



**NTNU – Trondheim**  
Norwegian University of  
Science and Technology

Department of Petroleum Engineering and Applied Geophysics

## **Examination paper for TPG4160 Reservoir Simulation**

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**Examination time (from-to): 9:00-13:00**

**Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.**

**Other information:**

**Language: English**

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**Number of pages enclosed: 0**

**Checked by:**

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Date

Signature

**Question 1** (5 points)

This question relates to the Gulltopp Field project work.

- The reservoir pressure of Gulltopp was slowly declining before production start in 2008. Why?
- Was the amount of outflow from Gulltopp before production start in 2008 significant compared to the total produced volumes until 2014? Why or why not?
- What was the main driving mechanism during production of Gulltopp?
- What were the main uncertainties in the simulation results?
- Discuss briefly how oil recovery from Gulltopp could have been improved using alternative development strategies.

Solution

Any answer that shows that they have been working actively with the group project

**Question 2** (10 points)

List all steps and standard relationships/formulas involved in deriving partial differential flow equations for flow in porous media. Black-Oil, one-phase, one-dimensional, horizontal flow is sufficient.

Solution

- Starting with a mass balance for the control element:

$$\left\{ \begin{array}{l} \text{Mass into the} \\ \text{element at } x \end{array} \right\} - \left\{ \begin{array}{l} \text{Mass out of the} \\ \text{element at } x+Dx \end{array} \right\} = \left\{ \begin{array}{l} \text{Rate of change of mass} \\ \text{inside the element} \end{array} \right\}$$

- then substitute for the relationship between velocity and pressure

- Darcy's:  $u = -\frac{k}{\mu} \frac{\partial P}{\partial x}$ ,

- or Forchheimer's (for high velocity flow):  $-\frac{\partial P}{\partial x} = u \frac{\mu}{k} + \beta u^n$ ,

- or Brinkman's (applies to both porous and non-porous flow):

$$-\frac{\partial P}{\partial x} = u \frac{\mu}{k} - \mu \frac{\partial^2 u}{\partial x^2}.$$

- Thirdly introducing the relationship between porosity and pressure:

$$c_r = \left( \frac{1}{\phi} \right) \left( \frac{\partial \phi}{\partial P} \right)_T$$

- and finally the relationship between fluid density and pressure:

- Oil density:  $\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{So}}{B_o}$ , which for one-phase flow becomes:  $\rho_o = \frac{\text{constant}}{B_o}$

- Real gas equation:  $PV = nZRT$ , which leads to:  $\rho_g = \rho_{gS} \frac{P}{Z} \frac{Z_S}{P_S}$

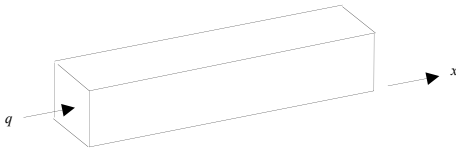
**Question 3** (10 points)

List all steps and standard approximations for converting a continuous partial differential equation to discrete form. Include a sketch of the continuous and the discrete (gridded) flow system. Include standard approximations needed for the simple diffusivity equation:

$$\frac{\partial^2 P}{\partial x^2} = \left(\frac{\phi\mu c}{k}\right) \frac{\partial P}{\partial t}$$

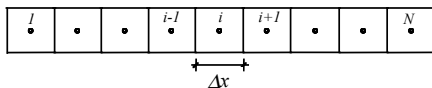
Solution

a) Continuous system:



$$\frac{\partial^2 P}{\partial x^2} = \left(\frac{\phi\mu c}{k}\right) \frac{\partial P}{\partial t}$$

b) Discrete system:



$$\frac{P_{i+1}^{t+\Delta t} - 2P_i^{t+\Delta t} + P_{i-1}^{t+\Delta t}}{\Delta x^2} \approx \left(\frac{\phi\mu c}{k}\right) \frac{P_i^{t+\Delta t} - P_i^t}{\Delta t}, \quad i = 1, \dots, N$$

c) Taylor approximations:

$$P(x, t) = P(x, t + \Delta t) + \frac{-\Delta t}{1!} P'(x, t + \Delta t) + \frac{(-\Delta t)^2}{2!} P''(x, t + \Delta t) + \dots$$

leading to:

$$\left(\frac{\partial P}{\partial t}\right)_i^{t+\Delta t} = \frac{P_i^{t+\Delta t} - P_i^t}{\Delta t} + O(\Delta t).$$

$$P(x + \Delta x, t + \Delta t) = P(x, t + \Delta t) + \frac{\Delta x}{1!} P'(x, t + \Delta t) + \frac{(\Delta x)^2}{2!} P''(x, t + \Delta t) + \dots$$

$$P(x - \Delta x, t + \Delta t) = P(x, t + \Delta t) + \frac{(-\Delta x)}{1!} P'(x, t + \Delta t) + \frac{(-\Delta x)^2}{2!} P''(x, t + \Delta t) + \dots$$

leading to:

$$\left(\frac{\partial^2 P}{\partial x^2}\right)_i^{t+\Delta t} = \frac{P_{i+1}^{t+\Delta t} - 2P_i^{t+\Delta t} + P_{i-1}^{t+\Delta t}}{(\Delta x)^2} + O(\Delta x^2).$$

**Question 4** (9 points)

The discretized form of the left hand side of the oil equation may be written in terms of transmissibility and pressure differences, as:

$$\frac{\partial}{\partial x} \left( \frac{k k_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right)_i \approx T_{x_{oi+1/2}} (P_{oi+1} - P_{oi}) + T_{x_{oi-1/2}} (P_{oi-1} - P_{oi})$$

Using the following transmissibility as example,

$$T_{x_{oi+1/2}} = \frac{2k_{i+1/2} \lambda_{oi+1/2}}{\Delta x_i (\Delta x_{i+1} + \Delta x_i)}$$

- What is the averaging method normally applied to absolute permeability between grid blocks ( $k_{i+1/2}$ )? Why? Write the expression for average permeability between grid blocks (i+1) and (i).
- Write an expression for the selection of the conventional upstream mobility term ( $\lambda_{oi+1/2}$ ) for use in the transmissibility term of the oil equation above for flow between the grid blocks (i+1) and (i).
- Make a sketch of a typical Buckley-Leverett saturation profile resulting from the displacement of oil by water (i.e., analytical solution). Then, show how the corresponding profile, if calculated in a numerical simulation model, typically is influenced by the choice of mobilities between the grid blocks (Sketch curves for saturations computed with upstream or average mobility terms, respectively).

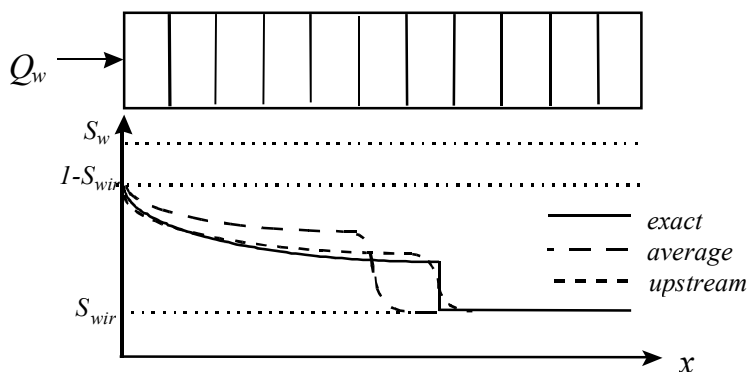
Solution

- Harmonic average is used because it properly represents flow in series across blocks of different permeabilities. It may be derived from Darcy's law (steady flow).

$$k_{i+1/2} = \frac{\Delta x_i + \Delta x_{i+1}}{\frac{\Delta x_i}{k_i} + \frac{\Delta x_{i+1}}{k_{i+1}}}$$

$$b) \lambda_{oi+1/2} = \begin{cases} \lambda_{oi+1} & \text{if } P_{oi+1} \geq P_{oi} \\ \lambda_{oi} & \text{if } P_{oi+1} < P_{oi} \end{cases}$$

c)



