NTNU – Trondheim
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Department of Petroleum Engineering and Applied Geophysics

SOLUTION

Examination paper for TPG4160 Reservoir Simulation

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Examination date: June 5, 2015

Examination time (from-to): 0900-1300

Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

Language: English

Number of pages: 5

Number of pages enclosed: 0

Checked by:

Date Signature

Question 1 (12 points)

a) Make sketches of *Black Oil* fluid properties B_o, B_g, B_w, R_{so}, μ_o, μ_g, μ_w. Label bubble point pressure, and saturated and undersaturated regions.

b) Express reservoir densities for the three fluids in terms of B_o, B_g, B_w, R_{so}, ρ_{oS}, ρ_{gS}, ρ_{wS}.

c) Express the density of the part of the reservoir oil that remains liquid at the surface.

d) Express the density of the part of the reservoir oil that becomes gas at the surface.

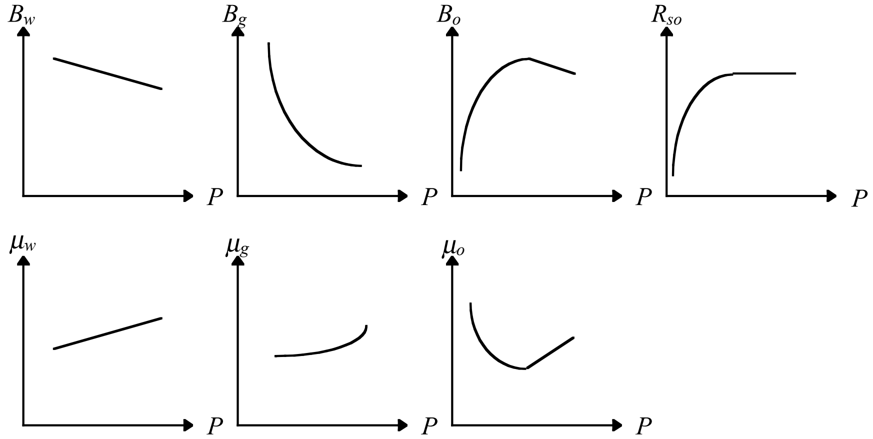
e) Express the gas density using real gas equation.

f) Write the definition for fluid compressibility.

g) Write an expression for pore compressibility

Solution

a)



b)

$$\rho_o = \frac{\rho_{oS} + \rho_{gs} R_{so}}{B_o}$$

$$\rho_g = \frac{\rho_{gS}}{B_g}$$

$$\rho_w = \frac{\rho_{wS}}{B_w}$$

c) $$\rho_{oL} = \frac{\rho_{oS}}{B_o}$$

d) $$\rho_{oG} = \frac{\rho_{gs} R_{so}}{B_o}$$

e) $PV = nZRT$.

$$\Rightarrow \rho_g = \rho_{gS} \frac{P}{Z} \frac{Z_S}{P_S}$$

f) $$c_f = -(\frac{1}{V})(\frac{\partial V}{\partial P})_T$$

g) $$c_r = (\frac{1}{\phi})(\frac{\partial \phi}{\partial P})_T$$

Question 2 (12 points)

For a completely water-wet system, make sketches of saturation functions (including labels for important points/areas)

a) Oil-water system: imbibition and drainage k_{rw}, k_{row}, P_{cow} vs. S_w

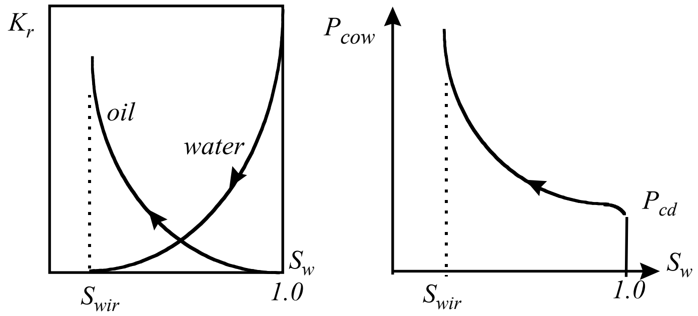
b) Oil-gas system: imbibition and drainage k_{rg}, k_{rog}, P_{cog} vs. S_g

c) Typical contours of three-phase k_{ro} in a ternary (triangular) diagram (axes S_o, S_w, S_g)

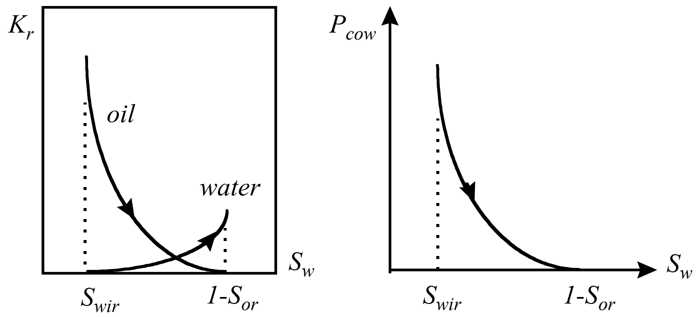
Solution

a)

