NTNU – Trondheim
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Science and Technology

Department of Petroleum Engineering and Applied Geophysics

SOLUTION

Examination paper for TPG4160 Reservoir Simulation

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Other information:

Language: English

Number of pages: 6

Number of pages enclosed: 0

Checked by:

Date Signature

Question 1 (3+3+3 points)

This question relates to the Gullfaks H1 Segment project work.

a) Which geological factors are causing the good communication in the Lower Brent Group of the H1 Segment of the Gullfaks Field
b) Describe briefly how chemical injection is accounted for in the Eclipse simulations that you did
c) What are the main uncertainties in the simulation results?

Solution

The student should show that he/she really participated in the group work

Question 2 *(4,5+2+4+2+3+5+2 points)*

The simple, one-dimensional, linear, horizontal, one-phase diffusivity equation may be written as:

$$\frac{\partial^2 P}{\partial x^2} = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}.$$

a) List the steps involved in deriving the diffusivity equation
b) Sketch the one-dimensional, horizontal porous system that the equation applies to, in both continuous and discrete form.
c) Using Taylor series expansions, derive the finite difference approximations needed for the discretization of the equation (for constant grid block size).
d) What are the error terms associated with these approximations?
e) Write the difference equation on explicit form, and outline a procedure for pressure solution.
f) Write the difference equation on implicit form, and outline a procedure for pressure solution.
g) Why is the explicit form seldom used?

Solution

a) Only a list in text or using equations is required

-continuity equation $-\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial t}(\phi\rho)$

-Darcy´s equation $u = -\frac{k}{\mu}\frac{\partial P}{\partial x}$

-fluid compressibility $c_f = -(\frac{1}{V})(\frac{\partial V}{\partial P})_T = (\frac{1}{\rho})(\frac{\partial \rho}{\partial P})_T \Rightarrow \rho c_f = (\frac{d\rho}{dP})$

-rock compressibility $c_r = (\frac{1}{\phi})(\frac{\partial \phi}{\partial P})_T \Rightarrow \frac{d\phi}{dP} = \phi c_r$

-assume constant permeability and viscosity

-substitution: $\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}(\phi\rho)$

-right side: $\frac{\partial}{\partial t}(\phi\rho) = \rho\frac{d\phi}{dP}\frac{\partial P}{\partial t} + \phi\frac{d\rho}{dP}\frac{\partial P}{\partial t} = \left(\rho\phi c_r + \phi\rho c_f\right)\frac{\partial P}{\partial t} = \rho\phi c\frac{\partial P}{\partial t}$

-left side: $\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{k}{\mu}\frac{\partial}{\partial x}\left(\rho\frac{\partial P}{\partial x}\right)$

-then:

$$\frac{\partial}{\partial x}\left(\rho\frac{\partial P}{\partial x}\right) = \rho\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right) + \left(\frac{\partial P}{\partial x}\right)\frac{d\rho}{dP}\frac{\partial P}{\partial x} = \rho\left(\frac{\partial^2 P}{\partial x^2}\right) + \left(\frac{\partial P}{\partial x}\right)^2\frac{d\rho}{dP} = \rho\left(\frac{\partial^2 P}{\partial x^2}\right) + \left(\frac{\partial P}{\partial x}\right)^2\rho c_f$$

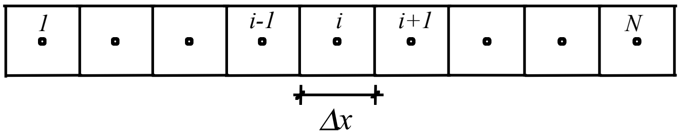
-assume that

-so that $\frac{\partial}{\partial x}\left(\rho\frac{\partial P}{\partial x}\right) \approx \rho\left(\frac{\partial^2 P}{\partial x^2}\right)$

-thus, the final equation becomes: $\frac{\partial^2 P}{\partial x^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t}$

b) -continuous system

-discrete system



c) ***Space derivative***

At constant time, $t+\Delta t$ (alternatively, t may be used), the pressure function may be expanded in forward and backward directions:

$$P(x+\Delta x,t+\Delta t)=P(x,t+\Delta t)+\frac{\Delta x}{1!}P'(x,t+\Delta t)+\frac{(\Delta x)^2}{2!}P''(x,t+\Delta t)+\frac{(\Delta x)^3}{3!}P'''(x,t+\Delta t)+.....$$

$$P(x-\Delta x,t+\Delta t)=P(x,t+\Delta t)+\frac{(-\Delta x)}{1!}P'(x,t+\Delta t)+\frac{(-\Delta x)^2}{2!}P''(x,t+\Delta t)+\frac{(-\Delta x)^3}{3!}P'''(x,t+\Delta t)+....$$

