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ABSTRACT
Borehole instability during drilling in shale is more pronounced than in any other formations. No well is drilled in shale without problems. A major instability risk is borehole shear failure. This paper evaluates different shear failure modes under in-situ stress state during underbalanced drilling (UBD). The following parallel objectives were studied:

- Generation of input data for the “geomechanical model” by presenting extensively used correlations for estimating rock strength, in-situ acting stresses and formation pore pressure. Such generated data were used as input to an upgraded analytical model to estimate borehole shear failures.
- Borehole sensitivity analysis were extended to evaluating the borehole collapse risk from effects like differential stress and loads, stress anisotropy, cohesion, pore pressure and friction angle.

The analytical model relies on the Mohr-Coulomb (M-C) failure criterion. Matlab codes were developed to simulate the analytical model and to validate it through Gullfaks well data.

The results showed that the developed geomechanical model is capable of assessing in-situ stresses with a certain degree of quality. The sensitivity analysis results showed that the mud weight and rock strength are the most critical parameters for determining borehole collapse risks for UBD candidates. The borehole collapse model is quantifying the risk of shear failure modes with acceptable accuracy. The generality of this study is to provide a standard workflow to assess in-situ stresses along with borehole failure risks for vertical and horizontal wells.

Keywords: oil well drilling, shale, geomechanical model, borehole instability.

1.0 INTRODUCTION
Shale is specifically mentioned in this setting, due to the fact that borehole instability is more pronounced in such formations than in any other formation [9,10,11,13 14 & 16]. From field experience, it was found that shales (hard rock) make up of more than 80 % of the sediments and rocks in siliclastic environments and about three quarters of the borehole problems are caused by shale instability, troubles such as sloughing shale and stuck pipe. At best, an unstable wellbore would mean that drilling performance is impeded through lost time. At worst it could mean a hole collapse and total loss of a well. All this means extra costs. A significant amount of lost time and extra cost (about 20 billion USD/year) is accounted to overcome shale related problems worldwide.

The problem addressed in this study is the borehole shear failures risks in shale during UBD through in-situ stress regimes. Based on in-situ stress magnitudes, Anderson (1951) classified three types of earth’s in-situ stress states: extensional (σi>σh>σv), strike-slip (σh>σi>σv) and compressional (σh>σv>σi). Borehole instability is in most of the cases, a direct reflection of these stress states. An anisotropic stress pattern is characterized by a specific failure position in the borehole circumference, and this position is controlled by in-situ stresses [9-14]. A brief description of these stresses and their impact on stable drilling were presented through previous publications [9, 11 & 15].

This paper aims to evaluate the shear failure risk in shale through in-situ stress regimes. A stress field model which varies with depth is therefore presented in Fig.1. This model represents wellbores drilled in shallow (case-I), medium-deep (case-II) and deep basins (case-III). These three cases are defined based on the in-situ stress magnitudes vs. depth of investigation.

Wellbore stability models that include some aspects of shear failure analysis in shale have already been developed [9-15]. Inspection to these models, found that the assessment of in-situ stresses is the focal weak side in borehole instability analysis [9 & 11-15]. A standard geomechanical model is essential for evaluating in-situ stresses. Estimated in-situ stresses may be used as input into shear failure models to evaluate different shear failure modes with reasonable accuracy. This paper presents a geomechanical model based on extensively used correlations for estimating rock strength, in-situ acting stresses and formation pore pressure. This current investigation enhanced the insight on borehole...
collapse risk.

It is not known exactly how rock fails. The processes associated with failure are complex and not subjected to convenient characterization through simplified models. The Collapse criterion defines a state where the borehole is no longer stable, but becomes unstable to a degree where it is defined as collapsing. Many different arguments can be used to define the collapse criteria, e.g. scientific arguments based on mechanical criteria or operational argument based on practical limitations. Operational arguments are related to type and amount of cavings or breakouts present in the drilling fluid, degree of wellbore instability with respect to section of angle & length, and the inclination of the borehole. Scientific arguments are fulfillment of a failure criterion, choice of failure criteria with respect to stress conditions, type of formation, and type of analysis method (analytical or numerical).

