NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Petroleum Engineering and Applied Geophysics

SOLUTION - Examination paper for TPG4160 Reservoir Simulation

Academic contact during examination: Jon Kleppe

Phone: 91897300/73594925

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Question 1 (26x0,5 points)

Explain briefly the following terms as applied to reservoir simulation (short sentence and/or a formula for each):

a) Control volume
b) Mass balance
c) Taylor series
d) Numerical dispersion
e) Explicit
f) Implicit
g) Stability
h) Upstream weighting
i) Variable bubble point
j) Harmonic average
k) Transmissibility
l) Storage coefficient
m) Coefficient matrix
n) IMPES
o) Fully implicit
p) Cross section
q) Coning
r) PI
s) Stone´s relative permeability models
t) Discretization
u) History matching
v) Prediction
w) Black Oil
x) Compositional
y) Dual porosity
z) Dual permeability

Solution

a) Control volume **small volume used in derivation of continuity equation**
b) Mass balance **principle applied to control volume in derivation of continuity equation**
c) Taylor series **expansion formula used for derivation of difference approximations (or formula:** $f(x+h)=f(x)+\frac{h}{1!}f'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+....$**)**
d) Numerical dispersion **error term associated with finite difference approximations derived by use of Taylor series**
e) Explicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t**
f) Implicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t+Δt**
g) Stability **as applied to implicit and explicit discretization of diffusivity equation: explicit form is conditional stable for** $\Delta t \le \frac{1}{2}\frac{\phi\mu c}{k}(\Delta x)^2$**, while implicit form is unconditionally stable**

h) Upstream weighting **descriptive term for the choice of mobility terms in transmissibilities**
i) Variable bubble point **term that indicates that the discretization og undersaturated flow equation includes the possibility for bubble point to change, such as for the case of gas injection in undersaturated oil**
j) Harmonic average **averaging method used for permeabilities when flow is in series**
k) Transmissibility **flow coefficient in discrete equations that when muliplied with pressure difference between grid blocks yields flow rate.**
l) Storage coefficient **flow coefficient in discrete equations that when muliplied with pressure change or saturation change in a time step yields mass change in grid block**
m) Coefficient matrix **the matrix of coefficient in the set of linear equations**
n) IMPES **an approximate solution method for two or three phase equations where all coefficients and capillary pressures are computed at time level of previous time step when generating the coefficient matrix**
o) Fully implicit **an solution method for two or three phase equations where all coefficients and capillary pressures are computed at the current time level generating the coefficient matrix. Thus, iterations are required on the solution.**
p) Cross section **an x-z section of a reservoir**
q) Coning **the tendency of gas and water to form a cone shaped flow channel into the well due to pressure drawdown in the close neigborhood.**
r) PI **the productivity index of a well**
s) Stone´s relative permeability models **methods for generating 3-phase relative permeabilities for oil based on 2-phase data**
t) Discretization **converting of a contineous PDE to discrete form**
u) History matching **in simulation the adjustment of reservoir parameters so that the computed results match observed data.**
v) Prediction **computing future performance of reservoir, normally following a history matching.**
w) Black Oil **simplified hydrocarbon description model which includes two phases (oil, gas) and only two components (oil, gas), with mass transfer between the components through the solution gas-oil ratio parameter.**
x) Compositional **detailed hydrocarbon description model which includes two phases but N components (methane, ethane, propane, ...).**
y) Dual porosity **denotes a reservoir with two porosity systems, normally a fractured reservoir**
z) Dual permeability **denotes a reservoir with two permeabilities (block-to-block contact) in addition to two porosities, normally a fractured reservoir**

Question 2 (*1+2+2+2+4+4 points*)

Answer the following questions related to the derivation of reservoir fluid flow equations:
a) Write the mass balance equation (one-dimensional, one-phase)
b) List 3 commonly used expressions for relating fluid density to pressure
c) Write the most common relationship between velocity and pressure, and write an alternative relationship used for high fluid velocities.
d) Write the expression for the relationship between porosity and pressure.
e) Derive the following partial differential equation (show all steps):

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

f) Reduce the equation in e) to the simple diffusivity equation:

$$\frac{\partial^2 P}{\partial x^2}=(\frac{\varphi\mu c}{k})\frac{\partial P}{\partial t}$$