By adding these two expressions, and solving for the second derivative, we get the following approximation:

$$P''(x,t+\Delta t)=\frac{P(x+\Delta x,t+\Delta t)-2P(x,t+\Delta t)+P(x+\Delta x,t+\Delta t)}{(\Delta x)^2}+\frac{(\Delta x)^2}{12}P''''(x,t+\Delta t)+.....$$

or, by employing the grid index system, and using superscript to indicate time level:

$$(\frac{\partial^2 P}{\partial x^2})_i^{t+\Delta t}=\frac{P_{i+1}^{t+\Delta t}-2P_i^{t+\Delta t}+P_{i-1}^{t+\Delta t}}{(\Delta x)^2}+O(\Delta x^2).$$

Time derivative

At constant position, x, the pressure function may be expanded in backward direction in regard to time:

$$P(x,t)=P(x,t+\Delta t)+\frac{\Delta t}{1!}P'(x,t+\Delta t)+\frac{(\Delta t)^2}{2!}P''(x,t+\Delta t)+\frac{(\Delta t)^3}{3!}P'''(x,t+\Delta t)+.....$$

By solving for the first derivative, we get the following approximation:

$$P'(x,t+\Delta t)=\frac{P(x,t+\Delta t)-P(x,t)}{\Delta t}+\frac{(\Delta t)}{2}P''(x,t+\Delta t)+.....$$

or, employing the index system:

$$(\frac{\partial P}{\partial t})_i^{t+\Delta t}=\frac{P_i^{t+\Delta t}-P_i^t}{\Delta t}+O(\Delta t).$$

d) Error terms are derived above:

$$(\frac{\partial^2 P}{\partial x^2})_i^{t+\Delta t}:\quad O(\Delta x^2)$$

$$(\frac{\partial P}{\partial t})_i^{t+\Delta t}:\quad O(\Delta t)$$

e) ***Explicit***

Using the approximations above at time level t we get the explicit difference equation (shown only in general form; end blocks will be different):

$$\frac{P_{i+1}^t-2P_i^t+P_{i-1}^t}{\Delta x^2}\approx(\frac{\phi\mu c}{k})\frac{P_i^{t+\Delta t}-P_i^t}{\Delta t},\quad i=1,...,N$$

Since the equation contains only one unknown, it may be solved explicitly:

$$P_i^{t+\Delta t} = P_i^t + (\frac{\Delta t}{\Delta x^2})(\frac{k}{\phi\mu c})(P_{i+1}^t - 2P_i^t + P_{i-1}^t), \quad i = 1,...,N$$

f) ***Implicit***

Using the approximations above at time level t+Δt we get the explicit difference equation:

$$\frac{P_{i+1}^{t+\Delta t} - 2P_i^{t+\Delta t} + P_{i-1}^{t+\Delta t}}{\Delta x^2} = (\frac{\phi\mu c}{k})\frac{P_i^{t+\Delta t} - P_i^t}{\Delta t}, \quad i = 1,...,N$$

which is a set of N equations with N unknowns, which may be solved simultaneously, using a number of solution methods, for instance Gaussian elimination:

$$a_i P_{i-1}^{t+\Delta t} + b_i P_i^{t+\Delta t} + c_i P_{i+1}^{t+\Delta t} = d_i, \quad i = 1,..N$$

g) The explicit formulation is seldom used because it becomes unstable for large time steps. It has the following stability requirement:

$$\Delta t \le \frac{1}{2}(\frac{\phi\mu c}{k})\Delta x^2,$$

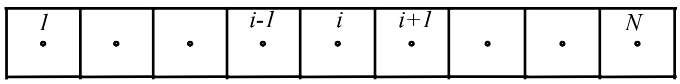
Question 3 (2+3+3+4 points)

Sketch the coefficient matrix for the following systems, indicating non-zero diagonals with approximate lines. Label the diagonals. What is the bandwidth?