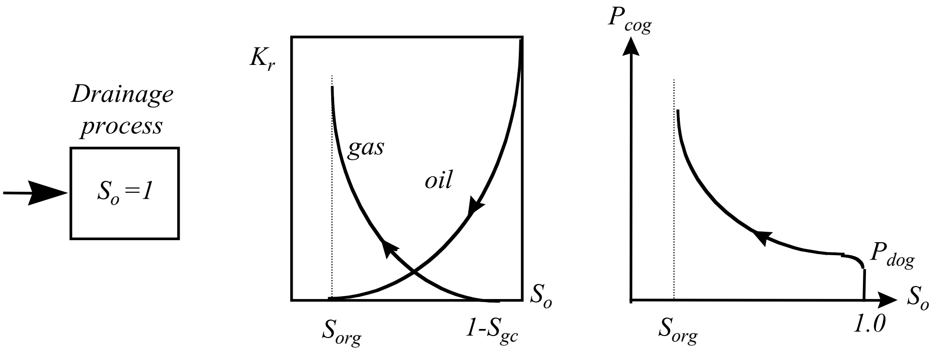
Drainage curves

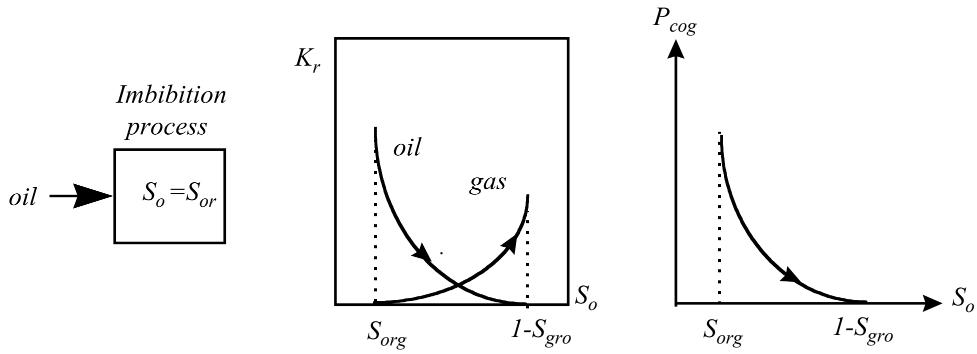


Imbibition curves

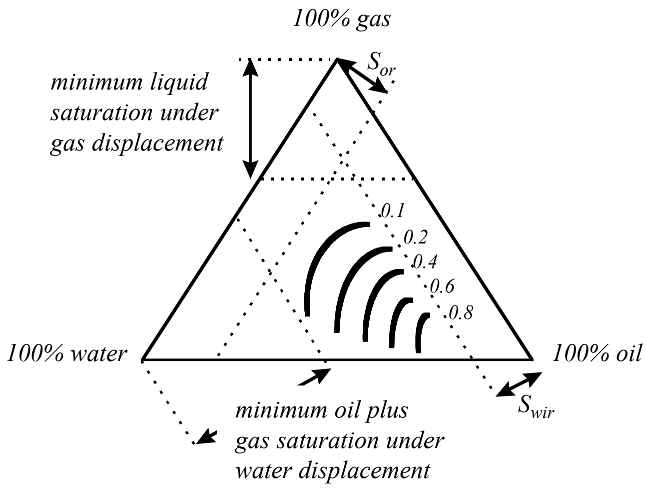


b)





c)



Question 3 (8 points)
Explain briefly the following terms as applied to reservoir simulation (short sentence and/or a formula for each):

a) Control volume
b) Mass balance
c) Taylor series
d) Numerical dispersion
e) Explicit
f) Implicit
g) Stability
h) Upstream weighting
i) Variable bubble point
j) Harmonic average
k) Transmissibility
l) Storage coefficient
m) Coefficient matrix
n) IMPES
o) Fully implicit
p) Cross section
q) Coning
r) PI
s) Stone´s relative permeability models
t) Discretization

u) History matching
v) Prediction
w) Black Oil
x) Compositional
y) Dual porosity
z) Dual permeability