Solution

a) For constant cross sectional area, the continuity equation simplifies to:

$$-\frac{\partial}{\partial x}(\rho u)=\frac{\partial}{\partial t}(\phi\rho)$$

b) Compressibility definition:

$$c_f=-(\frac{1}{V})(\frac{\partial V}{\partial P})_T\,.$$

Real gas law:

$$PV=nZRT.$$

The gas density may be expressed as:

$$\rho_g=\rho_{gS}\frac{P}{Z}\frac{Z_S}{P_S}$$

Black Oil description:

$$\rho_o=\frac{\rho_{oS}+\rho_{gs}R_{so}}{B_o}\,.$$

c) Darcy's equation, which for one dimensional flow is:

$$u=-\frac{k}{\mu}\left(\frac{\partial P}{\partial x}-\rho g\sin\alpha\right).$$

An alternative equations is the Forchheimer equation, for high velocity flow (horizontal):

$$-\frac{\partial P}{\partial x}=u\frac{\mu}{k}+\beta u^n$$

where n is proposed by Muscat to be 2

d) Rock compressibility:

$$c_r=(\frac{1}{\phi})(\frac{\partial\phi}{\partial P})_T$$

e) Substitution for Darcy´s eq.:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\partial}{\partial t}(\phi\rho)$$

Fluid density: $\rho_o=\dfrac{\rho_{oS}+\rho_{gS}R_{so}}{B_o}=\dfrac{\text{constant}}{B_o}$

Right side

$$\frac{\partial}{\partial t}(\phi\rho)=\frac{\text{constant}}{B}\frac{\partial\phi}{\partial P}+\phi\frac{\partial(\text{constant}/B)}{\partial t}=\text{constant}\left(\frac{1}{B}\frac{d\phi}{dP}+\phi\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}=\text{constant }\phi\left(\frac{c_r}{B}+\phi\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

or

$$\frac{\partial}{\partial t}(\phi\rho)=\text{constant }\phi\left[\frac{c_r}{B}+\frac{d(1/B)}{dP}\right]\frac{\partial P}{\partial t}$$

Left side (horizontal)

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\text{constant}}{B}\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\text{constant}\frac{\partial}{\partial x}\left(\frac{k}{B\mu}\frac{\partial P}{\partial x}\right)$$

Thus, the flow equation becomes:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left[\frac{c_r}{B}+\frac{d(1/B)}{dP}\right]\frac{\partial P}{\partial t}$$

f) Starting with the equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

Assume that the group $k/\mu B$ is approximately constant:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)\Rightarrow\frac{k}{\mu B}\frac{\partial^2 P}{\partial x^2}$$

Since

$$\frac{d(1/B)}{dP}=-\frac{c_f}{B},$$

$$\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)=\frac{1}{B}\left(c_r+c_f\right)=\frac{c}{B}$$

Thus, the equation becomes

$$\frac{\partial^2 P}{\partial x^2}=(\frac{\varphi\mu c}{k})\frac{\partial P}{\partial t}$$

<u>Question 3</u> (10 points)

Use Taylor series and show <u>all steps</u> in the discretization of the following equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

Solution

Right side:

$$P(x,t)=P(x,t+\Delta t)+\frac{-\Delta t}{1!}P'(x,t+\Delta t)+\frac{(-\Delta t)^2}{2!}P''(x,t+\Delta t)+\frac{(-\Delta t)^3}{3!}P'''(x,t+\Delta t)+.....$$

Solving for the time derivative, we get:

$$(\frac{\partial P}{\partial t})_i^{t+\Delta t}=\frac{P_i^{t+\Delta t}-P_i^t}{\Delta t}+O(\Delta t).$$

Thus,

$$\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}\approx\phi_i\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)_i\frac{P_i^{t+\Delta t}-P_i^t}{\Delta t}$$

Left side:

$$\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_{i+1/2}=\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+\frac{\Delta x/2}{1!}\frac{\partial}{\partial x}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+\frac{(\Delta x/2)^2}{2!}\frac{\partial^2}{\partial x^2}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+.....$$

$$\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_{i-1/2}=\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+\frac{-\Delta x/2}{1!}\frac{\partial}{\partial x}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+\frac{(-\Delta x/2)^2}{2!}\frac{\partial^2}{\partial x^2}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i+.....$$

combination yields

$$\frac{\partial}{\partial x}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i=\frac{\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_{i+1/2}-\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_{i-1/2}}{\Delta x}+O(\Delta x^2).$$

