrm{qg}^{*}$, the corresponding available wellhead pressure pwh will be smaller for diameter 1 than for diameter 2.
a) False
b) True

THIS PROBLEM IS CORRECTED AUTOMATICALLY BY INSPERA

PROBLEM 6 (2 POINTS). For a production system consisting of a single well producing to a separator through a flowline, the curve of required pressure at the inlet of the flowline consists of pressures calculated counter-current from separator to pipeline inlet considering pressure drop in the pipeline for several well rates.
a) True
b) False

THIS PROBLEM IS CORRECTED AUTOMATICALLY BY INSPERA

ANSWER THIS PROBLEM USING PEN AND PAPER. YOUR ANSWER WILL BE SCANNED AND UPLOADED TO INSPERA:

PROBLEM 7 (10 POINTS).
You are considering drilling a well in a undersaturated oil reservoir of thickness 40 m , with a horizontal permeability $\left(k_{H}\right)$ equal to 15 md , and a permeability anisotropy equal to $9\left(\frac{k_{H}}{k_{V}}\right)$.

Task 1. Determine how long should you make a horizontal well to have the same productivity index as a vertical well that is completed all through the reservoir thickness.

## Additional information

Expressions for productivity index of vertical well and horizontal well (in units of $\left.\frac{S m 3}{d} / b a r\right)$ are given below:

- Vertical undersaturated oil well perforated through all the reservoir thickness (h)

$$
J_{v e r t i c a l w e l l}=\frac{k_{H} \cdot h}{18.68 \cdot\left(\mu_{o} \cdot B_{o}\right)_{@ p_{a v}} \cdot\left[\ln \left(\frac{r_{e}}{r_{w}}\right)\right]}
$$

Where permeability is in md, $h$ in $m$, viscosity in $c P$ and Bo in m3/Sm3

- Horizontal well located in the middle of the layer with thickness h, width D and length L.

$$
J_{\text {horizontal well }}=\frac{k_{H} \cdot h}{6.22 \cdot\left(\mu_{o} \cdot B_{o}\right)_{@ p_{a v}} \cdot\left[\frac{\pi \cdot D}{2 \cdot L_{w}}+\frac{3 \cdot h \cdot \beta}{L_{w}} \cdot \ln \left(\frac{h \cdot \beta}{\pi \cdot r_{w} \cdot(1+\beta)}\right)\right]}
$$

With

- $\beta=\sqrt{\frac{k_{H}}{k_{V}}}$
- $L_{w}$ is wellbore length
- Assume wellbore radius $\left(r_{w}\right)$ equal to 0.15 m
- Assume $\left(\mu_{o} \cdot B_{o}\right)_{@ p_{a v}}=2.15$
- For the vertical well J expression assume that $r_{e}=1500 \mathrm{~m}$
- For the horizontal well J expression assume that $D=1500 \mathrm{~m}$ and $\mathrm{L}=1500 \mathrm{~m}$

Solution
Making equal both analytical expressions gives

$$
\begin{aligned}
& \frac{k_{H} \cdot h}{18.68 \cdot\left(\mu_{o} \cdot B_{o}\right)_{@ p_{a v} \cdot\left[\ln \left(\frac{r_{e}}{r_{w}}\right)\right]}} \begin{array}{l}
=\frac{k_{H} \cdot h}{6.22 \cdot\left(\mu_{o} \cdot B_{o}\right)_{@ p_{a v}} \cdot\left[\frac{\pi \cdot D}{2 \cdot L_{w}}+\frac{3 \cdot h \cdot \beta}{L_{w}} \cdot \ln \left(\frac{h \cdot \beta}{\pi \cdot r_{w} \cdot(1+\beta)}\right)\right]}
\end{array}
\end{aligned}
$$

Cancelling out common terms and clearing out the well length

$$
L_{w}=\frac{6.22 \cdot\left[\frac{\pi \cdot D}{2}+3 \cdot h \cdot \beta \cdot \ln \left(\frac{h \cdot \beta}{\pi \cdot r_{w} \cdot(1+\beta)}\right)\right]}{18.68 \cdot\left[\ln \left(\frac{r_{e}}{r_{w}}\right)\right]}
$$

Substituting all factors, this gives $L_{w}=156 \mathrm{~m}$, i.e. In order to make the horizontal well more productive than the vertical well it has to be more than 3 times longer than the thickness of the reservoir.