Solution

a) Control volume **small volume used in derivation of continuity equation**
b) Mass balance **principle applied to control volume in derivation of continuity equation**
c) Taylor series **expansion formula used for derivation of difference approximations (or formula:** $f(x+h)=f(x)+\frac{h}{1!}f'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+....$**)**
d) Numerical dispersion **error term associated with finite difference approximations derived by use of Taylor series**
e) Explicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t**
f) Implicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t+Dt**
g) Stability **as applied to implicit and explicit discretization of diffusivity equation: explicit form is conditional stable for** $\Delta t \le \frac{1}{2}\frac{\phi\mu c}{k}(\Delta x)^2$, **while implicit form is unconditionally stable**
h) Upstream weighting **descriptive term for the choice of mobility terms in transmissibilities**
i) Variable bubble point **term that indicates that the discretization og undersaturated flow equation includes the possibility for bubble point to change, such as for the case of gas injection in undersaturated oil**
j) Harmonic average **averaging method used for permeabilities when flow is in series**
k) Transmissibility **flow coefficient in discrete equations that when muliplied with pressure difference between grid blocks yields flow rate.**
l) Storage coefficient **flow coefficient in discrete equations that when muliplied with pressure change or saturation change in a time step yields mass change in grid block**
m) Coefficient matrix **the matrix of coefficient in the set of linear equations**
n) IMPES **an approximate solution method for two or three phase equations where all coefficients and capillary pressures are computed at time level of previous time step when generating the coefficient matrix**
o) Fully implicit **an solution method for two or three phase equations where all coefficients and capillary pressures are computed at the current time level generating the coefficient matrix. Thus, iterations are required on the solution.**
p) Cross section **an x-z section of a reservoir**
q) Coning **the tendency of gas and water to form a cone shaped flow channel into the well due to pressure drawdown in the close neigborhood.**
r) PI **the productivity index of a well**

s) Stone´s relative permeability models **methods for generating 3-phase relative permeabilities for oil based on 2-phase data**
t) Discretization **converting of a contineous PDE to discrete form**
u) History matching **in simulation the adjustment of reservoir parameters so that the computed results match observed data.**
v) Prediction **computing future performance of reservoir, normally following a history matching.**
w) Black Oil **simplified hydrocarbon description model which includes two phases (oil, gas) and only two components (oil, gas), with mass transfer between the components through the solution gas-oil ratio parameter.**
x) Compositional **detailed hydrocarbon description model which includes two phases but N components (methane, ethane, propane, ...).**
y) Dual porosity **denotes a reservoir with two porosity systems, normally a fractured reservoir**
z) Dual permeability **denotes a reservoir with two permeabilities (block-to-block contact) in addition to two porosities, normally a fractured reservoir**

Question 4 (12 points)

The discretized form of the left hand side of the oil equation may be written in terms of transmissibility and pressure differences, as:

$$\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)_i \approx T_{xoi+1/2}(P_{oi+1}-P_{oi})+T_{xoi-1/2}(P_{oi-1}-P_{oi})$$

Using the following transmissibility as example,

$$T_{xoi+1/2}=\frac{2k_{i+1/2}\lambda_{oi+1/2}}{\Delta x_i\left(\Delta x_{i+1}+\Delta x_i\right)}$$

a) What is the averaging method normally applied to absolute permeability between grid blocks ($k_{i+1/2}$)? Why? Write the expression for average permeability between grid blocks (i+1) and (i).
b) Write an expression for the selection of the conventional upstream mobility term ($\lambda_{oi+1/2}$) for use in the transmissibility term of the oil equation above for flow between the grid blocks (i+1) and (i).
c) Make a sketch of a typical Buckley-Leverett saturation profile resulting from the displacement of oil by water (i.e., analytical solution). Then, show how the corresponding profile, if calculated in a numerical simulation model, typically is influenced by:
d) choice of mobilities between the grid blocks (Sketch curves for saturations computed with upstream or average mobility terms, respectively).
e) time step size
f) capillary pressure

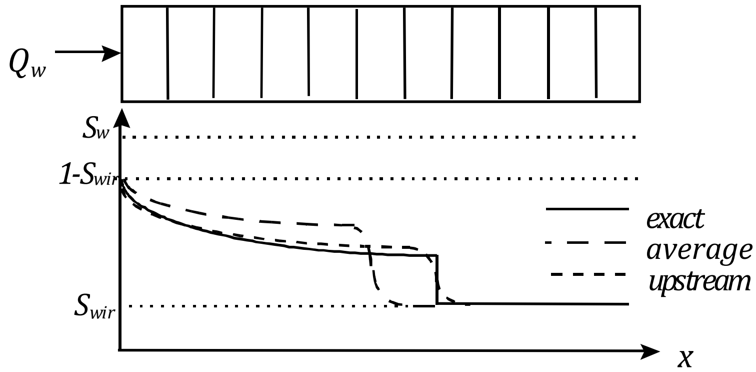
Solution

a) Harmonic average is used because it properly represents flow in series across blocks of different permeabilities. It may be derived from Darcy's law (steady flow).

$$k_{i+1/2} = \frac{\Delta x_i + \Delta x_{i+1}}{\dfrac{\Delta x_i}{k_i} + \dfrac{\Delta x_{i+1}}{k_{i+1}}}$$

b) $\lambda_{o_{i+1/2}} = \begin{cases} \lambda_{o_{i+1}} & if \;\; P_{o_{i+1}} \geq P_{o_i} \\ \lambda_{o_i} & if \;\; P_{o_{i+1}} < P_{o_i} \end{cases}$

c)
d)



e) On their sketch: the smaller time step, the closer to the exact solution
f) On their sketch: capillary pressure will give dispersion at the front, and a minor deviation from the exact solution behind the front