Using similar central difference approximations for the two pressure gradients:

$$\left(\frac{\partial P}{\partial x}\right)_{i+1/2}=\frac{P_{i+1}-P_i}{\Delta x}+O(\Delta x)$$

and

$$\left(\frac{\partial P}{\partial x}\right)_{i-1/2}=\frac{P_i-P_{i-1}}{\Delta x}+O(\Delta x).$$

the expression becomes:

$$\frac{\partial}{\partial x}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i\approx\frac{\left[(\frac{k}{\mu B})\frac{P_{i+1}-P_i}{\Delta x}\right]_{i+1/2}-\left[(\frac{k}{\mu B})\frac{P_i-P_{i-1}}{\Delta x}\right]_{i-1/2}}{\Delta x}$$

or

$$\frac{\partial}{\partial x}\left[(\frac{k}{\mu B})\frac{\partial P}{\partial x}\right]_i\approx(\frac{k}{\mu B})_{i+1/2}\frac{P_{i+1}-P_i}{\Delta x^2}-(\frac{k}{\mu B})_{i-1/2}\frac{P_i-P_{i-1}}{\Delta x^2}$$

Thus, the difference equation becomes:

$$(\frac{k}{\mu B})_{i+1/2}\frac{P_{i+1}-P_i}{\Delta x^2}-(\frac{k}{\mu B})_{i-1/2}\frac{P_i-P_{i-1}}{\Delta x^2}\approx\left[\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\right]_i^{t+\Delta t}\frac{P_i^{t+\Delta t}-P_i^t}{\Delta t}$$

The terms

$(\frac{k}{\mu B})_{i+1/2}$ and $(\frac{k}{\mu B})_{i-1/2}$ are then computed using harmonic averages of properties of blocks i-1, i and i+1, respectively.

<u>Question 4</u> *(3+5+5 points)*

a) Show <u>all steps</u> in the derivation of the simple, one dimensional, radial, horizontal, one-phase diffusivity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r})=(\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$

b) Derive the numerical approximation for this equation using the transformation:

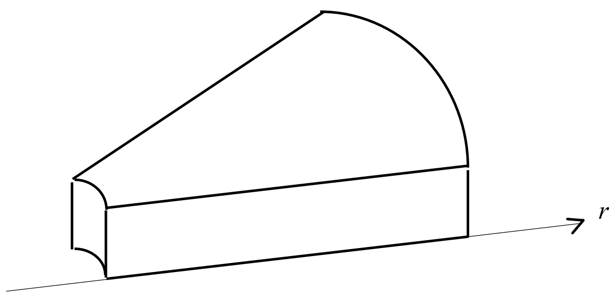
$$u=\ln(r)$$

c) Explain why the radial grid dimensions in cylindrical coordinates often are selected according to the formula:

$$\frac{r_{i+1/2}}{r_{i-1/2}}=(\frac{r_e}{r_w})^{1/N}$$

Solution

a)



In a radial system, the flow area is a function of radius, and for a full cylinder (360 degrees) the area is:

$A=2\pi rh$.

Thus, the continuity equation may be written (derivation is not required):

$$\frac{1}{r}\frac{\partial}{\partial r}(u\rho r)=\frac{\partial}{\partial t}(\rho\phi)\cdot$$

Substituting for Darcy´s eqn,:

$$u=-\frac{k}{\mu}\frac{\partial P}{\partial r}$$

we get

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(-\rho r\frac{k}{\mu}\frac{\partial P}{\partial r}\right)=\frac{\partial}{\partial t}(\rho\phi)$$

Using definitions of compressibilities (at constant temperature)

$$c_r=\frac{1}{\phi}\frac{d\phi}{dP}$$

$$c_f=\frac{1}{\rho}\frac{d\rho}{dP}$$

we rewrite the right side as:

$$\frac{\partial}{\partial t}(\rho\phi)=\rho\frac{\partial\phi}{\partial t}+\phi\frac{\partial\rho}{\partial t}=\rho(\frac{d\phi}{dP}+\phi\frac{d\rho}{dP})\frac{\partial P}{\partial t}=\rho\phi(c_r+c_f)\frac{\partial P}{\partial t}$$