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The selection of a failure criterion for borehole stability analysis is a challenging task [5]. Proper selection of failure criteria for borehole stability analysis is therefore unclear to drilling engineers. Rock mechanics experts have applied several failure criteria in an attempt to relate rock strength measured in different simple tests to borehole stability. Some of the predicting methods are M-C criterion, Hoek- Brown criterion, Drucker-Prager criterion, Yield Zone criterion, Cam-Clay model, Modified Lade criterion and Griffith failure criterion. The theoretical backgrounds and limitations of these models have been extensively covered in the literatures [5, 6, 7, 9 & 12]. This study applied the M-C criterion. The theoretical backgrounds and limitations of these models have been extensively covered in the literatures [5, 6, 7, 9 & 12]. This study applied the M-C criterion. These models have been extensively used, and should always be checked or calibrated against proper data from each field. To get a better accuracy of 

![Fig. 1 Stress Vs. Depth.](image)

The selection of a failure criterion for borehole stability analysis is a challenging task [5]. Proper selection of failure criteria for borehole stability analysis is therefore unclear to drilling engineers. Rock mechanics experts have applied several failure criteria in an attempt to relate rock strength measured in different simple tests to borehole stability. Some of the predicting methods are M-C criterion, Hoek- Brown criterion, Drucker-Prager criterion, Yield Zone criterion, Cam-Clay model, Modified Lade criterion and Griffith failure criterion. The theoretical backgrounds and limitations of these models have been extensively covered in the literatures [5, 6, 7, 9 & 12]. This study applied the M-C criterion due to its simplicity and level of acceptability. For quantifying borehole failure risk the total work is divided into the following phases:

- Develop and investigate the geomechanical model to estimate in-situ stresses.
- Quantify borehole shear failure risk for UBD in shales.
- Conduct sensitivity analysis to define critical parameters for the borehole failure state by accounting for the effect of differential stress and loads magnitudes, cohesion, friction angle, pore pressure and well trajectory.

The M-C linear elastic failure model was used to quantify shear failure risk. Matlab codes were developed to simulate both the geomechanical and the shear failure models dynamically. This study gives an integrated workflow accomplished with Geomechanical model to borehole shear failure risk.

### 2.0 CONSTRUCTION OF GEOMECHANICAL MODEL (GMM)

#### 2.1 Generation of Input Parameters

One challenge for constructing a geomechanical model is the generation of consistent input data. Many of the required parameters can be inferred from different sources, using some of empirical correlations, theoretical expressions, or analogue data previously experienced. Both stress field and rock mechanical properties are part of the GMM. Various methods and techniques have been used to calculate necessary input to generate GMM. This study developed a standard GMM based on updated published work [3, 8, 12 & 16]. Details of present GMM along with data integration techniques are presented through Table A1.

#### 2.2 Assessment of the In-Situ Stresses

For typical depths of oil reservoirs, the ratio of the 

\[ \sigma_h / \sigma_v \] 

ranges from 0.3 to 1.5, and 

\[ \sigma_h / \sigma_t \] 

ranges from 1 to 2 [1,2, 4, 6 & 8]. In particular, the horizontal stress magnitude and orientation are not usually measured. This will definitely increase the uncertainty in the results. This work assesses in-situ stresses based on the developed geomechanical model, validated through Gullfaks well data. Estimating in-situ stresses are presented through Fig. 2. It is seen that the in-situ stress regimes is identified and varies with depth. For example, at shallow and medium deep formations (1000-1800 m), the \( \sigma_h \) is largest while at deep formation (2000-3000 m), \( \sigma_v \) dominates. Over-pressured zones are identified between 1500-23000 m. This specific zone of overpressure have been verified through others publications [9, 16-18]. These publications have been extensively used, containing core data and lithological study from several wells. Our project did not have scope to analyze cores and formation lithology; we had to rely on comparing our result with previous published results within the same geological area.

