A Generic Model for Calculation of Frictional Losses in Pipe and Annular Flows

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Abstract

A theoretical foundation of a new simulator for hydraulic calculations of mud transport is presented. It is demonstrated that it is possible to design a simulator based on theoretical concepts taken from fluid dynamics. The theory is based on a simple turbulence model, which is calibrated against direct simulations of turbulent flows, and a simple expression for suppression of turbulence at low wall strain-rates. The model predicts turbulent non-Newtonian flows, provided rheology data is available. This is an attractive feature of the model as no general non-Newtonian flow correlations are available in the literature.

The output from the model is wall shear stresses and pressure drops, while the most important input is the fluid data, and in particular the fluid rheology and its dependency on pressure and temperature. Due to instabilities caused by the drill-string and variable surface roughness at the contacting walls, it is assumed that the flow is unstable or quasi-turbulent, even for low Reynolds numbers well below 1,000. Also, effects of rotation and heat transport are treated in the same fundamental manner. The effects of rotation are included by solving for the tangential velocity in addition to the axial velocity. It is shown that rotation may have a profound impact on the pressure drop.

The suggested methodology is expected to increase the precision of predictions and reduce the costs of lab testing and traditional curve fitting to analytical models. The model is verified against experimental data. For most situations, we obtained good agreement with data, as well as for laminar/turbulent transitional flows.

Finally, the paper indicates that traditional Fann readings, using 4 points, are inadequate for an accurate rheology determination. In order to improve the hydraulic model, more focus on rheology data is needed.

Introduction

An important aspect of hydraulic optimization is an accurate estimation of the equivalent circulating density (ECD), especially in long, narrow wells. For this purpose, several simulators for hydraulic calculations of drilling wells have been developed.

One frequent problem in such simulations is the quality of rheological data that affects the selection of the rheological model. A common approach is to let the user select a rheological model from a set of predefined rheologies. Another approach is to curve fit the data to a best fit with a selected rheology model. Denis and Guillot found that the Fann viscometer systematically seems to overpredict pressure losses. They tested one pipe viscometer and two rotational viscometers and recommended applying a high quality viscometer instead of the inaccurate Fann viscometer. In another work, Efaghi et al. had access to the training of mud in a 2 7/8 in., 3,000 ft. deep tubing, comparing Bingham and Power law rheological models. Laminar pressure losses were not predicted accurately since the viscometer shear rate was not representative of the actual laminar flow in the tubing.

Valuable data of turbulent flow of non-Newtonian fluids in rough pipes was recorded by Szilas, Bobok, and Navratil. They compared results for crude production in a 12 in. ID, 161 km long pipeline and obtained good correspondence with turbulent model predictions using direct analogy to non-Newtonian flows. Effect of rotation on pressure drop was studied by Walker and Al-Rawi. They recorded pressure losses of bentonite-water mixtures in laminar, helical flow, where the inner wall in the annulus rotates. At low shear rates, they also observed "gelling" of the fluid under steady state conditions. Under the actual laminar shear thinning conditions, they found that the pressure drop decays with increasing drill-string rotation. Their prediction methodology (laminar flow only) was similar to the model concepts presented in this paper. Both these latter works indicate that turbulence and drill-string rotation can be treated in a comprehensive manner.

Many people working with drilling operations still use their own "rules of thumb," even if they have access to various simulation tools. It is well known that for some situations, none of the simulators may give as much as a 50 bar error in downhole pressure. This was further supported by a survey conducted by Saga Petroleum. This work concluded that none of the four hydraulic simulators tested by Saga predicted results resembling ECD and only one gave an acceptable accuracy. This model was comprehensive and had accordingly a high user level. A common problem with all the existing simulators seems to be that combined effects of rheology, turbulence, and drill-string rotation is not treated in a consistent manner.

Based on this background, it was decided to develop a new approach to ECD simulation. The model should be based on a solid theoretical framework with a minimum of empirical input. In addition, the model should be easy to use and fast to operate.