The left side may be rewritten as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r\frac{\partial P}{\partial r}\right)=\frac{k}{\mu}\frac{1}{r}\left[\rho\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)+\left(r\frac{\partial P}{\partial r}\right)\frac{d\rho}{dP}\frac{\partial P}{\partial r}\right]=\frac{k}{\mu}\rho\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)+rc_f\left(\frac{\partial P}{\partial r}\right)^2\right]$$

It may be shown that:

$$\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)>>rc_f\left(\frac{\partial P}{\partial r}\right)^2$$

Thus, the left side may be approximated by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r\frac{\partial P}{\partial r}\right)\approx\frac{k}{\mu}\rho\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)$$

The simple form of the radial equation then becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r})=(\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$

b) For the radial flow equation, we will first make the following transformation of the r-coordinate into a u-coordinate:

$$u=\ln(r).$$

Thus,

$$\frac{du}{dr}=\frac{1}{r}$$

and

$$r=e^u\ .$$

The PDE may then be written:

$$e^{-u}\frac{\partial}{\partial u}\left(e^u\frac{\partial P}{\partial u}\frac{du}{dr}\right)\frac{du}{dr}=\frac{\phi\mu c}{k}\frac{\partial P}{\partial t},$$

or

$$e^{-2u}\frac{\partial^2 P}{\partial u^2}=\frac{\phi\mu c}{k}\frac{\partial P}{\partial t}.$$

Using the difference approximations above, we may write the numerical for of the left side as:

$$e^{-2u}\frac{\partial^2 P}{\partial u^2}\approx e^{-2u_i}\frac{P_{i+1}-2P_i+P_{i-1}}{\Delta u_i^2}$$

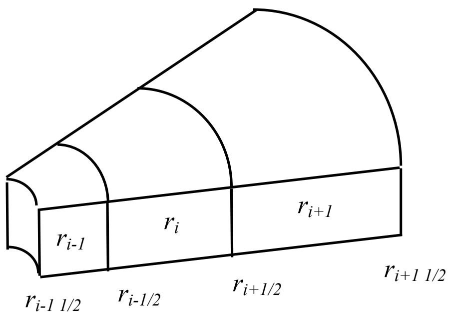
After back-substitution of r, we write the left side as:

$$e^{-2u_i}\frac{P_{i+1}-2P_i+P_{i-1}}{\Delta u_i^2}=\frac{1}{r_i^2}\frac{P_{i+1}-2P_i+P_{i-1}}{[\ln(r_{i+1/2}/r_{i-1/2})]^2}$$

The right side is approximated as for the linear equation. Thus, the complete difference equation becomes (no superscript means t+Δt):

$$\frac{1}{r_i^2}\frac{P_{i+1}-2P_i+P_{i-1}}{[\ln(r_{i+1/2}/r_{i-1/2})]^2}=\frac{P_i-P_i^t}{\Delta t},\quad i=1,...,N$$

The formula applies to the radial grid block system shown below:



The position of the grid block centers, relative to the block boundaries, may be computed using the midpoint between the *u*-coordinate boundaries:

$$u_i=(u_{i+1/2}+u_{i-1/2})/2,$$

or, in terms of radius:

$$r_i=\sqrt{r_{i+1/2}r_{i-1/2}}\ .$$

This is the geometric average of the block boundary radii.

c) Frequently in simulation of flow in the radial direction, the grid blocks sizes are chosen such that:

$$\Delta u_i=(u_{i+1/2}-u_{i-1/2})=\text{constant}$$

or

$$\ln\left(\frac{r_{i+1/2}}{r_{i-1/2}}\right)=\text{constant},$$

which for a system of N grid blocks and well and external radii of r_w and r_e, respectively, implies that

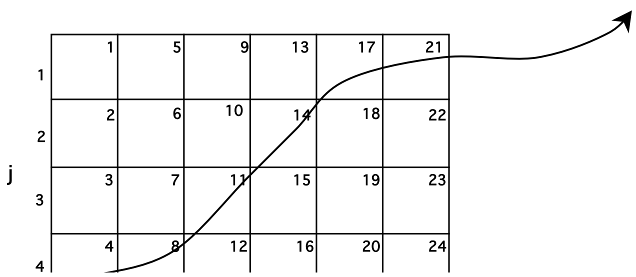
$$N\cdot\ln\left(\frac{r_{i+1/2}}{r_{i-1/2}}\right)=\ln\left(\frac{r_e}{r_w}\right)$$

or

$$\frac{r_{i+1/2}}{r_{i-1/2}}=\left(\frac{r_e}{r_w}\right)^{1/N}=\text{constant}.$$