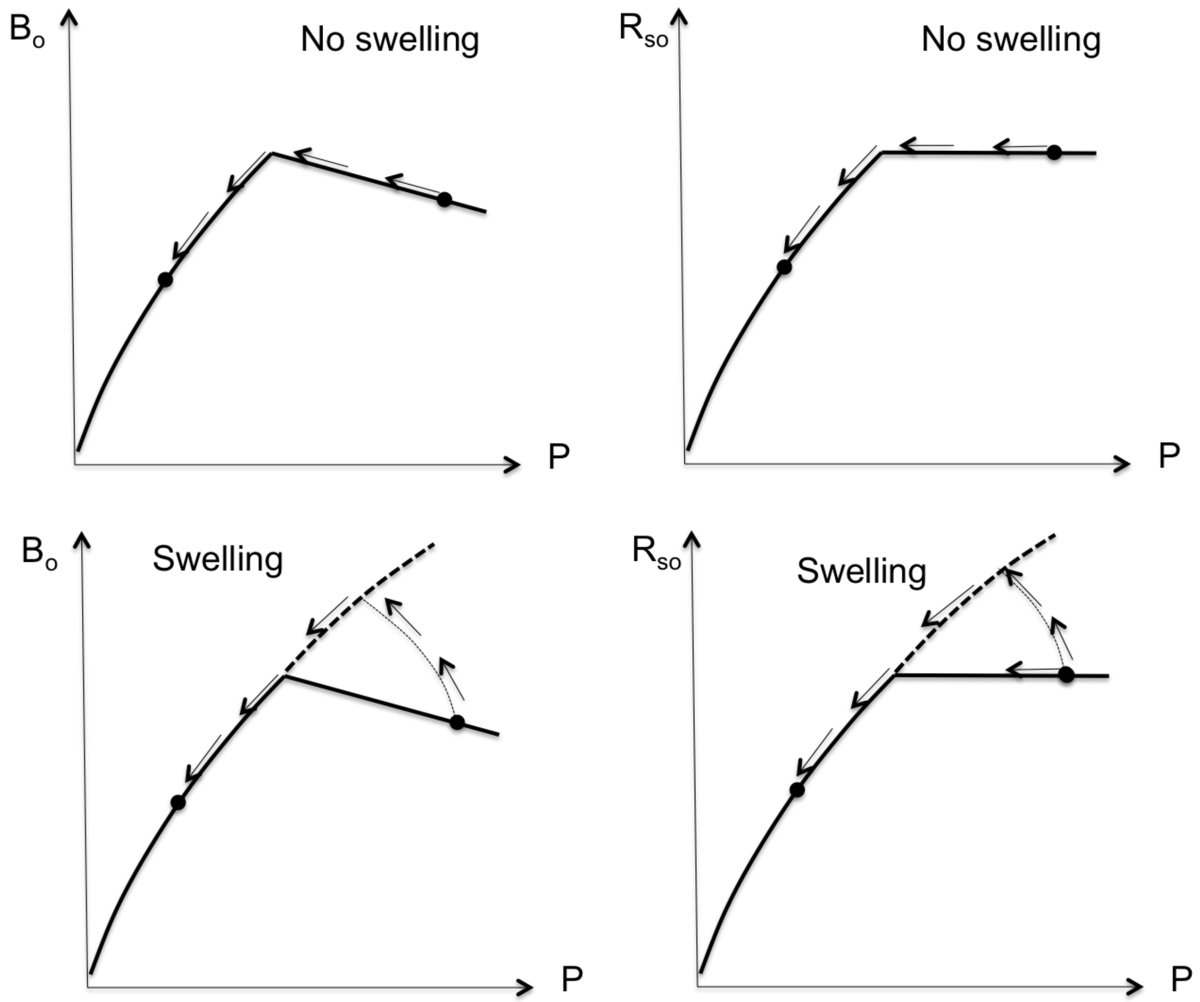
Question 5 (6 points)

In the Exercise 3, the effect of gas injection in under-saturated oil is investigated. Two different cases (swelling and no-swelling case) are compared.