The minimum horizontal stress is estimated by using Breckels and Van Eeklelen [3] correlation (developed & tested for US Gulf Coast wells). However, the author’s (Breckels and Van Eeklelen) experience was that the relation for depths down to 2500 m gave fairly good estimates in most parts of the North Sea with water depths up to approximately 300 m. Though the water depth of our studied well was greater than 300 m, estimation of \( \sigma_h \) in shallow formation depth up to 1500 m did not show a good trend with \( \sigma_v \) (Fig. 2). Thus, estimation of \( \sigma_h \) relations provided by (Breckels and Van Eeklelen) should only be considered as a first estimate and should always be checked or calibrated against proper data from each field. To get a better accuracy of \( \sigma_h \) we need some adjustment into Breckel’s empirical correlation. Regression analysis on several well data sets may provide updated correlations to estimate \( \sigma_h \) in
North Sea wells.

The pore pressure assessment is the most critical part of our GMM because until now, very limited options are found for predicting pore pressure in shale. The Eaton (1975) correlation was used in this study. The exponent (i.e., $n = 3, 3.5, 4, 5$ & $6$) is playing a major role to assess pore pressure in the North Sea area. The assessment of pore pressure vs. different exponents is presented in Fig. 3. The pore pressure trend for the exponent $n = 4.0$ gives reasonable accuracy for Gullfaks well, as supported by others studied also [9, 17 & 18].

The pore pressure assessment is the most critical part during drilling (unless the far field stresses would change much during a drilling period). Depending on the prediction of magnitudes and direction of wellbore stresses, prior indications whether a borehole will fail or not would be available.

A near borehole stress model ($\sigma_r, \sigma_\theta, \sigma_z$) is essential to evaluate shear failure risks in shale. Many publications [5, 9-15] have been focused on borehole stress modeling. From their studies, it was found that stress related failures are the major reasons for borehole instabilities. Having anisotropic horizontal stresses, which is the common situation, will change the borehole stress. Detailed and in-depth discussions on the near borehole stress model in shale have been extensively covered through many publications [9-16].

It is seen that the developed geomechanical model is capable of assessing the in-situ stresses. The accuracy of present GMM can be obtained by verifying it through further investigations. The GMM can be revised and upgraded through field data, lab investigation or more case studies throughout this research project. On the other hand, pore pressure prediction in shale is of critical concern, and needs to be focused on separately. The combined use of well logs and experimental compaction trends may improve the ability to predict trends of porosity, permeability, density, or velocity versus depth, enabling prediction of overpressure in shales. Fig. 2 was used as calibration chart for estimating the in-situ stresses at depth of interest. These calibrated data were used for conducting the sensitivity analysis to evaluate the shear failure risks in terms of minimum MW for avoiding borehole collapse. A complete set of wellbore stability analysis data are presented in Table A2. Further verification is essential to improve the confidence of the developed GMM.

2.0 EVALUATION OF BOREHOLE FAILURE MODES

The evaluation of shear failure modes is dependent on the assessment of near borehole stresses. Near wellbore stress are generated after a wellbore is drilled to support the rock that was originally supported by the removed solids in the borehole. Near wellbore stresses ($\sigma_r, \sigma_\theta, \sigma_z$) are normally of higher magnitudes and act on the formation at the wellbore wall. It is believed that, the near wellbore stress concentration is created immediately depending on the prediction of magnitudes and direction of wellbore stresses, prior indications whether a borehole will fail or not would be available.

A near borehole stress model ($\sigma_r, \sigma_\theta, \sigma_z$) is essential to evaluate shear failure risks in shale. Many publications [5, 9-15] have been focused on borehole stress modeling. From their studies, it was found that stress related failures are the major reasons for borehole instabilities. Having anisotropic horizontal stresses, which is the common situation, will change the borehole stress. Detailed and in-depth discussions on the near borehole stress model in shale have been extensively covered through many publications [9-16].