Question 5 *(4+4+6 points)*

In the following 2-dimensional cross-section of a reservoir (one fluid only), a well is producing at a constant rate Q (st. vol. oil/unit time) and perforates the grid blocks 4, 8, 11, 14, 17 and 21 in the x-z grid system shown:



The (unknown) bottom hole pressure P_{bh} is specified at a reference depth d_{ref}. Assume that hydrostatic pressure equilibrium exists inside the well tubing.

a) Write the expression for oil rate from each perforated block (in terms of productivity indices, mobility terms, pressure differences and hydrostatic pressure differences)
b) Write the expression for the total oil flow rate for the well (group the constants into parameters A, B, C, D, F, G, H, representing a constant term and the contribution to flow from the 6 grid block pressures involved)
c) The standard pressure equation for this grid system, without the well terms, is:

$$e_{i,j}P_{i,j-1} + a_{i,j}P_{i-1,j} + b_{i,j}P_{i,j} + c_{i,j}P_{i+1,j} + f_{i,j}P_{i,j+1} = d_{i,j} \quad i = 1,...,N_1, \quad j = 1,...,N_2$$

Sketch the coefficient matrix for this system, including the well. Indicate how the coefficient matrix is altered by the well (approximately, with x´s and lines labeled with the appropriate coefficient name).

Solution

a) Write the expression for oil rate from each perforated block (in terms of productivity indices, mobility terms, pressure differences and hydrostatic pressure differences)

$$q_{ij} = PI_{ij}\lambda_{ij}\left[P_{ij} - P_{bh} - (d_{ij} - d_{ref})\rho g\right]$$
$$ie.$$
$$q_4 = PI_4\lambda_4(P_4 - P_{bh} - \Delta d_4\rho g)$$
$$q_8 = PI_8\lambda_8(P_8 - P_{bh} - \Delta d_8\rho g)$$
$$...$$
$$...$$

b) Write the expression for the total oil flow rate for the well (group constants into parameters A, B, C, D, E, F, G, H)

$$Q_{tot} = \sum_{perf\ blocks} q_{ij} = A + BP_4 + CP_8 + DP_{11} + EP_{14} + FP_{17} + GP_{21} + HP_{bh}$$
$$or$$
$$BP_4 + CP_8 + DP_{11} + EP_{14} + FP_{17} + GP_{21} + HP_{bh} = d_{25}$$

c) The standard pressure equation for this grid system, without the well terms, is:

$$e_{i,j}P_{i,j-1} + a_{i,j}P_{i-1,j} + b_{i,j}P_{i,j} + c_{i,j}P_{i+1,j} + f_{i,j}P_{i,j+1} = d_{i,j} \quad i = 1,...,N_1, \quad j = 1,...,N_2$$

Sketch the coefficient matrix for this system, including the well. Indicate how the coefficient matrix is altered by the well (approximately, with x´s and lines labelled with the appropriate coefficient name).

```
b f     c
e b f     c
  e b f     c
    e b       c                                 x
a       b f     c
  a     e b f     c
    a     e b f     c
      a     e b       c                         x
        a       b f     c
          a     e b f     c
            a     e b       c                   x
              a       b f     c
                a     e b f     c
                  a     e b f     c             x
                    a     e b f     c
                      a     e b       c
                        a       b f     c       x
                          a     e b f     c
                            a     e b f     c
                              a     e b       c
                                a       b f     x
                                  a     e b f
                                    a     e b f
                                      a     e b
      B       C     D     E     F       G       H
```

<u>Question 6</u> *(3+2+3+5 points)*

The discretized form of the oil equation may be written as

$$T_{xo_{i+1/2}}(P_{oi+1} - P_{oi}) + T_{xo_{i-1/2}}(P_{oi-1} - P_{oi}) - q'_{oi} = C_{poi}(P_{oi} - P^t_{oi}) + C_{soi}(S_{wi} - S^t_{wi})$$

a) What is the physical significance of each of the 5 terms in the equation?