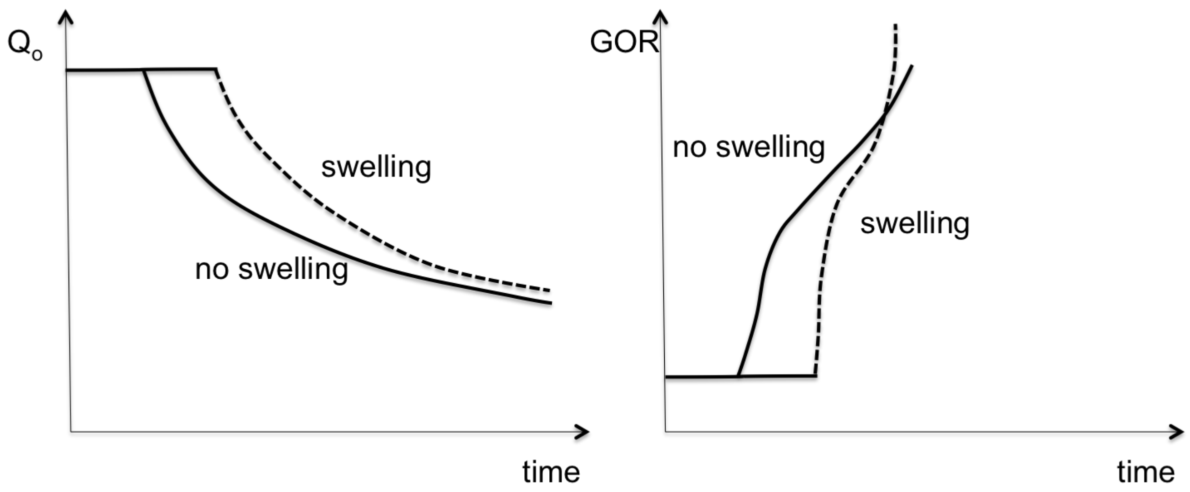
a) What is the difference between these two cases in terms of R_{so} (Explain with plots of B_o and R_{so} vs. pressure).
b) How do these two cases influence the GOR and field oil production (Explain with plots of GOR and field oil production vs. time).

Solution

a)


B_o
No swelling
P
R_{so}
No swelling
P
B_o
Swelling
P
R_{so}
Swelling
P

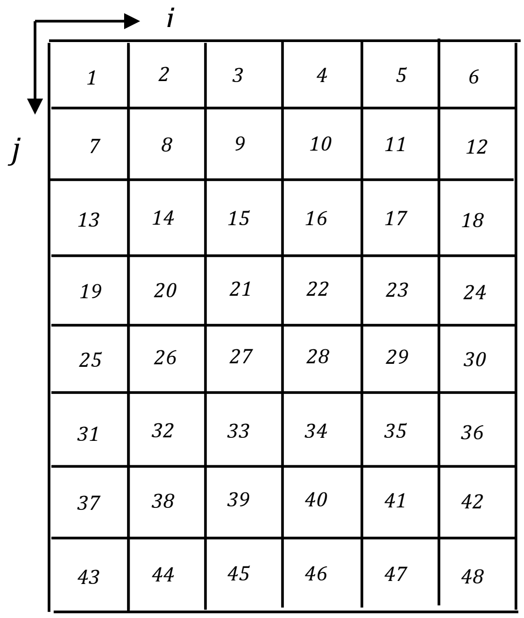
b)


Q_o
swelling
no swelling
time
GOR
no swelling
swelling
time

Question 6 (5 points)
For two-dimensional (x,y), one phase flow, the pressure equation is:

$$e_{i,j}P_{i,j-1} + a_{i,j}P_{i-1,j} + b_{i,j}P_{i,j} + c_{i,j}P_{i+1,j} + f_{i,j}P_{i,j+1} = d_{i,j} \qquad i = 1,N_x, j = 1,N_y$$

applicable to the following grid system:



List the block number for all gridblocks where:

$a_{i,j} = 0$
$c_{i,j} = 0$
$e_{i,j} = 0$
$f_{i,j} = 0$

Solution

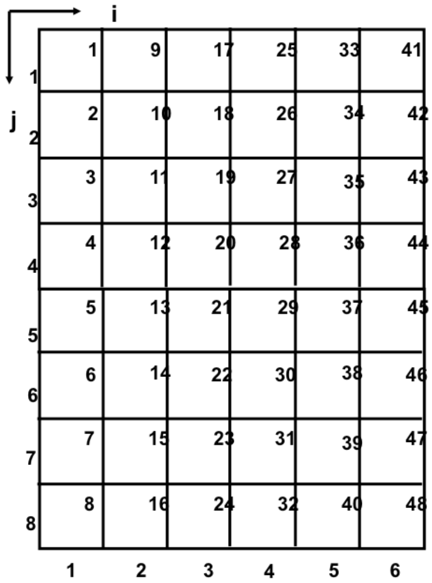
$a_{i,j} = 0$: 1,7,13,19,25,31,37,43
$c_{i,j} = 0$: 6,12,18,24,30,36,42,48
$e_{i,j} = 0$: 1,2,3,4,5,6
$f_{i,j} = 0$: 43,44,45,46,47,48