From their studies it was found that hoop stress, rock strength and mud weight design are the most influential parameters to cause shear failure during UBD in shale. Classical Kirsch solution [7] explains how hoop stress does lead to shear failure at circular borehole walls for UBD candidates. At a later part of this paper hoop stress based on Kirsch solution are estimated. By arranging permutation and combination of the near borehole stresses, six possible shear failure modes may exist. All modes influence borehole instability, but the following three shear failure modes are the most applicable in this study:

- **Mode A**: $\sigma_r \geq \sigma_\theta \geq \sigma_z$; axial stress is the intermediate concern; M-C failure state $= f(\sigma_r, \sigma_\theta)$.
- **Mode B**: $\sigma_\theta \geq \sigma_z \geq \sigma_r$; tangential stress is the intermediate concern; M-C failure state $= f(\sigma_\theta, \sigma_z)$.
- **Mode C**: $\sigma_z \geq \sigma_r \geq \sigma_\theta$; radial stress is the intermediate concern; M-C failure state $= f(\sigma_z, \sigma_r)$.

Modes “A” and “B” appear as collapse of the wellbore, but mode “C” appears as collapse of the wellbore first where subjected to excessive internal pressure when compared to the external stress, either in shear or extension mode. This theoretical interpretation was evaluated through analysis simulation based on the M-C failure criteria. The mathematical formulations and
stresses at the borehole wall are \[7\];

\[
\begin{align*}
\sigma_r &= P_w \\
\sigma_\theta &= \sigma_h + \sigma_b - 2(\sigma_h - \sigma_b)\cos 2\theta - P_w \\
\sigma_z &= \sigma_v - 2\nu b(\sigma_b - \sigma_h)\cos 2\theta \\
\tau_{\theta r} &= \tau_{\theta z} = \tau_{\theta b} = 0
\end{align*}
\] [1]

Azimuth “\(\theta\)” is the relative position of horizontal stresses. The tangential (\(\sigma_\theta\)) and radial (\(\sigma_r\)) stress in Eq.1 are a functions of the mud pressure (\(P_w\)). Hence, any changes in the mud pressure will only affect \(\sigma_\theta\) and \(\sigma_r\). When \(P_w\) decreases, \(\sigma_\theta\) increases towards the compressive strength, at which \(\sigma_\theta\) should be less than or equal to \(P_w\). This is the concern for underbalanced drilling with respect to borehole design. Thus, the lower limit of the mud pressure is associated with borehole collapse and \(\sigma_\theta > \sigma_r\). It is therefore an effective and useful approach to focus on tangential stress, which incorporates borehole failure mode, regulated by the in-situ stress magnitudes & direction, mud pressure and material intrinsic properties [15]. Investigation of Eq. 1, near borehole stresses were evaluated and presented in Figs. 4 a & b. From Fig. 4b, it is affirmed that horizontal strength anisotropy has significant influence on near borehole stresses to create a strong anisotropic stress environment at borehole wall and leads to borehole instability.

II. Hoop Stress and Borehole Instability

It can be seen from Figs. 5 (a, b & c) that at \(\theta = 90^\circ\) or \(270^\circ\), that the predicted hoop stress is maximum. The large hoop stress at \(\theta = 90^\circ\) together with magnitudes and direction of the anisotropy in-situ stresses turned the borehole instable. The first borehole breakout may be initiated at the peak of the hoop stress. On the other hand, at \(\theta = 180^\circ\), the borehole stresses are lower where formation fracture may initiates. However, depending of the direction of the in-situ stresses and well trajectory, breakout and fracture initiation positions may be changing [15]. This sensitivity analysis indicates that Mode C gives unrealistic results (i.e. see Fig. 5a) which is only valid in the strong tectonic stress regimes. Others observation is that horizontal wells are more vulnerable under mode B (Fig. 5c). In case of a vertical well, both modes A and B become closer and equally dominant to lead to borehole instability. The axial stress is not a function of mud weight; all three modes are changing with similar trend, where only the anisotropy intensity of the in-situ stresses is exposed to transformed axial stresses. It is recommended that for predicted shear failure at the borehole wall, “\(\theta\)” should be equal to \(90^\circ\) while equals to \(180^\circ\) for fracturing of the formation.