**Question 5** (12 points)

For two-phase flow of oil and water in a horizontal, one-dimensional porous medium, the flow equations can be written (including well terms):

$$\frac{\partial}{\partial x} \left( \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{rw}}{\mu_w B_w} \frac{\partial P_w}{\partial x} \right) - q'_w = \frac{\partial}{\partial t} \left( \frac{\phi S_w}{B_w} \right),$$

where

$$P_w = P_o - P_{cow}$$

$$S_o + S_w = 1$$

- Write the two flow equations on discretized forms in terms of transmissibilities, storage coefficients and pressure and saturation differences (**Do not derive**).
- List the assumptions for IMPES solution, and outline **briefly** how we solve for pressures and saturations
- What are the limitations of the IMPES solution?

Solution

$$a) \quad T^{xo_{i+1/2}}(P_{o_{i+1}} - P_{o_i}) + T^{xo_{i-1/2}}(P_{o_{i-1}} - P_{o_i}) - q'_{oi} = C_{poo_i}(P_{o_i} - P_{o_i}^t) + C_{swo_i}(S_{w_i} - S_{w_i}^t), \quad i = 1, N$$

$$T^{xw_{i+1/2}}[(P_{o_{i+1}} - P_{o_i}) - (P_{cow_{i+1}} - P_{cow_i})] + T^{xw_{i-1/2}}[(P_{o_{i-1}} - P_{o_i}) - (P_{cow_{i-1}} - P_{cow_i})] - q'_{wi} = C_{pow_i}(P_{o_i} - P_{o_i}^t) + C_{sww_i}(S_{w_i} - S_{w_i}^t), \quad i = 1, N$$

b) ALL DETAILS BELOW ARE NOT NEEDED, BUT A PROPER OUTLINE IS REQUIRED. In the IMPES solution, all coefficients and capillary pressures are evaluated at time=t. The two equations are combined so that the saturation terms are eliminated. The resulting equation is the pressure equation:

$$a_i P_{o_{i-1}} + b_i P_{o_i} + c_i P_{o_{i+1}} = d_i, \quad i = 1, N$$

which may be solved for pressures in all grid blocks by Gaussian Elimination Method, or some other method. Then, the saturations may be solved for explicitly by using one of the equations. Using the oil equation, yields:

$$S_{w_i} = S_{w_i}^t + \frac{1}{C_{swo_i}} [T^{xo_{i+1/2}}(P_{o_{i+1}} - P_{o_i}) + T^{xo_{i-1/2}}(P_{o_{i-1}} - P_{o_i}) - q'_{oi} - C_{poo_i}(P_{o_i} - P_{o_i}^t)] \quad , \quad i = 1, N$$

c) The approximations made in the IMPES method, namely the evaluation of coefficients at old time level when solving for pressures and saturations at a new time level, puts restrictions on the solution which sometimes may be severe. Obviously, the greatest implications are on the saturation dependent parameters, relative permeability and capillary pressure. These change rapidly with changing saturation, and therefore IMPES may not be well suited for problems where rapid variations take place.

IMPES is mainly used for simulation of field scale systems, with relatively large grid blocks and slow rates of change. It is normally not suited for simulation of rapid changes close to wells, such as coning studies, or other systems of rapid changes.

However, provided that time steps are kept small, IMPES provides accurate and stable solutions to a long range of reservoir problems.

**Question 6** (12 points)

For a one-dimensional, horizontal, 3-phase oil, water, gas system, the general flow equations are (including well terms):

$$\frac{\partial}{\partial x} \left( \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{rg}}{\mu_g B_g} \frac{\partial P_g}{\partial x} + R_{so} \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_g - R_{so} q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_g}{B_g} + R_{so} \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{rw}}{\mu_w B_w} \frac{\partial P_w}{\partial x} \right) - q'_w = \frac{\partial}{\partial t} \left( \frac{\phi S_w}{B_w} \right)$$

- Explain briefly the physical meaning of each term in all three equations.
- What are the criteria for **saturated** flow? What are the functional dependencies of  $R_{so}$  and  $B_o$ ?
- What are the primary unknowns when solving the **saturated** equations?
- What are the criteria for **undersaturated** flow? What are the functional dependencies of  $R_{so}$  and  $B_o$ ?
- What are the primary unknowns when solving the **undersaturated** equations?
- Rewrite the equations above for **undersaturated** flow conditions.