ANSWER THIS PROBLEM USING PEN AND PAPER. YOUR ANSWER WILL BE SCANNED AND UPLOADED TO INSPERA.

PROBLEM 8 (5 POINTS). IPR for CO2/water injector
Make a plot of the IPR for a CO2 (or water) injector. Indicate the effect of skin on the plot.

## Solution

For both cases the IPR is fairly linear


This is because of the behavior of the product of Bw and viscw versus p (for water injector):


Bw viscosity
Or the behavior of $1 /$ viscg*Bg at high pressures (for the CO2 injector)


Positive skin causes the slope of the IPR to become bigger:


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PROBLEM 9 (5 POINTS). Plot at a fixed temperature, for an oil, the following black oil properties versus pressure:

- Oil volume factor, $\mathrm{B}_{0}$
- Solution gas-oil ratio, $\mathrm{R}_{\mathrm{s}}$
- Gas volume factor, $\mathrm{B}_{\mathrm{g}}$
- Solution oil-gas ratio, $r_{s}$

Indicate the proper units of each variable.

Solution


Bo and Bg in m3/Sm3
Rs and rs in Sm3/Sm3

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## PROBLEM 10 (6 POINTS).

Consider a horizontal well producing from an undersaturated oil layer. The horizontal section of the well has a total length of 2 km and a inner diameter of $6^{\prime \prime}$. The pressure at the heel of the well is higher than the bubble point pressure of the oil at reservoir temperature.

Plot the IPR of the well (pressure at the heel versus total oil well rate at standard conditions) considering that:
a) There is no pressure drop along the wellbore
b) There is pressure drop along the wellbore

Explain the behavior plotted.

## Solution:

It will look like the figure below:


When one considers the pressure drop in the wellbore, it is mostly due to frictional pressure drop, which are a nonlinear function of the well rate. Because of this, the curve becomes nonlinear.

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PROBLEM 11 (12 POINTS).
Consider the dry gas production system shown in the figure below


Task 1: what is the choke pressure drop required (in bar) for the system to deliver a rate of 2.0 E6 Sm³/d?

Task 2: Will the choke operate in the critical or subcritical regime? Assume the critical pressure ratio is equal to 0.5.

## Additional information:

- Neglect the pressure drop in the flowline, i.e. assume $p_{i n}=p_{\text {sep }}=40$ bara
- Use the following equations:

| Inflow equation: |
| :--- |
| $q_{\bar{g}}=C_{R} \cdot\left(p_{R}^{2}-p_{w f}^{2}\right)^{n}$ |
| With |
| $C_{R}=104 \quad \mathrm{Sm}^{3} / \mathrm{d} / \mathrm{bar}^{\wedge} 2 \mathrm{n}$ |
| $\mathrm{n}=0.9$ |
| $\mathrm{p}_{\mathrm{R}}=304$ bara |
| Tubing equation: |
| $q_{\bar{g}}=C_{T} \cdot\left(\frac{p_{w f}^{2}}{e^{S}}-p_{w h}^{2}\right)^{0.5}$ |
| $C_{\mathrm{T}}=4.41 \mathrm{E} 4 \quad \mathrm{Sm}^{3} / \mathrm{d} / \mathrm{bar}$ |
| $\mathrm{S}=0.31$ |

## SOLUTION

Task 1:

1. Calculate the available pressure at the wellhead departing from reservoir pressure:

- With the rate provided, use the IPR equation and calculate the flowing bottomhole pressure:
$P_{w f}=\left(p_{R}{ }^{2}-\left(q_{g} / C_{R}\right)^{(1 / n)}\right)^{0.5}=186.7$ bara
- With the rate provided and the flowing bottom-hole pressure, use the tubing equation and calculate the available wellhead pressure:
$P_{w h}=\left(\left(p_{w f}{ }^{2} / e^{S}\right)-\left(q_{g}{ }^{2} / C_{T}{ }^{2}\right)\right)^{0.5}=153.4$ bara
The choke pressure drop required for the system to deliver a rate of 2 e $6 \mathrm{Sm}^{\wedge} 3 / \mathrm{d}$ is therefore: $p_{w h}-p_{i n}=153.4-40=113.4$ bar


## SOLUTION

Task 2
Solution: The pressure ratio is $40 / 150=0.266$, with is very low. Critical flow usually occurs at pressure ratios of 0.5 , so it is highly likely that the choke is operating in the critical regime.

ANSWER THIS PROBLEM USING PEN AND PAPER. YOUR ANSWER WILL BE SCANNED AND UPLOADED TO INSPERA.

## PROBLEM 12 (10 POINTS).

Consider an oil and gas mixture flowing upwards in a vertical well. The local rates of oil and gas are $\mathrm{q}_{\mathrm{o}}=0.07 \mathrm{~m}^{3} / \mathrm{s}$ and $\mathrm{q}_{\mathrm{g}}=0.3 \mathrm{~m}^{3} / \mathrm{s}$. The inner diameter of the tubing is 0.15 m . The density of the oil and the gas are $700 \mathrm{~kg} / \mathrm{m}^{3}$ and $200 \mathrm{~kg} / \mathrm{m}^{3}$ respectively.

Task 1. Calculate the non-slip gas volume fraction.
Task 2. Calculate the hydrostatic pressure gradient ( $\mathrm{dp} / \mathrm{dx}$ in bara/m) using the value of the density of the mixture. To calculate the density of the mixture use the non-slip gas volume fraction calculated in Task 1.
Task 3. Assume that the real velocity of the gas is twice the real velocity of the liquid. Calculate the gas holdup (gas volume fraction of the mixture considering slip).
Task 4. For the condition presented in Task 3, will the hydrostatic pressure gradient of the mixture be higher than the value calculated in task 2 or lower? Explain your answer

## Task 1.

The non-slip gas volume fraction is lamba_g $=\mathrm{qg} /(\mathrm{gg}+\mathrm{qo})=0.81$
Task 2
Density of the mixture rho_m = rho_। * lamda_| + rho_g* lamgda_g = $700 * 0.19+200 * 0.81=294.6 \mathrm{~kg} / \mathrm{m} 3$

Hydrostatic pressure gradient $=$ rho_m*g $=294.6$ * $9.81 / 1 \mathrm{e} 5=0.0289 \mathrm{bar} / \mathrm{m}$
Task 3.
$\mathrm{vg}=2^{*} \mathrm{vl}$
$\mathrm{qg} /(\mathrm{Ag})=2^{*}(\mathrm{q} / / \mathrm{AL})$
but AL $=A-A g$, then
$\mathrm{qg} / \mathrm{Aq}=2^{*} \mathrm{ql} /(\mathrm{A}-\mathrm{Ag})$
Using $\mathrm{Hg}=\mathrm{Ag} / \mathrm{A}$
$q g(1 / \mathrm{Hg}-1)=2^{*} q \mathrm{l}$
$H g=1 /\left(2^{*} q / / q g+1\right)$
Substituting the numbers, this gives
$\mathrm{Hg}=0.68$
Task 4
The gas occupies less of the cross section when compared with the non-slip case. This means that the density of the mixture will be higher (more similar to the liquid).
Therefore the hydrostatic pressure gradient will be higher also.
Density of the mixture rho_m = rho_। * HI + rho_g* H_g = 700*0.32+200*0.68 = 359 kg/m3
Hydrostatic pressure gradient $=$ rho_m*g $=359$ * 9.81/1e5 $=0.035 \mathrm{bar} / \mathrm{m}$

ANSWER THIS PROBLEM USING EXCEL. WRITE THE PROCEDURE AND THE ANSWER TO TASK 2 IN THE EXCEL FILE.