Requirements of the New Model

In this new model concept, two important criteria should be fulfilled:

i) High accuracy in the simulated data; and,

ii) A user friendly interface, built on top of a robust equation solver.

The input and interpretation of the results should not need the assistance of an expert in the models or the program, and the
accuracy of the predicted pressures should be high with no need to adjust empirical model parameters.

The model is characterized as quasi two-dimensional as it solves for the coupled axial and tangential velocity profiles. A preliminary version of the model is implemented in the simulator and has the following capabilities:

- Computes velocity profiles and pressure drops without empirical correlations from flow-loop experiments. This can be done for any fluid and any flow situation. This feature is unique for the present model.
- Simultaneous solution of the axial and rotational flow due to drill-string rotation allows for prediction of coupled effects on pressure drop.
- Computes pressure distribution in drill-string and annulus.
- Adapts a simple turbulence model, which can deal with any rheology of the fluid.
- Local viscosities are computed over the entire flow cross-section. These local informations may be used for calculation of particle sedimentation velocities.
- Simple graphical representation of the simulated results.
- Steady state simulations.

**Description of the Model**

The model is mainly based on solving standard fluid mechanic transport equations. The concept is one dimensional in the way that we solve the velocity profiles over the cross-section of the pipe or the annulus. By predicting pressure drops in straight pipes, it is possible to simulate an expanding pipe diameter if the real pipe is replaced by a series of straight pipes with different cross-sections. It is assumed that errors due to local radial flow expansion is a second order effect regarding the pressure drop and the ECD. The details are elaborated below.

### Non-Newtonian Flow

The momentum balance provides a relationship between pressure drop and shear stress. Shear stress of a non-Newtonian fluid may, for developed flows in straight pipes, be expressed by:

\[
\tau_{ij} = \mu^{\text{app}}(\dot{\gamma}) \left( \frac{\partial}{\partial r} V_i + \frac{\partial}{\partial x_j} V_j \right) = \mu^{\text{app}}(\dot{\gamma}) \frac{\partial}{\partial x_j} V_j
\]

where the total strain rate \(\dot{\gamma}\) is defined by:

\[
\dot{\gamma} = \sqrt{2\dot{\gamma}_y \dot{\gamma}_y}; \quad \dot{\gamma}_y = \frac{1}{2} \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)
\]

The tensor notation in Equation (2) is retained due to a later incorporation of drill-string rotation. For pure axial, developed tube flow \(\tau_{rr} = \frac{\partial p}{\partial r}\). Further, we assume that the effective viscosity, \(\mu(\dot{\gamma})\), is known from experiments and is available in table form. In the case of a Power-law fluid, \(\mu(\dot{\gamma}) = K \cdot \dot{\gamma}^n\). If \(n < 1\), the fluid is a shear thinning fluid. In the case of a Bingham type fluid, we will have large, but finite, \(\mu(\dot{\gamma})\) at small \(\dot{\gamma}\) with \(\mu(\dot{\gamma})\) falling off at increasing \(\dot{\gamma}\). By numerical solution of the axial momentum equation and calculation of the flow rate, we can find the wall shear stress and the pressure drop in an iterative manner.

The effective viscosity \(\mu(\dot{\gamma})\) can be found from the experimental data for a real mud, as shown in Figure 1. In this case, the effective viscosity is found between experimental points by linear interpolation. A better technique may be interpolation by natural splines.

**Turbulent Flow**

In the case of turbulent flow, the axial momentum equation has to include the turbulent Reynolds stress \(\rho\nu'\nu'\):

\[
0 = -\frac{\partial p}{\partial x} + \rho g_x + \frac{\partial}{\partial r} \left[ r \tau_{rr} - r \rho \nu' \nu' \right] - \frac{\partial}{\partial r} \left[ r \tau_{rr} - r \rho \nu' \nu' \right]
\]

where, in this work, we have chosen to model the Reynolds stress by an isotropic turbulent viscosity \(\mu_t\):

\[
\rho \nu' \nu' = -\mu_t \frac{\partial}{\partial r} V_r
\]