Solution

$$\begin{aligned} \text{a) } & \frac{\partial}{\partial x} \left( \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right) \\ & \text{transport of oil} \quad \text{oil prod.} \quad \text{accumulation of oil} \\ & \frac{\partial}{\partial x} \left( \frac{kk_{rg}}{\mu_g B_g} \frac{\partial P_g}{\partial x} + R_{so} \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_g - R_{so} q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_g}{B_g} + R_{so} \frac{\phi S_o}{B_o} \right) \\ & \text{transport of free gas} \quad \text{transport of sol. gas} \quad \text{free gas prod.} \quad \text{sol. gas prod.} \quad \text{accumulation of free gas} \quad \text{accumulation of solution gas} \\ & \frac{\partial}{\partial x} \left( \frac{kk_{rw}}{\mu_w B_w} \frac{\partial P_w}{\partial x} \right) - q'_w = \frac{\partial}{\partial t} \left( \frac{\phi S_w}{B_w} \right) \\ & \text{transport of water} \quad \text{water prod.} \quad \text{accumulation of water} \end{aligned}$$

- Saturated flow: criteria  $P_o = P_{bp}$  and  $S_g > 0$ .  
dependencies  $B_o = f(P_o)$  and  $R_{so} = f(P_o)$ .
- $P_o$  and  $S_g$
- Undersaturated flow: criteria  $P_o > P_{bp}$  and  $S_g = 0$ .  
dependencies  $B_o = f(P_o, P_{bp})$  and  $R_{so} = f(P_{bp})$ .
- $P_o$  and  $P_{bp}$

$$\text{f) } \frac{\partial}{\partial x} \left( \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_o = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial x} \left( R_{so} \frac{kk_{ro}}{\mu_o B_o} \frac{\partial P_o}{\partial x} \right) - q'_g - R_{so} q'_o = \frac{\partial}{\partial t} \left( R_{so} \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{rw}}{\mu_w B_w} \frac{\partial P_w}{\partial x} \right) - q'_w = \frac{\partial}{\partial t} \left( \frac{\phi S_w}{B_w} \right)$$

**Question 7** (12 points)

For two-phase flow (constant flow area) the right hand side of the oil and gas equations may be written (saturated oil case):

$$\frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

$$\frac{\partial}{\partial t} \left( \frac{\phi S_g}{B_g} + R_{so} \frac{\phi S_o}{B_o} \right)$$

The corresponding discretized forms are:

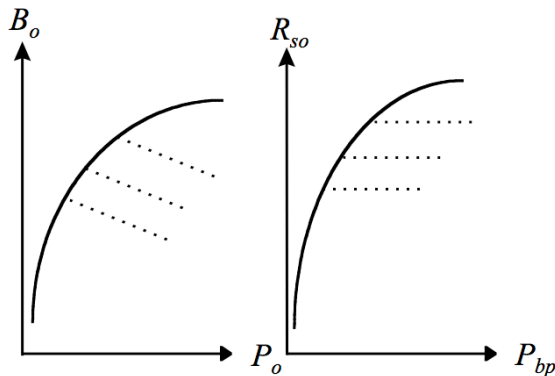
$$C_{poo_i} (P_{o_i} - P_{o_i}^t) + C_{sgo_i} (S_{g_i} - S_{g_i}^t)$$

$$C_{pog_i} (P_{o_i} - P_{o_i}^t) + C_{sgg_i} (S_{g_i} - S_{g_i}^t)$$

- Sketch typical curves that show the pressure dependencies of  $B_o$ ,  $B_g$ ,  $R_{so}$ .
- Show the complete derivations of the four coefficients  $C_{poo_i}$ ,  $C_{sgo_i}$ ,  $C_{pog_i}$ ,  $C_{sgg_i}$ .