4.2. Prediction of Borehole Collapse

From Fig. 6 the borehole collapse pressure is found through various formations under modes A & B. The predictions indicate that the collapse pressure (CP) is determined by the largest in-situ stresses and the material intrinsic properties. In a later sensitivity analysis, it is shown how in-situ stress and material intrinsic properties influence the CP.

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4.3 Factors Associated with Collapse Pressure

Material intrinsic properties have a significant influence on the M-C failure state. The impacts of the following sensitive parameters on the borehole collapse model are:

(a) Vertical well
(b) horizontal well
(c) comparison between vertical and horizontal wells. Data from Table A2 (case III, deep well).

Fig. 5 Evaluation of hoop stresses for vertical & horizontal wells based on Kirsch solution. a) Vertical well b) horizontal well c) comparison between vertical and horizontal wells. Data from Table A2 (case III, deep well).

Fig. 6 Prediction of the borehole collapse pressure in vertical wells through an integrated approach by using GMM & CPM. Data from Gullfaks field (well #34/10-16).

I. Effect of Friction Angle and Cohesion on CPM

It is seen that material friction angle and cohesion have remarkable influence on the shear failure modes (Figs. 7 a & b). The required minimum MW to prevent borehole collapse is decreasing considerably with increasing material strength & its internal friction. These results indicate that the accuracy of the borehole instability model to obtain minimum MW strongly depends on the rock strength and material friction. The particular field case study shows that horizontal wells require higher mud weight in normal stress regimes (Fig. 7 a & b). Fig 7 a indicated that AMW in between horizontal and vertical wells is reduced with increasing material friction angle. In friction angle, $\alpha = 1$ degree with fixed formation strength (10 MPa), the minimum MW difference between horizontal and vertical wells is maximum ($\Delta$ MW 0.40 s.g). The MW difference is reducing with increasing friction angle (i.e., @ $\alpha = 30$ degree, $\Delta$ MW = 0.2 s.g). It is not surprising to see; minimum MW to prevent borehole collapse is gradually reducing with increasing formation strength (Fig 7b). In ultra deep and strong tectonic geological regions, the horizontal stress anisotropy requires relatively higher collapse pressure than vertical [9, 11, 13 & 15]. Mode C gives unrealistic result in normal stress field [Fig. A1]. The phase diagram is showing (Fig. A2) normally stresses that the material friction angle and cohesion may determine the domination of shear failure modes in CPM.

II. Effect of Pore Pressure on CPM

Pore pressure prediction in shale, serve as an input to CPM, and produce leading uncertainties of the model together with determining appropriate mud design to avoid collapse. This is the most unpredictable parameter that was used in GMM, eventually its influence on the mud design is vital. Fig. 7c is indicating that MW needs to increase significantly with increasing pore pressure to avoid instability. It is interesting to see that estimated MW for different wells and modes (except horizontal well & mode A) became close to close with increasing $P_r$. 

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and it is insignificant when $P_f$ reach 40 MPa. The horizontal well under mode A and the normal in-situ stress condition required maximum mud weight to prevent borehole collapse. But under strong tectonic stress regimes, vertical wells or well drilled through $\sigma_H$ are considered as critical with respect to borehole instability [9, 15]. The borehole trajectory has a trivial impact on mud design while drilling through strong tectonic geological regions, horizontal stress anisotropy determine the collapse risk [9-15].