Using the following transmissibility as example,

$$T_{xo_{i-1/2}} = \frac{2k_{i-1/2}\lambda_{oi-1/2}}{\Delta x_i\left(\Delta x_i + \Delta x_{i-1}\right)}$$

b) What type of averaging method is normally applied to absolute permeability between grid blocks? Why? Write the expression for average permeability between grid blocks (i-1) and (i).

c) Write an expression for the selection of the conventional *<u>upstream mobility term</u>* for use in the transmissibility term of the oil equation above for flow between the grid blocks (i-1) and (i).

d) Make a sketch of a typical Buckley-Leverett saturation profile resulting from the displacement of oil by water (ie. analytical solution). Then, show how the corresponding profile, if calculated in a numerical simulation model, typically is influenced by the choice of mobilities between the grid blocks (sketch curves for saturations computed with upstream or average mobility terms, respectively).

Solution

a)

$T_{xo_{i+1/2}}(P_{oi+1} - P_{oi})$ = flow between grid blocks i and i+1

$T_{xo_{i-1/2}}(P_{oi-1} - P_{oi})$ = flow between grid blocks i and i-1

q'_{oi} = production term

$C_{poi}(P_{oi} - P^t_{oi})$ = fluids compression/expansion term

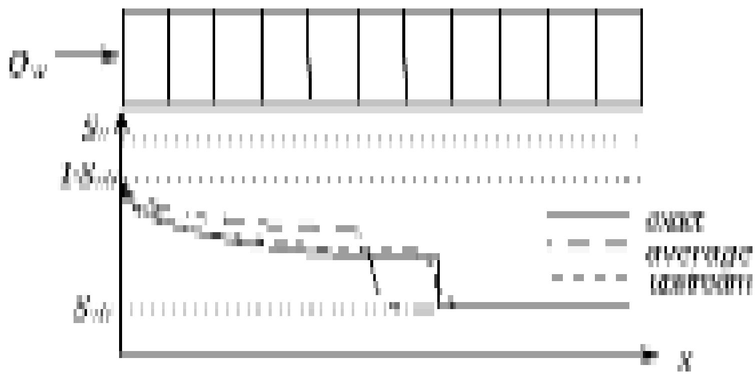
$C_{soi}(S_{wi} - S^t_{wi})$ = volume change due to saturations

b) Harmonic average is used, based on a derivation of average permeability of series flow, assuming steady flow and Darcy´s equation

$$\bar{k}_{i-1/2} = \frac{\Delta x_{i-1} + \Delta x_i}{\dfrac{\Delta x_{i-1}}{k_{i-1}} + \dfrac{\Delta x_i}{k_i}}$$

c) $\lambda o_{i-1/2} = \begin{cases} \lambda o_{i-1} & if \quad Po_{i-1} \geq Po_i \\ \lambda o_i & if \quad Po_{i-1} < Po_i \end{cases}$

d)



<u>Question 7</u> (5x2 points)

Normally, we use either a *Black Oil* fluid description or a *compositional* fluid description in reservoir simulation.

a) What are the *components* and the *phases* used in *Black Oil* modeling?
b) What are the *components* and the *phases* used in *compositional* modeling?
c) Write the standard flow equations for the components required for *Black Oil* modeling (one dimensional, horizontal, constant flow area).
d) Write the standard flow equations the components required for *compositional* modeling (one dimensional, horizontal, constant flow area). Let

C_{kg} = *mass fraction of component k present in the gas phase*

C_{ko} = *mass fraction of component k present in the oil phase*.

e) A *Black Oil* fluid description may be regarded as a subset of a *compositional* fluid description. Define the pseudo-components required in order to reduce the *compositional* equations to *Black Oil* equations (one dimensional, horizontal, constant flow area)

Solution

a) ***Components***: oil and gas, ***phases***: oil and gas

b) ***Components***: hydrocarbons ($C_1H_4, C_2H_6, C_3H_8,....$) and non-hydrocarbons ($CO_2, H_2S, C_2,....$), ***phases***: oil and gas

c)