We can now find the wall shear stress by solving Equation (3). However, first we need an acceptable model for the turbulent viscosity. Such a model was proposed by Johansen[10], who calibrated a simple eddy viscosity model with data from direct simulations of Newtonian turbulent flows. A simplified model version, which is sufficient to reproduce momentum transport, reads:

\[
\frac{\mu_t}{\rho} = \nu_t = \left\{ \begin{array}{ll}
\left( \frac{y^+}{11.4} \right)^2 & ; y^+ < 51.98 \\
0.4y^+ & ; y^+ \geq 51.98
\end{array} \right.
\]

where the dimensionless wall distance is given by:

\[
y^+ = y\sqrt{y^+} \nu_t
\]
Numerical Calculations and Turbulent Transitional Modification

If we solve Equation (3), employing Equations (4) to (6), we can compute a wall friction coefficient defined by:

\[ f = \frac{\frac{\partial p}{\partial x}}{\frac{1}{2} \rho V^2} \] ...................................................(10)

We can now solve the equation for different Reynolds numbers and compare the results with available data in Figure 2. In this computation, we use the turbulence model for all Reynolds numbers. In drilling operations, it is possible that the flow is turbulent even at very low \( Re \) due to imperfect walls. Hence, it may be possible to apply the turbulence model for all \( Re \). The consequence is larger pressure drops for \( Re \) between 200 and 10,000. This may turn out to be in good agreement with field conditions where the walls are rough and turbulence may exist, even at very low Reynolds numbers.

We see that the agreement between the analytic and the numerical solution is excellent for \( Re \) up to 100. For \( Re \) larger than 10,000, the numerical solution is in excellent agreement with the data compiled by Prandtl\(^{(11)}\). In the transition region, where \( Re \) range from 1,600 to 10,000, our model overpredicts the Prandtl results.

Transitional Modification of the Turbulent Viscosity

For wall shear rates close to the transition to turbulence, it is necessary to reduce the effect of turbulent momentum transport for low Reynolds numbers. This is done by converting the Reynolds number criterion to a wall strain-rate condition. We define the wall Reynolds number as:

\[ Re_{wall} = \frac{\rho V D}{\mu_{eff}} = \left( \frac{8V}{D} \right) \frac{D^2}{8\nu_{eff}} = \frac{S}{8\nu_{eff}} \] ...................................................(11)

where turbulence is damped out below some critical \( Re_{wall} \). The next step is to introduce some damping function \( \mu_{eff} \), which we want to use together with the fully turbulent viscosity [Equation (5)] to represent the Reynolds stress, by:

\[ \rho \nu_{eff} = -\mu_{eff} \frac{\partial \nu}{\partial r} \] ...................................................(12)

Our recommended model for \( f_{mu} \) is:

\[ f_{mu} = \left( 1.0 \cdot \exp \left( \text{amax1} [\hat{\gamma}_1/N_1, \hat{\gamma}_2/N_2] \right)^{gap^{*2}} / 2.4e4 \right) \] ...................................................(13)

In this model, \( \hat{\gamma} \) and \( \nu \) represent the strain-rate and effective laminar kinematic viscosity at walls 1 and 2, respectively. The gap is the tube radius, or the half gap width in the case of annular flow. The model translates a critical \( Re \) for transition to a critical \( \hat{\gamma} \) at the wall. Furthermore, the model smoothly damps the effect of turbulence for low Reynolds numbers.

As seen in Figure 3, the prediction compares more favourably with the Prandtl data for low \( Re \). This justifies the application of the transitional model, and this model is used further in this work. This means that \( \mu_{eff} \) is now replaced by \( \mu_{eff} = f_{mu} \mu_{0} \) Equation (5). However, turbulent transition is complex with effects of hysteresis, and will certainly depend on operational conditions and physical condition of the surfaces confining the fluid. The suggested model cannot cope with all these complexities, therefore it should be expected that the predicted pressure drop can both under- and over-estimate...
experimental data if we have transitional flows. It should be noted that real transitional flows can be intermittent and show periods with either laminar or turbulent flow.