5.0 CONCLUSIONS

- A geomechanical model is under development and has reached a certain level of quality. Predicted in-situ stresses were used as input to the borehole shear failure model to quantify failure risks. The developed model can be revised and updated by using calibrated lab test data, or best fit drilling data.
- Eaton’s empirical correlation can be used to predict pore pressure in North Sea wells with $n = 4$ or higher as the best fitting of exponent.
- M-C shear failure criterion can be used for minimum MW design to prevent borehole collapse under the modes A and B but not for mode C. Mode C gives unrealistic results which may only be applicable in strong tectonic geological regions. Under mode A which is the common situation, horizontal wells are most risks.
- Material friction angle and cohesion may determine the domination of shear failure modes in CPM. Mode A is dominating within friction angle 0-35° while for mode B it is greater than 35°.
- Hoop stress, mud weight and rock strength are the most influential elements in borehole collapse modeling.
- Shale is a most heterogeneous substance. Never expect much accuracy of predictive results through any borehole collapse model. It is impossible to capture the total characteristics of shale behaviour into one stability analysis model. This study gives confidence to optimize MW for balanced drilling. The efficient integrated approach (GMM through CPM) in this study may therefore be useful in design of OBD wells. The techniques used in this study may apply equally to others wells to design MW. This study results does not concern the mud design of UBD, because UBD is a critical issue in shale, which knowledge is necessary for further investigation to capture shale heterogeneity.

6. REFERENCES

6. Grauls D “Overpressure assessment using a minimum
7. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>σv</td>
<td>Vertical stress</td>
<td>MPa</td>
</tr>
<tr>
<td>σh</td>
<td>Min. horizontal stress</td>
<td>MPa</td>
</tr>
<tr>
<td>σH</td>
<td>Max. horizontal stress</td>
<td>MPa</td>
</tr>
<tr>
<td>σ0</td>
<td>Tangential or hoop stress</td>
<td>MPa</td>
</tr>
<tr>
<td>σa</td>
<td>Axial stress</td>
<td>MPa</td>
</tr>
<tr>
<td>σr</td>
<td>Radial stress</td>
<td>MPa</td>
</tr>
<tr>
<td>Pm</td>
<td>Pore pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>Ph</td>
<td>Hydrostatic pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>Pw</td>
<td>Wellbore pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>ν</td>
<td>Poissons ratios</td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>Cohesion strength</td>
<td>MPa</td>
</tr>
</tbody>
</table>

Abbreviation:
- GMM : Geomechanical Model
- CPM : Collapse Pressure Model
- CPM : Collapse Pressure Model
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- CPM : Collapse Pressure Model

8.0 ACKNOWLEDGEMENTS

The authors want to thank NTNU for supporting and giving permission to write this paper. We would like to express our appreciation to Prof. Rune Martin Holt, Erling Fjær, Sintef petroleum research, Per Horsrud and Ole Kristian Søreide Statoil, for their time to discuss critical issues in this work.

9.0 APPENDIX A

Table A1: Main parameter and sources of information used to build the geomechanical model [3,4, 8, 11, 12].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source</th>
<th>Correlation used in this study</th>
</tr>
</thead>
<tbody>
<tr>
<td>σv</td>
<td>Density log</td>
<td>σv = \frac{L}{R} \rho_s g h</td>
</tr>
<tr>
<td>σH</td>
<td>Best gauge</td>
<td>σH = 1.2 \times \rho_s g h (best guess)</td>
</tr>
<tr>
<td>σ0</td>
<td>Breckels, 1982</td>
<td>σ0 = 0.0053D^{1.45} + 0.46(\rho_s - \rho_w)</td>
</tr>
<tr>
<td>D</td>
<td>Eaton, 1975</td>
<td>D = 3000 m</td>
</tr>
<tr>
<td>C0</td>
<td>Horsrud, 98</td>
<td>C0 = 0.77 \times V_{\text{sh}}^{0.93}</td>
</tr>
<tr>
<td>ν</td>
<td>DSI tool &amp; Wang, 98</td>
<td>ν = \frac{1}{2}(\Delta\sigma_v/\Delta\tau)^2 - 1</td>
</tr>
<tr>
<td>E</td>
<td>DSI tool &amp; Wang, 98</td>
<td>E = \frac{\rho_s (4\Delta\tau - 3\Delta\sigma_v)}{1 - \Delta\sigma_v^2}</td>
</tr>
</tbody>
</table>