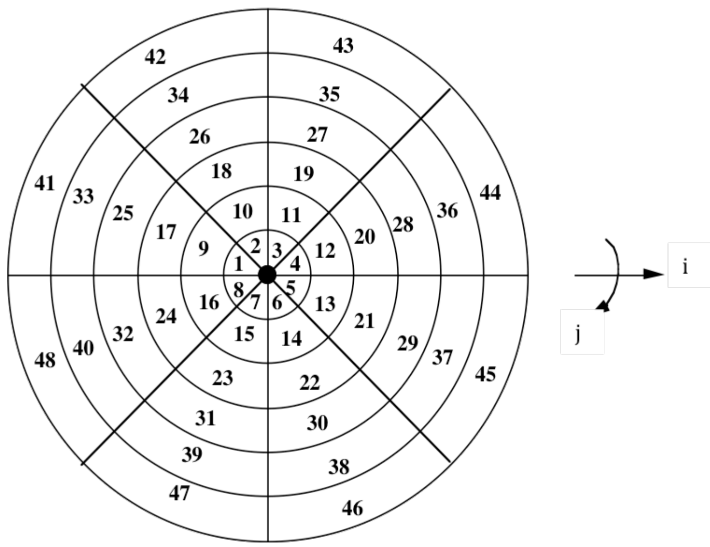
Question 7 *(10 points)*

Below are grids for two 6 by 8 grids, one in x-y rectangular coordinates, and one in r-Q cylindrical coordinates. The pressure equation for one-phase flow for both systems is:

$$e_{i,j}P_{i,j-1} + a_{i,j}P_{i-1,j} + b_{i,j}P_{i,j} + c_{i,j}P_{i+1,j} + f_{i,j}P_{i,j+1} = d_{i,j} \quad i = 1,...,N_1, \quad j = 1,...,N_2$$

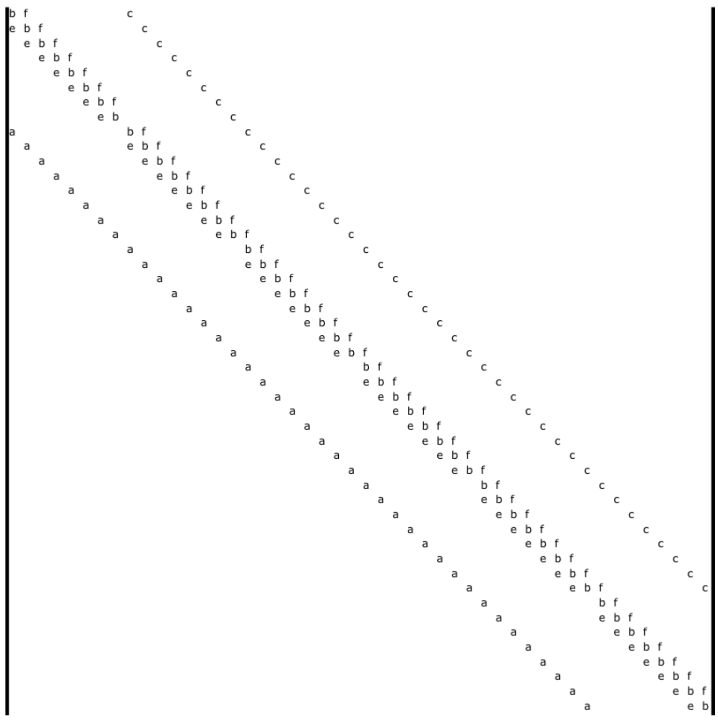
Sketch the coefficient matrix for both systems (approximately, with lines and the appropriate coefficient name). Show the key differences between the two on the cylindrical coefficient matrix.



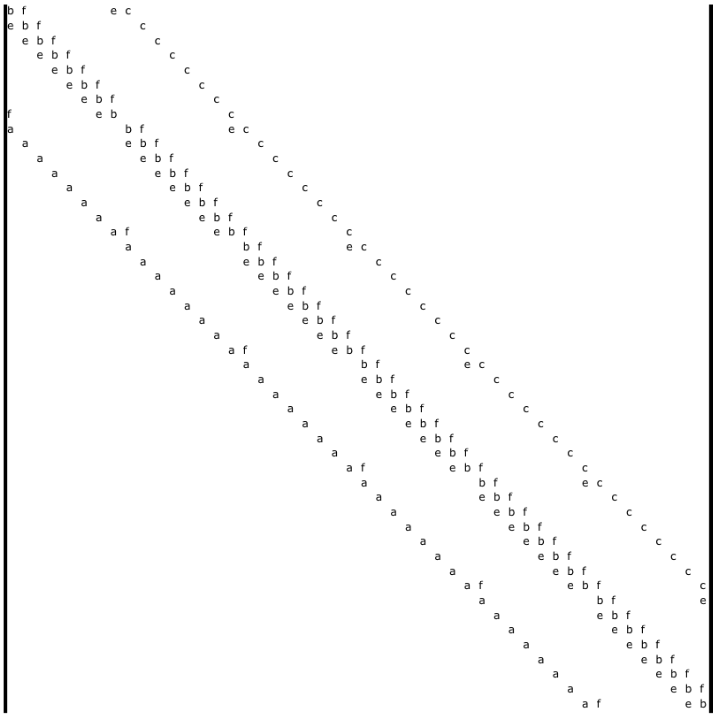


Solution

Coefficient matrix for rectangular system



Coefficient matrix for cylindrical system. Key difference from rectangular system are the connections between blocks where j=1 with blocks where j=8