Non-Newtonian Flows

In Figure 4, we see annular flow profiles of velocity and viscosity predicted by the model. Both the mud presented in Figure 1 and water were predicted. The water velocity profile is characteristically turbulent, while the mud profile looks more “laminar” due to its non-Newtonian rheology. We can further see the disadvantage with the linear interpolation of the rheology data shown in Figure 1. The mud viscosity distribution over the flow cross-section is now represented by a step-wise function. This is clearly seen from Figure 4. Even with our step-wise apparent mud viscosity representation, the velocity profile still appears as smooth. However, by employing spline interpolations, we can make the mud viscosity smooth.

Treatment of Wall Roughness

The idea of the inclusion of a wall roughness model into the simulator has been discussed in the literature\(^\text{12}\). In such models, the roughness is modelled as well-defined geometrical objects protruding from the wall, as shown in Figure 5. The protrusions may be seen as cylinders that are described by diameter, height, and area distribution. The cylinders are formally treated as a medium, which impose a drag force on the liquid. In this manner, very accurate models for surface roughness may be adopted. This type of modelling is easily implemented into our modelling concept. One for steel pipes (surface piping, drill string, casing, etc.) and one for the open hole wall. Hence, surface roughness effects can be included without empirical model input. The only requirement is that a description of the surface exists. By developing methods for automatic characterization of surfaces, it is believed that significant resources may be saved compared to full flow-loop investigations of pressure drops for each type of surface. Such a model may readily be implemented into the present formalism. Regardless, it remains to be investigated how important this mechanism is in the case of mud transport.

Treatment of Drill-String Rotation

It is known that drill-string rotation may have a significant impact on the pressure drops during pumping of drilling fluids. In laminar shear thinning flow, it may be expected that rotation will increase total shear, reduce viscosity, and thereby reduce the dynamic pressure drop. Experience indicates that the opposite often takes place. This can be explained both by re-dispersion of sedimented cuttings and turbulence in the annulus. Rotation will form secondary vortices and contribute to turbulence generation in the annulus. All enhancement of axial-radial mixing will contribute to the momentum transport, as is the case in the large majority of turbulent flows. It is reasonable to assume that by applying the turbulence model to the rotational velocity component, we may capture these phenomena to a large extent. Notice that Newtonian turbulent flow is always shear thickening, leading to an increased pressure drop due to rotation. If the flow is turbulent and the fluid is shear thinning, we may have either increased or reduced pressure drops caused by the rotation. So if we solve for the rotational component in addition to the axial, we may include the main features of these effects in a relatively fundamental way.

The transport equation for the rotational velocity component for steady state and developed flow can now be written as:

\[
0 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 V_{\text{eff}} \frac{\partial}{\partial r} \left( \frac{V_\theta}{r} \right) \right)
\]

FIGURE 4: Predicted radial distribution of water velocity, mud velocity, and the apparent “laminar” mud viscosity in an annulus. Average velocity is 0.8 m/s for both fluids.

FIGURE 5: Model for wall roughness interaction with the flow.

The consequence of the additional solution of Equation (14) is that the effective strain-rates and viscosities [Equations (1), (5), and (12)] are modified by the tangential velocity components. Hence, rotation will influence the axial pressure drops. However, for laminar developed Newtonian flows, such effects do not exist.

Typical predicted velocity profiles for a non-Newtonian turbulent flow are shown in Figure 6. Here the average axial mud velocity in the annulus is 0.4 m/s, while the rotational velocity of

FIGURE 6: Predicted axial and rotational velocity distribution across the annulus. The mud rheology is given in Figure 1.
the outer drill-string wall is 1.0 m/s, corresponding to 190 RPM. The effects on the pressure drops are seen in Figure 7, where the drill-string rotation is varied for three given flow situations (water at 10 l/s and mud at 10 and 50 l/sec, respectively).

As seen in Figure 7, rotation has a larger relative effect on pressure drop in water than for a typical mud with rheology, as shown in Figure 1. The water result is explained by the drill-string rotation, which increase turbulence and results in a thinner laminar sub-layer. For the mud flow rate of 50 l/sec, rotation increases the pressure drop, while for the mud flow rate of 10 l/sec, the pressure drop has a local minimum for a drill-string rotation at approximately 150 RPM.