Solution

a)



b) Students should show complete derivations

$$C_{poo} = \frac{\phi_i (1 - S_{g_i})}{\Delta t} \left[ \frac{c_r}{B_o} + \frac{d(1/B_o)}{dP_o} \right]_i$$

$$C_{sgo} = -\frac{\phi_i}{B_{oi} \Delta t}$$

$$C_{pog} = \frac{\phi_i}{\Delta t} \left[ S_g \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dP_g} \right) + R_{so} (1 - S_g) \left( \frac{c_r}{B_o} + \frac{d(1/B_o)}{dP_o} \right) + \frac{(1 - S_g)}{B_o} \frac{dR_{so}}{dP_o} \right]_i$$

$$C_{sgg} = \frac{\phi_i}{\Delta t} \left[ S_g \left( \frac{c_r}{B_g} + \frac{d(1/B_g)}{dP_g} \right) \frac{dP_{cog}}{dS_g} - \frac{R_{so}}{B_o} + \frac{1}{B_g} \right]_i$$

**Question 8** (10 points)

Normally, we use either a *Black Oil* fluid description or a *compositional* fluid description in reservoir simulation.

- What are the *components* and the *phases* used in *Black Oil* modeling?
- What are the *components* and the *phases* used in *compositional* modeling?
- Write the standard flow equations for the components required for *Black Oil* modeling (one dimensional, horizontal, constant flow area).
- Write the standard flow equations the components required for *compositional* modeling (one dimensional, horizontal, constant flow area). Let  
 $C_{kg}$  = mass fraction of component  $k$  present in the gas phase  
 $C_{ko}$  = mass fraction of component  $k$  present in the oil phase.
- A *Black Oil* fluid description may be regarded as a subset of a *compositional* fluid description. Define the pseudo-components required in order to reduce the *compositional* equations to *Black Oil* equations (one-dimensional, horizontal, constant flow area)

Solution

- Components:** oil and gas, **phases:** oil and gas
- Components:** hydrocarbons ( $C_1H_4, C_2H_6, C_3H_8, \dots$ ) and non-hydrocarbons ( $CO_2, H_2S, C_2, \dots$ ), **phases:** oil and gas

c)

$$\frac{\partial}{\partial x} \left( \frac{kk_{rg}}{B_g \mu_g} \frac{\partial P_g}{\partial x} + \frac{kk_{ro} R_{so}}{B_o \mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + \frac{S_o R_{so}}{B_g} \right) \right]$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{ro}}{B_o \mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

d)

$$\frac{\partial}{\partial x} \left( C_{kg} \rho_g \frac{kk_{rg}}{\mu_g} \frac{\partial P_g}{\partial x} + C_{ko} \rho_o \frac{kk_{ro}}{\mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left[ \phi (C_{kg} \rho_g S_g + C_{ko} \rho_o S_o) \right], \quad k = 1, N_c$$

e)

The *Black Oil* model may be considered to be a *pseudo-compositional* model with two components. Define the components and the fractions needed to convert the compositional equations to Black-Oil equations.

*component 1: oil – k=o*

$$\frac{\partial}{\partial x} \left( C_{og} \rho_g \frac{kk_{rg}}{\mu_g} \frac{\partial P_g}{\partial x} + C_{oo} \rho_o \frac{kk_{ro}}{\mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left[ \phi (C_{og} \rho_g S_g + C_{oo} \rho_o S_o) \right]$$

*component 2: gas – k=g*



$$\frac{\partial}{\partial x} \left( C_{gg} \rho_g \frac{kk_{rg}}{\mu_g} \frac{\partial P_g}{\partial x} + C_{go} \rho_o \frac{kk_{ro}}{\mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left[ \phi (C_{gg} \rho_g S_g + C_{go} \rho_o S_o) \right]$$

*Question: what are the fractions needed to get the Black Oil equations:*

$$\frac{\partial}{\partial x} \left( \frac{kk_{rg}}{B_g \mu_g} \frac{\partial P_g}{\partial x} + \frac{kk_{ro} R_{so}}{B_o \mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + \frac{S_o R_{so}}{B_g} \right) \right]$$

$$\frac{\partial}{\partial x} \left( \frac{kk_{ro}}{B_o \mu_o} \frac{\partial P_o}{\partial x} \right) = \frac{\partial}{\partial t} \left( \frac{\phi S_o}{B_o} \right)$$