Question 8 (16 points)

The pressure equation for single-phase slightly compressible flow is

$$TP^{t+\Delta t} = B$$

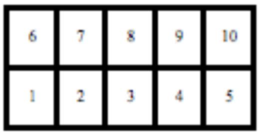
Where T is the transmissibility matrix. For an n-cell simulation the pressure vector is

$$P^{t+\Delta t} = \begin{pmatrix} P_1 \\ . \\ . \\ . \\ . \\ P_n \end{pmatrix}^{t+\Delta t}$$

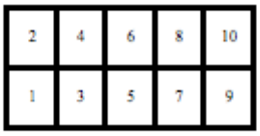
In the following you are to show the structure of T for cases a – d below. Show the nonzero parts of T with an x and the zero parts with an o.

(make a 10x10 grid for each case and enter x or o in the squares)

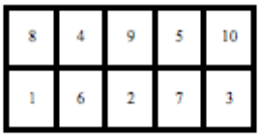
a. Standard ordering by rows:



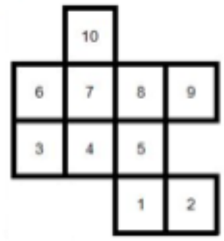
b. Standard ordering by columns:



c. Checkerboard (Cyclic-2) Ordering:



d. Irregular Ordering:



Solution

a)

x	x	o	o	o	x	o	o	o	o
x	x	x	o	o	o	x	o	o	o
o	x	x	x	o	o	o	x	o	o
o	o	x	x	x	o	o	o	x	o
o	o	o	x	x	o	o	o	o	x
x	o	o	o	o	x	x	o	o	o
o	x	o	o	o	x	x	x	o	o
o	o	x	o	o	o	x	x	x	o
o	o	o	x	o	o	o	x	x	x
o	o	o	o	x	o	o	o	x	x

c)

x	o	o	o	o	x	x	o	o	o
o	x	o	o	o	x	x	o	x	o
o	o	x	o	o	o	x	o	o	x
o	o	o	o	o	x	o	x	x	o
o	o	o	o	o	o	x	o	x	x
x	x	o	x	o	o	o	o	o	o
o	x	x	o	x	o	x	o	o	o
x	o	x	o	o	o	o	x	o	o
o	x	o	x	x	o	o	o	x	o
o	o	x	o	x	o	o	o	o	x

b)

x	x	x	o	o	o	o	o	o	o
x	x	o	x	o	o	o	o	o	o
x	o	x	x	x	o	o	o	o	o
o	x	x	x	o	x	o	o	o	o
o	o	x	o	x	x	x	o	o	o
o	o	o	x	x	x	o	x	o	o
o	o	o	o	x	o	x	x	x	o
o	o	o	o	o	x	x	x	o	o
o	o	o	o	o	o	x	o	x	x
o	o	o	o	o	o	o	x	x	x

d)

x	x	o	o	x	o	o	o	o	o
x	x	o	o	o	o	o	o	o	o
o	o	x	x	o	x	o	o	o	o
o	o	x	x	x	o	x	o	o	o
x	o	o	x	x	o	o	x	o	o
o	o	x	o	o	x	x	o	o	o
o	o	o	x	o	x	x	x	o	x
o	o	o	o	x	o	x	x	x	o
o	o	o	o	o	o	o	x	x	o
o	o	o	o	o	o	x	o	o	x

Question 9 (11 points)

a) Write the oil-water equations (one-dimensional, horizontal flow) in discrete form using transmissibilities and storage coefficients, including production/injection terms.
b) What are the primary variables (unknowns)?
c) What is the main challenge in solving the equations for pressures and saturations?
d) What are the assumptions for solving the equations using the simplified IMPES method?
e) List the advantages and disadvantages of IMPES reservoir simulation compared to fully implicit reservoir simulation in terms of accuracy, stability and cost.
f) In what kind of problems may the IMPES method not be well suited for?

Solution

a)

$$Tx o_{i+1/2}\left(Po_{i+1} - Po_i\right) + Tx o_{i-1/2}\left(Po_{i-1} - Po_i\right) - q'_{oi} = Cpoo_i\left(Po_i - Po_i^t\right) + Cswo_i\left(Sw_i - Sw_i^t\right), \qquad i = 1, N$$

$$Txw_{i+1/2}\left[\left(Po_{i+1} - Po_i\right) - \left(Pcow_{i+1} - Pcow_i\right)\right] + Txw_{i-1/2}\left[\left(Po_{i-1} - Po_i\right) - \left(Pcow_{i-1} - Pcow_i\right)\right] - q'_{wi} = Cpow_i\left(Po_i - Po_i^t\right) + Csww_i\left(Sw_i - Sw_i^t\right), \qquad i = 1, N$$

b) Oil pressure and water saturation
c) Coefficients and capillary pressures in the equations for each grid block are dependent on the pressures and saturations that we are solving for
d) All coefficients (transmissibilities and storage coeffiocients) and capillary pressures in the equations for each grid block are dependent on the pressures and saturations that we are solving for
e) **Advantages**: simpler solution routines and not so computer intensive per time step; less discretization error since time steps must be small. **Disadvantages**: small time steps are required in order to avoid large errors in coefficients and capillary pressures (due to assumptions); large time steps may lead to instability
f) Not well suited for systems where rapid changes in saturations and pressures take place