Principles of the Simulator

The pressures along the flow path are simply calculated by using Equation (9), and start at the surface where the mud leaves the flow-loop. The principles of the calculation are illustrated in Figure 8. Numerically, we first solve the velocity profiles in each defined pipe section to obtain the wall shear stresses. Then, we compute the pressures in the flow-loop by the following algorithm (steady state):

\[
p_L = p_{L-1} + \Delta X \left( \rho_{L-1} \left( \frac{2R_1 \tau_{1w}}{R_2^2 - R_1^2} - 2R_1 \frac{\tau_{w}}{R_2^2 - R_1^2} \right) \right)
\]

where \( L \) is the index which starts with \( L = 1 \) in the annulus at the top of the site, and increase by one for each element (each 12.5 m long) down to the bit, continuing inside the drill-string until the last element is reached at the pump. Here, \( g_s \) is the component of gravity in the opposite flow direction and the element length \( \Delta X \) is positive. The gravity component \( g_s \) is positive in the annulus and negative inside the drill-string. Note that the wall shear stress \( \tau_{w} \) is negative and \( \tau_{1w} \) is positive or zero in the drill-string.

The depth \( Z_L \) from the pump to element \( L \) is given by:

\[
Z_L = \sum_{i=L}^{L-1} \Delta X \left( \frac{g_s}{r_i^{1.5}} \right)
\]

The local densities are computed from composition and total pressure.

Verification of Newtonian Turbulent Flow

The turbulent flow data of Skalle(13) are now compared to the model prediction. This is done to assure consistency with the non-Newtonian data used in the next section, which were recorded with the same experimental set-up(13). The flow rate in a 0.11 m diameter tube is varied, while the viscosity (\( \mu = 75 \cdot 10^{-4} \text{Pas} \)) was kept constant. As seen in Figure 9, the agreement between the model prediction and the experimental data is excellent. The Reynolds number varies between 10,000 and 450,000.

Verification of Laminar, Non-Newtonian Flow

For laminar, non-Newtonian flows, we first compared predictions and analytical solutions of powerlaw fluids. This gave identical results which are not presented here. In the following, we focus on Bingham fluids. The shear stress \( \tau \) in a Bingham fluid is characterized by:

\[
\tau = \tau_0 + \mu_{pl} \gamma \quad \text{...........................................}(17)
\]

where \( \tau_0 \) is the yield stress and \( \mu_{pl} \) is the plastic viscosity. This can be translated into an apparent viscosity:

\[
\mu_{app} = \frac{\tau_0}{\gamma} + \mu_{pl} \quad \text{...........................................}(18)
\]

One practical tool for the determination of non-Newtonian flows is the tube rheometer (also called the capillary viscometer). The momentum equation for a tube after primary integration is written as:

\[
\frac{\partial p}{\partial x} = \left( -\tau_0 \frac{\partial v}{\partial r} + \mu_{pl} \frac{\partial v}{\partial r} \right) \quad \text{...........................................}(19)
\]

The local densities are computed from composition and total pressure.
After one more integration, we obtain:

\[
V(r) = \begin{cases} 
-\frac{\tau_0}{\mu_p} (R-r) \frac{\partial p}{\partial x} \frac{R^2-r^2}{4\mu_p} & ; \ r < r_0 \\
-\frac{\tau_0}{\mu_p} (R-r_0) \frac{\partial p}{\partial x} \frac{R^2-r_0^2}{4\mu_p} & ; \ r \geq r_0 
\end{cases}
\]

where the radius \( r_0 \) is given by:

\[
r_0 = \frac{2 \tau_0}{\frac{\partial p}{\partial x}}
\]

If we introduce the global strain rate \( \dot{\gamma}_g \) defined by:

\[
\dot{\gamma}_g = \frac{V}{R} \frac{4}{\gamma}
\]

and the wall shear stress \( \tau_w \):

\[
\tau_w = \frac{\Delta p}{\Delta x} \frac{R}{2}
\]