*Answer:*

fraction of "gas in gas":  $C_{gg} = 1$

fraction of "oil in gas":  $C_{og} = 0$

fraction of "gas in oil":  $C_{go} = \frac{\rho_g S R_{so}}{\rho_o B_o}$

fraction of "oil in oil":  $C_{oo} = \frac{\rho_o S}{\rho_o B_o}$

### **Question 9** (10 points)

Normally, we use either a *conventional single porosity* model or a *fractured* dual porosity model in simulation of a reservoir.

- Describe the main differences between a *conventional* reservoir and a *fractured* reservoir, in terms of the physics of the systems.
- How can we identify a fractured reservoir from standard reservoir data?
- Explain **briefly** the primary concept used in deriving the flow equations for a dual-porosity model.
- Write the basic equations (one-phase, one-dimension) for
  - a two-porosity, two-permeability system
  - a two-porosity, one-permeability system
- In terms of the physics of reservoir flow, what is the key difference between the two formulations in question d)?
- How is the fluid exchange term in the flow equations in question d) represented? What are the shortcomings of this representation?

### Solution

- Conventional: One porosity, one permeability system, with one flow equation for each component flowing.  
Fractured: Two porosities, two permeabilities system, with most of the fluids in the matrix system, and most of the transport capacity in the fracture system. Requires two flow equations for each component flowing.
- $K_{core} \ll K_{welltest}$
- The matrix system supplies fluids to the fracture system, by whatever mechanisms present (depletion, gravity drainage, imbibition, diffusion,...), and the fracture system

transports the fluids to the wells. Some transport may also occur in the matrix system, from block to block, provided that there is sufficient contact.

- d) Dual porosity, dual permeability model:

$$\frac{\partial}{\partial x} \left( \frac{k}{\mu B} \frac{\partial P}{\partial x} \right)_f + q'_{mf} = \frac{\partial}{\partial t} \left( \frac{\phi}{B} \right)_f$$

$$\frac{\partial}{\partial x} \left( \frac{k}{\mu B} \frac{\partial P}{\partial x} \right)_m - q'_{mf} = \frac{\partial}{\partial t} \left( \frac{\phi}{B} \right)_m$$

One porosity, one permeability model (fracture eqn.):

$$\frac{\partial}{\partial x} \left( \frac{k}{\mu B} \frac{\partial P}{\partial x} \right)_f + q'_{mf} = \frac{\partial}{\partial t} \left( \frac{\phi}{B} \right)_f$$

Conventional fracture models represent the exchange term by

$$-q'_{mf} = \sigma \lambda (P_m - P_f)$$

where  $\sigma$  is a geometric factor,  $\lambda$  is the mobility term, and  $P_m$  and  $P_f$  represent matrix and fracture pressures, respectively.

- e) In the dual porosity, dual permeability model, fluid flow may occur from one matrix block to another. In the one porosity, one permeability model, all flow occurs in the fracture system
- f) The exchange term is conventionally defined as

$$-q'_{mf} = \sigma \lambda (P_m - P_f)$$

where  $\sigma$  is a geometric factor,  $\lambda$  is the mobility term, and  $P_m$  and  $P_f$  represent matrix and fracture pressures, respectively.

Obviously, this term cannot adequately represent the flow mechanisms present, such as depletion, gravity drainage, imbibition, diffusion, ... In addition, an average pressure for the matrix block is used in the expression, so that pressure gradients inside the block are not accounted for.

**Question 10** (10 points)

For a one-dimensional, vertical ( $z$ ), 3 phase oil, water, gas system, outline how initial pressures and saturations may be computed in a simulation model, assuming that equilibrium conditions apply:

- Sketch the reservoir, with a grid superimposed, including gas-oil-contact (GOC) and water-oil-contact (WOC).
- Sketch the oil-gas and oil-water capillary pressure curves, and show how the initial equilibrium pressures and saturations are determined in the continuous system.
- Sketch the initial saturations as they are applied to the grid blocks.

Solution