Question 10 *(8 points)*

For two-phase flow (constant flow area) the right hand side of the gas equation may be written (undersaturated oil case):

$$\frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right)$$

The corresponding discretized form is:

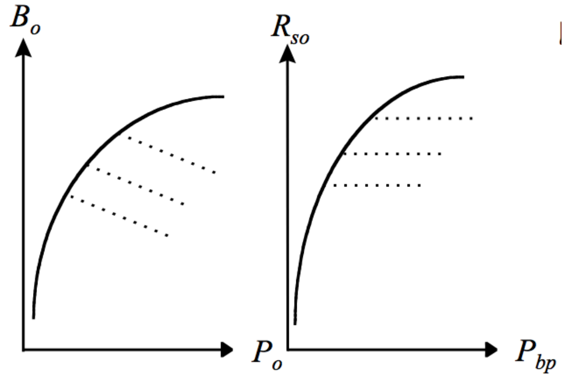
$$Cpog_i\left(Po_i - Po_i^t\right) + Cpbg_i\left(Pbp_i - Pbp_i^t\right) + Cswg_i\left(Sw_i - Sw_i^t\right)$$

a) Sketch typical curves that show the pressure dependencies of B_o and R_{so}.

b) Show the complete derivations of the three coefficients $C_{pog_i}, C_{pbg_i}, C_{swg_i}$.

Solution

a)



In the figures, the solid lines represent saturated conditions, while the dotted lines represent under-saturated behavior, with the bubble point pressure being defined by the intersection of the dotted line and the saturated line. Thus, the bubble point pressure depends on the amount of gas present in the system. The more gas, the higher the bubble point pressure.

For undersaturated oil the functional dependencies are: $B_o(P_o, P_{bp}), R_{so}(P_{bp})$

b) First, since we have two-phase flow it means that oil and water are flowing, while the gas is in the form of solution gas. We expand the right hand side of the gas equation as follows:

$$\frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right) = R_{so}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) + \frac{\phi S_o}{B_o}\frac{dR_{so}}{dP_{bp}}\frac{\partial P_{bp}}{\partial t}$$

The first term becomes:

$$R_{so}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = R_{so}\left\{\frac{\phi}{B_o}\frac{\partial S_o}{\partial t} + \phi S_o\left[\frac{\partial}{\partial P_o}\left(\frac{1}{B_o}\right)\frac{\partial P_o}{\partial t} + \frac{\partial}{\partial P_{bp}}\left(\frac{1}{B_o}\right)\frac{\partial P_{bp}}{\partial t}\right] + \frac{S_o}{B_o}\frac{d\phi}{dP_o}\frac{\partial P_o}{\partial t}\right\},$$

and in discrete form:

$$\left[R_{so}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)\right]_i \approx -\frac{(\phi R_{so})_i}{B_{oi}\Delta t}(S_{wi} - S_{wi}^t) + \frac{(\phi R_{so} S_o)_i}{\Delta t}\left[\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right]_i \left(P_{o_i} - P_{o_i}^t\right) + \frac{(\phi R_{so} S_o)_i}{\Delta t}\left[\frac{\partial(1/B_o)}{\partial P_{bp}}\right]_i \left(P_{bp_i} - P_{bp_i}^t\right)$$

The second term in discrete form becomes:

$$\left[\frac{\phi S_o}{B_o}\frac{dR_{so}}{dP_{bp}}\frac{\partial P_{bp}}{\partial t}\right]_i \approx \frac{(\phi S_o)_i}{B_{o_i}\Delta t}\left[\frac{dR_{so}}{dP_{bp}}\right]_i \left(P_{bp_i} - P_{bp_i}^t\right)$$

Collecting terms:

$$C_{pogi} = \frac{\left(\phi R_{so} S_o\right)_i}{\Delta t}\left[\frac{c_r}{B_o} + \frac{\partial\left(1/B_o\right)}{\partial P_o}\right]_i$$

$$C_{pbgi} = \frac{\left(\phi S_o\right)_i}{\Delta t}\left\{\left[R_{so}\frac{\partial\left(1/B_o\right)}{\partial P_{bp}}\right]_i + \frac{1}{B_{o_i}}\left(\frac{dR_{so}}{dP_{bp}}\right)_i\right\}$$

$$C_{swgi} = -\frac{\left(\phi R_{so}\right)_i}{B_{oi}\Delta t}$$