We may integrate the velocity profile to obtain the average velocity. The result is:

\[
\bar{\tau}_w = \left( \dot{\gamma}_g \cdot \mu_p \right) + 4 \tau_0 \left( 1 - \frac{\tau_0}{\tau_w} \right)
\]

We see that we may easily determine the plastic viscosity \( \mu_p \) and the yield stress by plotting \( \tau_w \) vs. \( \dot{\gamma}_g \). However, it is required that \( \dot{\gamma}_g \) is lower than \( -0.3 \cdot \tau_w \) in order to avoid effects from the denominator in the equation above. In this case, the apparent yield stress is given by:

\[
\tau_{app} = \frac{4 \tau_0}{3}
\]

The analytical solution, represented by Equation (20), is compared to the numerical solution in Figure 10. We see that, except for a minor deviation in the centre of the pipe, the agreement is excellent. This confirms both the analytical and the numerical solution. The minor deviation in the velocity profiles in the tube centre is explained by a numerical switch in the code which limits the maximum viscosity. This makes the code more stable.

It is obvious that the tube rheometer may give important rheology information if the data is processed in a correct manner. It is further obvious that if the obtained data is processed by Equation (22), we can get a true expression for the rheology. Equation (24) will be a better alternative for a fluid with yield stress.

In Figure 11, we plotted experimental data from Barry (14) \( \tau = 15.9 \text{ Pa} + 2.515 \times 10^{-2} \gamma \) extracted by using Equation (22) to calculate the strain-rate, together with our inferred rheology \( \tau = 11.9 \text{ Pa} + 2.515 \times 10^{-2} \gamma \) which was recalculated by Equation (24) and is valid for \( \tau_w \) lower than \( -0.3 \cdot \tau_w \). Only the expression for our inferred rheology was applied in the numerical model. This was done, and it turned out, that the experimental pressure drops cannot be reproduced if we use \( \tau = 15.9 \text{ Pa} + 2.515 \times 10^{-2} \gamma \) as the experimental rheology. However, with our inferred rheology, the laminar model prediction reproduces the experimental data closely for \( \gamma > 300 \text{ s}^{-1} \).

The model predictions are influenced by very small turbulence effects and give slightly higher wall shear stresses. For \( \gamma < 300 \text{ s}^{-1} \), the predicted curve is below the experimental curve. This is explained by Equation (25) which shows that our inferred
rheology is slightly underestimated for $\dot{\gamma} < 500 \text{ s}^{-1}$. We clearly see that the traditional use of Equation (22) to calculate rheology can give significant errors for fluids with a non-zero yield stress.

Verification of Turbulent Non-Newtonian Flow

Application of the full model with turbulence is seen in Figure 12 where the experimental data of Skalle\(^{(13)}\) are compared to the laminar prediction and the standard turbulent model prediction. We see that the turbulent part compares nicely to the experiment. However, at low flow rates and laminar flow, the pressure drop cannot be reasonably well predicted. This indicates that the measured rheology may be different from the rheology present in the flow experiment. The rheology in all the experiments by Skalle\(^{(13)}\) had six experimental points. The exception was the rheology in Test 2, which consisted of 11 points. These data are used directly in our prediction.

The precision in the prediction is not quite satisfactory. We believe this is mainly due to problems with the rheology since the predicted pressure drop is below the experimental results in a large portion of the laminar region. In the experiments, the rheology did change by time (thixotropic) and this can explain the unexpected discrepancy for laminar flow.

In experiments with heavy oil, Barry\(^{(14)}\) investigated the pressure drops for different flow rates in a 2-inch diameter tube. The oil was well represented through Bingham rheology. However, we used Equation (24) to translate their experimental yield stress to a real yield stress. As seen in Figure 13, the experiments are in good agreement with the model prediction. For the three lower temperatures, where there is a noticeable yield stress, the model predicts the data well.