a) One-dimensional (x), one phase flow, with the pressure equation:

$$a_i P_{i-1} + b_i P_i \ + c_i P_{i+1} = d_i, \quad i = 1, N$$

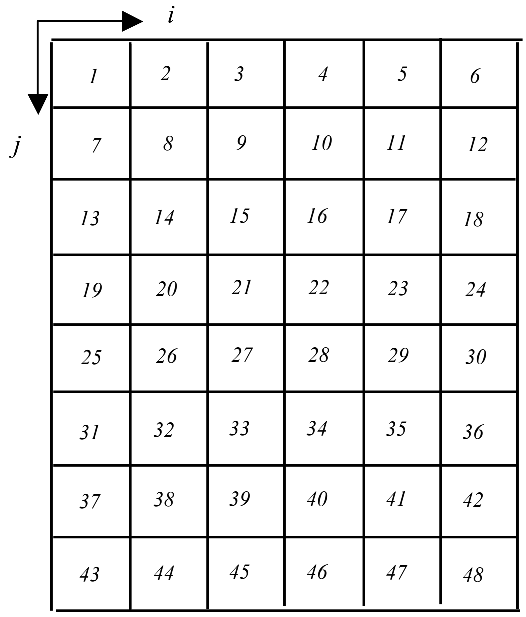
applicable to the following grid system:



b) Two-dimensional (x,y), one phase flow, with the pressure equation:

$$e_{i,j} P_{i,j-1} + a_{i,j} P_{i-1,j} + b_{i,j} P_{i,j} + c_{i,j} P_{i+1,j} + f_{i,j} P_{i,j+1} = d_{i,j} \qquad i = 1, N_x, j = 1, N_y$$

Applicable to the following grid system:



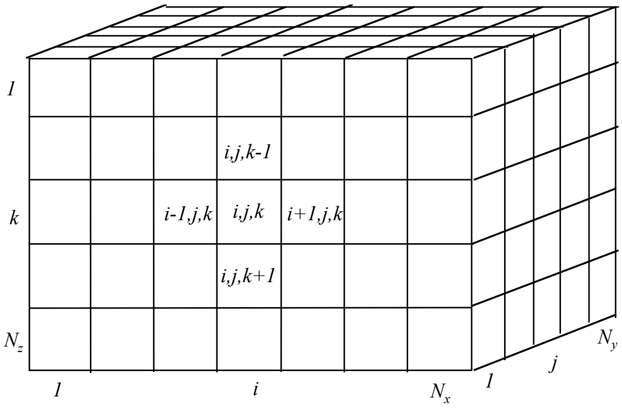
c) As question b) above, but now the numbering of the grid starts in the j-direction.

d) Three-dimensional (*x,y,z*), one phase flow, with the pressure equation:

$$g_{i,j,k} P_{i,j,k-1} + e_{i,j,k} P_{i,j-1,k} + a_{i,j,k} P_{i-1,j,k} + b_{i,j,k} P_{i,j,k}$$

$$+ c_{i,j,k} P_{i+1,j,k} + f_{i,j,k} P_{i,j+1,k} + h_{i,j,k} P_{i,j,k+1} = d_{i,j,k} \qquad i = 1, N_x, j = 1, N_y, k = 1, N_z$$

applicable to the following grid system (grid blocks numbered in the sequence of x,y,z)



Solution

a) Bandwidth=3

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&&&&& &&&&& && a & b & c \\
&&&&& &&&&& &&& a & b
\end{array}\right|
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b) Bandwidth=2Nx+1 =13

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&&& a & b & c & & & & & f \\
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\end{array}\right|
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c) Bandwidth= 2Ny+1=17

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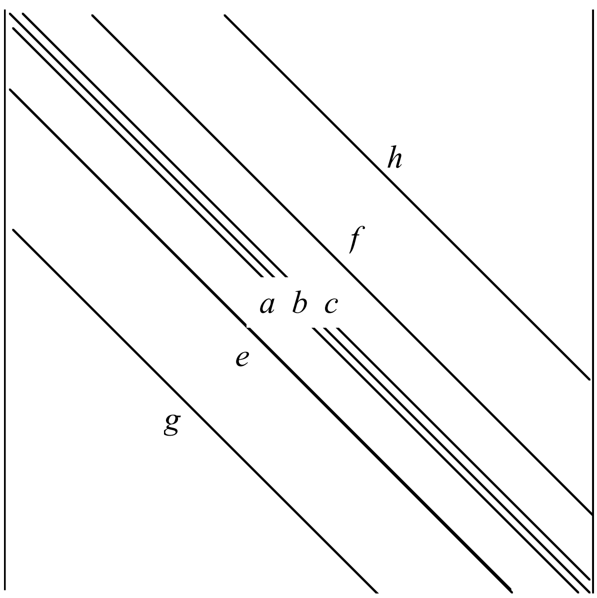
d) Bandwidth=2NxNy+1 or 2NxNz+1 or 2NyNz+1 depending on direction of numbering