Table A2: In- Situ Stress assessment data (from Fig.2) were
used in borehole analytical model to predict CP through a sensitivity analysis. Only most normal case (case III) is exampled.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Stress Criteria, MPa</th>
<th>Others parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-1; Shallow</td>
<td>$\sigma_H &gt; \sigma_v &gt; \sigma_r$</td>
<td>$P_f = 13 \text{ MPa}$, $C_0 = 5 \text{ MPa}$, depth (1200 m) 22&gt;19&gt;18 $T_v = 1 \text{ MPa}$, $u = 0.20$, $\alpha = 30^\circ$, $\theta = 90^\circ$</td>
</tr>
<tr>
<td>Case-II; Intermediate</td>
<td>$\sigma_H &gt; \sigma_v &gt; \sigma_r$</td>
<td>$P_f = 25.5 \text{ MPa}$, $C_0 = 10 \text{ MPa}$ depth (2000 m) 40 &gt; 36 &gt; 33</td>
</tr>
<tr>
<td>Case-III; Deep (2500 m); shale</td>
<td>$\sigma_H &gt; \sigma_v &gt; \sigma_r$</td>
<td>$P_f = 23 \text{ MPa}$, $C_0 = 10 \text{ MPa}$, depth (2000 m) $46.5 &gt; 46 &gt; 39$</td>
</tr>
</tbody>
</table>

A3: M-C Borehole collapse Model

For borehole collapse it is assumed a M-C shear failure model. This is governed by the maximum and minimum principle stresses. The failure model is:

**Mode A:** Considering the situation where $\sigma_H \geq \sigma_v \geq \sigma_r$. According to the M-C criterion, failure will occur when:

$$\sigma_i = \sigma_R + \sigma_i \tan^{-1} \beta$$

$$\sigma_i = \sigma_H + \sigma_i - 2(\sigma_H - \sigma_i)\cos \theta - P_v - P_f$$

and $\sigma_j = \sigma_i - P_w - P_f$;

By applying for minimum borehole pressure to prevent borehole collapse, above equations becomes:

$$\sigma_H + \sigma_R - 2(\sigma_H - \sigma_i)\cos \theta - P_v - P_f - C_0 - (P_v - P_f) \tan^2 \beta \leq 0$$

and resulting:

$$P_{w, \text{min}} \leq \frac{3\sigma_H - \sigma_R - C_0 + P_f (\tan^2 \beta - 1)}{1 + \tan^2 \beta}$$

Similarly for mode B and C, collapse pressure equations can be derived. A set of analytical solutions for shear failures is included in Table A3.

**Table A3:** An analytical solution of borehole collapse model (modes A, B & C) with the assumption of vertical well, impermeable borehole, perfect mud cake, and anisotropic horizontal stress.

<table>
<thead>
<tr>
<th>M</th>
<th>Condition</th>
<th>$P_{w, \text{min}}^{(a)}$</th>
<th>$P_{w, \text{min}}^{(b)}$</th>
<th>$P_{w, \text{min}}^{(c)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\sigma_H \geq \sigma_v \geq \sigma_r$</td>
<td>$P_{w, \text{min}}^{(a)}$</td>
<td>$P_{w, \text{min}}^{(b)}$</td>
<td>$P_{w, \text{min}}^{(c)}$</td>
</tr>
<tr>
<td>B</td>
<td>$\sigma_v \geq \sigma_H \geq \sigma_r$</td>
<td>$P_{w, \text{min}}^{(a)}$</td>
<td>$P_{w, \text{min}}^{(b)}$</td>
<td>$P_{w, \text{min}}^{(c)}$</td>
</tr>
<tr>
<td>C</td>
<td>$\sigma_r \geq \sigma_v \geq \sigma_H$</td>
<td>$P_{w, \text{min}}^{(a)}$</td>
<td>$P_{w, \text{min}}^{(b)}$</td>
<td>$P_{w, \text{min}}^{(c)}$</td>
</tr>
</tbody>
</table>

For **horizontal well**, $\sigma_i$ change to $\sigma_H$ and $\sigma_R$ change to $\sigma_r$. Mode C is only applicable in strong tectonic stress area where $\sigma_r$ will be greater than $\sigma_0$ and $\sigma_H$ or $\sigma_v$ also greater than $\sigma_i$.© ICME2009