In Figure 14, we plot experimental results from Langlinais et al.\(^{(15)}\) for two different muds and an annular pipe dimension of 0.0308 m and 0.0167 m for the outer and inner radius, respectively. In this case, we have used the power law rheology $\tau = 0.0551 \cdot \dot{\gamma}^{0.828}$, derived by the authors based on their Fann measurements. Using this rheology, we find that there is excellent agreement between the model and the experiments. We have also extracted rheology data direct from the Fann readings in the paper. Two different curves fit their Fann data (Own 1 and Own 2) giving almost identical results, but not as good as the power law rheology suggested by the paper.

In Figure 15, we use the rheology taken from four Fann readings in the paper, fitted to a yield power law model. We find that the agreement between the model and the experiment is not as good as seen in Figure 14. This is interesting as the density and rheology are almost identical for Mud 4 and Mud 5. The Fann readings for the two fluids at $\theta_{600}$, $\theta_{200}$, and $\theta_{100}$ were 31.0, 18.0, 13.0, and 8.0 for Mud 4 and 31.0, 17.5, 12.5, and 7.0 for Mud 5, respectively. These differences are within the margin of experimental error. However, the rheology data extracted from these two data sets give as much as 10% difference in predicted pressure drop. This indicates that the precision and methodology for extracting rheology information is highly inadequate.

In Figure 16, we see a yield power law rheology fitted to the Fann data for Mud 4\(^{(15)}\) and we see that the fitted curve gives an excellent fit. However, this fit is different from the rheology given by the paper\(^{(15)}\). This illustrates the problems of rheology determination from four Fann readings.
Note also that when converting the shear stress dial readings from a Fann viscometer to engineering units (lb/100 ft$^2$), a factor of 1.06 is usually truncated to 1.0. Since the dial reading generally cannot be made very precise, the factor 1.06 is not considered significant.$^{16}$

Drill-String Rotation

Walker and Al-Rawi$^{8}$ performed several tests on how drill-string rotation affects the pressure drop in an annulus. The rheology was taken from log plots in their paper, therefore we expect some error in our inferred rheology. In Figure 17, we see a comparison of predictions and experiments for different flow rates and drill-string rotation.

We see in Figure 17 that the pressure drop is somewhat underpredicted while the effect of rotation is well preserved. The discrepancy found between model prediction and the experiments are believed to be caused by inadequate rheology control and information. This is further supported by Denis and Gulluit$^{5}$ who found that different equipment for rheology determination gives different results. Wall slippage was also found to be a problem for small geometries, which is the case for the experiments of Walker and Al-Rawi$^{8}$.

Next, we considered results from Hansen and Sterri$^{17}$ who determined the rheology and pressure drops in concentric and eccentric annuli in their own experiments. They used the commercial 3D CFD-code PHOENIX to compute the pressure drop. Our prediction, which is done with a simpler model, is almost identical. This helps verify that our model gives consistent results based on the actual rheology applied.

In Figure 18, we see our model prediction for rotation in a concentric annulus, while the experiments by Hansen and Sterri$^{17}$ for an eccentric annulus are shown in Figure 19. Our predicted pressure drops are larger, as expected. This is explained by the fact that a concentric annulus gives more effective boundary layers than an eccentric situation. Our model predicts very well the relative effect of rotation on the pressure drop. We repeat that rotation may also increase the pressure drop, especially for turbulent Newtonian flows. This is inherent in our model, as was demonstrated in Figure 7.

Discussion

Our simulator is capable of solving stationary situations and takes into account the most important parameters. Dynamic situations, such as change of mud and start-up, time dependent rheology, drill-string eccentricity, cavings, barite sagging, and gas cut mud are all important parameters in critical situations. The model framework presented allows for the inclusion of several of these effects in a fundamental manner. However, we are limited to axisymmetric geometries. Still, the simulator can be designed in a user-friendly manner. However, the more complex physics that are added to the model, the more work will be required by the user.