## PROBLEM 13 (10 POINTS).

A test has been performed on a gas well and the following pairs of dry gas rate and flowing bottomhole pressure are reported:

| Test point | qg [Sm3/d] | pwf [bara] |
| :--- | :--- | :--- |
| 1 | 8.29 E06 | 484.3 |
| 2 | 20.01 E06 | 246.0 |

The reservoir pressure is 542.5 bara.
Your boss has suggested you to use the dry gas backpressure equation to represent the IPR of the formation:

$$
q_{\bar{g}}=C_{R} \cdot\left(p_{R}^{2}-p_{w f}^{2}\right)^{n}
$$

Task 1: provide the values of the backpressure coefficient $\left(C_{R}\right)$ and backpressure exponent ( n ) that match the test data. Explain the procedure you followed.

Task 2: Based on the theory given in class about dry gas IPR models, assess the validity of the equation given above.

Solution:
Task 1:
Apply natural logarithm to both sides of the backpressure equation:

$$
\ln \left(q_{\bar{g}}\right)=\ln \left(C_{R}\right)+\mathrm{n} \cdot \ln \left(p_{R}^{2}-p_{w f}^{2}\right)
$$

Applying the equation above for test points 1 and 2 gives

$$
\begin{aligned}
& \ln (8.29 \mathrm{E} 06)=\ln \left(C_{R}\right)+\mathrm{n} \cdot \ln \left(542.5^{2}-484.3^{2}\right) \\
& \ln (20.01 \mathrm{E} 06)=\ln \left(C_{R}\right)+\mathrm{n} \cdot \ln \left(542.5^{2}-246.0^{2}\right)
\end{aligned}
$$

This gives a system of two equations and two unknowns. The solution gives:
$C_{R}=6460 \mathrm{Sm} 3 / \mathrm{d} / \mathrm{bar}^{\wedge} 2 \mathrm{n}$ (units must be given!!)
$\mathrm{n}=0.651$

Task 2:
According to the theory covered in class, the equation provided is only valid for low pressure dry gas (e.g. below 100-150 bara). The reservoir pressure for this well is quite high, so one should use a linear PI or the IPR with the $m$ function.

ANSWER THIS PROBLEM USING THE EXCEL PROVIDED. WRITE THE PROCEDURE AND ANSWERS IN THE EXCEL FILE.

PROBLEM 14 (20 POINTS).
Consider a gas-lifted oil well. The injection point is very close to the bottom of the tubing, so it is reasonable to assume that the lift gas is injected at the end of the tubing (see the figure below). The end of the tubing is very close to the perforations.


The reservoir GOR is equal to 54. The reservoir IPR can be modeled with Vogel equation:

$$
q_{\bar{o}}=q_{\bar{o}, \max }\left[1-0.2 \cdot \frac{p_{w f}}{p_{R}}-0.8 \cdot\left(\frac{p_{w f}}{p_{R}}\right)^{2}\right]
$$

using a $p R=200$ bara, and a qomax $=3000 \mathrm{Sm} 3 / \mathrm{d}$.
The figure below shows the curves of Tubing performance relationship at a constant wellhead pressure of 30 bara, for different values of GOR in the tubing.


Task 1. Determine the optimal gas lift injection rate (i.e. the one that gives the highest reservoir oil rate). Use only the tubing GORs that are provided in the figure. Explain the procedure you followed.

## Additional information:

- Solve this problem graphically.

Solution
Using the Vogel equation, plot the IPR on top of the TPRs


The highest reservoir rate is obtain when the GOR of the tubing is equal to 300 . (qoR = $1125 \mathrm{Sm} 3 / \mathrm{d}$ ).


The amount of gas that comes from the reservoir is $q g R=54 * 1125 \mathrm{Sm} 3 / \mathrm{d}=60750 \mathrm{Sm} 3 / \mathrm{d}$. The tubing requires a total amount of gas equal to $q g T=300^{*} 1125=337500 \mathrm{Sm} 3 / \mathrm{d}$. Therefore the gas lift rate to inject is: 276750 Sm3/d