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{B_g\mu_g}\frac{\partial P_g}{\partial x} + \frac{kk_{ro}R_{so}}{B_o\mu_o}\frac{\partial P_o}{\partial x}\right) = \frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + \frac{S_oR_{so}}{B_g}\right)\right]$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{B_o\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

d)

$$\frac{\partial}{\partial x}\left(C_{kg}\rho_g\frac{kk_{rg}}{\mu_g}\frac{\partial P_g}{\partial x}+C_{ko}\rho_o\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left[\phi\left(C_{kg}\rho_g S_g+C_{ko}\rho_o S_o\right)\right],\qquad k=1,N_c$$

e)

The *Black Oil* model may be considered to be a *pseudo-compositional* model with two components. Define the components and the fractions needed to convert the compositional equations to Black-Oil equations.

component 1: oil – k=o

$$\frac{\partial}{\partial x}\left(C_{og}\rho_g\frac{kk_{rg}}{\mu_g}\frac{\partial P_g}{\partial x}+C_{oo}\rho_o\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left[\phi\left(C_{og}\rho_g S_g+C_{oo}\rho_o S_o\right)\right]$$

component 2: gas – k=g

$$\frac{\partial}{\partial x}\left(C_{gg}\rho_g\frac{kk_{rg}}{\mu_g}\frac{\partial P_g}{\partial x}+C_{go}\rho_o\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left[\phi\left(C_{gg}\rho_g S_g+C_{go}\rho_o S_o\right)\right]$$

Question: what are the fractions needed to get the Black Oil equations:

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{B_g\mu_g}\frac{\partial P_g}{\partial x}+\frac{kk_{ro}R_{so}}{B_o\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g}+\frac{S_oR_{so}}{B_g}\right)\right]$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{B_o\mu_o}\frac{\partial P_o}{\partial x}\right)=\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

Answer:

fraction of "gas in gas": $C_{gg}=1$

fraction of "oil in gas": $C_{og}=0$

fraction of "gas in oil": $C_{go}=\dfrac{\rho_{gS}R_{so}}{\rho_o B_o}$

fraction of "oil in oil": $C_{oo}=\dfrac{\rho_{oS}}{\rho_o B_o}$

Question 8 (6x2 points)

Normally, we use either a *conventional* model or a *fractured* model in simulation of a reservoir.

a) Describe the main differences between a *conventional* reservoir and a *fractured* reservoir, in terms of the physics of the systems.

b) How can we identify a fractured reservoir from standard reservoir data?
c) Explain **briefly** the primary concept used in deriving the flow equations for a dual-porosity model.
d) Write the basic equations (one-phase, one-dimension) for
   - a two-porosity, two-permeability system
   - a two-porosity, one-permeability system
e) In terms of the physics of reservoir flow, what is the key difference between the two formulations in question d)?
f) How is the fluid exchange term in the flow equations in question d) represented? What are the shortcomings of this representation?

Solution

a) Conventional: One porosity, one permeability system, with one flow equation for each component flowing.
   Fractured: Two porosities, two permeabilities system, withmost of the fluids in the matrix system, and most of the transport capacity in the fracture system. Requires two flow equations for each component flowing.
b) $K_{core} \ll K_{welltest}$
c) The matrix system supplies fluids to the fracture system, by whatever mechanisms present (depletion, gravity drainage, imbibition, diffusion,...), and the fracture system transports the fluids to the wells. Som transport may also occure in the matrix system, from block to block, provided that there is sufficient contact.

d) Dual porosity, dual permeability model:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)_f + q'_{mf} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_f$$

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)_m - q'_{mf} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_m$$

One porosity, one permeability model (fracture eqn.):

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)_f + q'_{mf} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_f$$

Conventional fracture models represents the exchange term by

$$-q'_{mf} = \sigma\lambda\left(P_m - P_f\right)$$

where σ is a geometric factor, λ is the mobility term, and P_m and P_f represent matrix and fracture pressures, respectively.

e) In the dual porosity, dual permeability model, fluid flow may occure from one matrix block to another. In the one porosity, one permeability model, all flow occurs in the fracture system
f) The exchange term is conventionally defined as

$$-q'_{mf} = \sigma\lambda\left(P_m - P_f\right)$$

where σ is a geometric factor, λ is the mobility term, and P_m and P_f represent matrix and fracture pressures, respectively.
Obviously, this term cannot adequately represent the flow mechanisms present, such as depletion, gravity drainage, imbibition, diffusion,... In addition, an average pressure for the matrix block is used in the exprexxion, so that pressure gradients inside the block is not accounted for.