Presently, we believe that the most important missing parameter is calculation of mud temperature. Temperature can sometimes affect rheology significantly and is an important quantity which may influence the predicted effective mud weight (ECD), which should be incorporated in order to increase the precision of the simulator. We have found that heat transfer can be treated in a fundamental way in the existing framework, without slowing down the simulator significantly. Heat transfer may be treated in close analogy to the momentum transfer and, by solving for the cross-stream temperature distribution, the heat transfer coefficients may be predicted. One other future expansion is to exchange the standard correlations for pressure loss through the BHA with a more general friction loss description. In this manner, the predicted pressure drops through any type of BHA may be related directly to the fluid physical properties, including the rheology.

The simulator is implemented in a Java-based framework that can be operated under the most commonly-known operating systems. However, commercialization of the simulator has not yet been decided.
Conclusions

1. A numerical model for predicting laminar and turbulent pressure drops during pumping of drilling fluids has been proposed. The model can deal with both pipe flows and annular flows.

2. The model concepts presented above allow hydraulic simulations to be performed without empirical input, such as correlations for pressure drops and heat transfer coefficients.

3. The model can reproduce any analytical solution based on known rheology. In addition, the model can import directly any rheology which can be determined experimentally. This is hard to obtain and is, in some situations, impossible with analytical models.

4. Verification studies indicate that the model computes pressure drops well for all laminar and turbulent Newtonian flows.

5. We have computed a large number of flows with a great variety of mud rheologies. The study shows that the model can treat both laminar and turbulent non-Newtonian flows. For some situations, we get excellent agreement with data. This is also the case for transitional flows (laminar-turbulent transition regions), as seen in Figures 13 and 16.

6. It is expected that modelled pressure drops should be in close agreement with experimental data if the quality of the fluid data is good. In particular, the pressures in the annulus above the BHA should be reproduced with high accuracy because here the pressure is only controlled by fluid density and shear stress in the annulus.

7. The discrepancy between model predictions and some of the friction loss data is believed to be mainly due to poor accuracy of the rheology measurement and representation. The missing implementation of effects of rough walls is another source of imperfect predictions.

8. It was found that the rheology based on four different Fann rheometer readings is inadequate for accurate rheology measurement and representation.

9. The model effectively reproduces the pure effect of drilling string rotation on pressure drops.

10. In some of the experiments, history effects in the rheology are present. The role of such phenomena should be taken into account if the mud has significant history effects. Wall slippage should also be considered more closely. The fluid rheology has to be sufficiently accurate to avoid experimental uncertainty in order to have significant effects on the predicted frictional losses. Accordingly, a more accurate determination of fluid rheology is wanted. Such an improved rheology determination is crucial to further model verification and improvement.

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NOMENCLATURE

\( V \) = averaged velocity of flow cross section (m/s)
\( V_r \) = radial flow velocity (m/s), assumed \( V_r = 0 \)
\( V_o \) = rotational velocity (m/s)
\( x \) = axial position (m)
\( y \) = distance to closest wall (m)
\( y^+ \) = dimensionless wall distance \( y^+ = \frac{y V_o}{\nu} \)
\( \rho \) = fluid density (kg/m³)
\( \Delta r \) = radial grid spacing (m)
\( \tau \) = fluid shear stress (Pa)
\( \mu_t \) = turbulent viscosity (Pas)
\( \mu_{nf} \) = effective viscosity \( \mu_{nf} = \mu_{ns} + \mu_t \)
\( \nu \) = kinematic viscosity (m²/s)

Subscripts

1 = inner wall in annulus
2 = outer wall in annulus
w = wall
\( \tau_w \) = wall shear stress
\( \rho_w \) = wall fluid density
\( \rho_r \) = radius grid spacing (m)

\( \rho' \) = fluctuating velocity of fluid in radial direction (m/s)
\( V_r' \) = fluctuating velocity of fluid in radial direction (m/s)
\( \nu' \) = fluctuating velocity of fluid in axial direction (m/s)
\( V_x \) = axial velocity (m/s)

REFERENCES

8. WALKER, R.E. and AL-RAWI, O., Helical Flow of Bentonite slurries; paper SPE 3108, Annual Fall Meeting, Houston, TX, p. 11, October 4 – 7, 1970.
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