Question 4 *(5+5+5 points)*

For two-phase flow of oil and gas in a horizontal, one dimensional, linear porous medium, the flow equations may be written as:

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)-q_o'=\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

and

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x}+R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)-q_g'-R_{so}q_o'=\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}+R_{so}\frac{\phi S_o}{B_o}\right),$$

where

$$P_{cog}=P_g-P_o$$

$$S_o+S_g=1.$$

a) Write the two flow equations on discretized forms in terms of transmissibilities, storage coefficients and pressure differences (no derivations).
b) List the assumptions for IMPES solution, and outline <u>briefly</u> how we solve for pressures and saturations
c) Outline <u>briefly</u> how we can solve for pressures and saturations by Newtonian iteration (ie. fully implicit solution).

Solution

a)

$$T_{xo_{i+1/2}}\left(P_{o_{i+1}}-P_{o_i}\right)+T_{xo_{i-1/2}}\left(P_{o_{i-1}}-P_{o_i}\right)-q_{oi}'$$
$$=C_{poo_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgo_i}\left(S_{g_i}-S_{g_i}^t\right),\qquad i=1,N$$

$$T_{xg_{i+1/2}}\left[\left(P_{o_{i+1}}-P_{o_i}\right)+\left(P_{cog_{i+1}}-P_{cog_i}\right)\right]+T_{xg_{i-1/2}}\left[\left(P_{o_{i-1}}-P_{o_i}\right)+\left(P_{cog_{i-1}}-P_{cog_i}\right)\right]-q_{gi}'$$
$$+\left(R_{so}T_{xo}\right)_{i+1/2}\left(P_{o_{i+1}}-P_{o_i}\right)+\left(R_{so}T_{xo}\right)_{i-1/2}\left(P_{o_{i-1}}-P_{o_i}\right)-\left(R_{so}q_o'\right)_i$$
$$=C_{pog_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgg_i}\left(S_{g_i}-S_{g_i}^t\right),\qquad i=1,N$$

b) IMPES solution
Assumptions:

$$T_{xo}^{\,t},T_{xg}^{\,t}$$
$$C_{poo}^{\,t},C_{pog}^{\,t}$$
$$C_{sgo}^{\,t},C_{sgg}^{\,t}\;.$$
$$P_{cog}^{\,t},R_{so}^{\,t}$$

Having made these approximations, the discretized flow equations become:

$$T_{xo_{i+1/2}}^{t}\left(P_{o_{i+1}}-P_{o_i}\right)+T_{xo_{i-1/2}}^{t}\left(P_{o_{i-1}}-P_{o_i}\right)-q_{oi}'$$
$$=C_{poo_i}^{t}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgo_i}^{t}\left(S_{g_i}-S_{g_i}^t\right),\qquad i=1,N$$

$$
\begin{aligned}
&Txg^{t}_{i+1/2}\Big[\big(P_{o_{i+1}}-P_{o_i}\big)+\big(P_{cog_{i+1}}-P_{cog_i}\big)^t\Big]\\
&+Txg^{t}_{i-1/2}\Big[\big(P_{o_{i-1}}-P_{o_i}\big)+\big(P_{cog_{i-1}}-P_{cog_i}\big)^t\Big]-q'_{gi}\\
&+\big(R_{so}Txo\big)^t_{i+1/2}\big(P_{o_{i+1}}-P_{o_i}\big)+\big(R_{so}Txo\big)^t_{i-1/2}\big(P_{o_{i-1}}-P_{o_i}\big)-\big(R_{so}{}^t q'_o\big)_i\\
&\qquad\qquad=Cpog^t_i\big(P_{o_i}-P^t_{o_i}\big)+Csgg^t_i\big(S_{g_i}-S^t_{g_i}\big),\qquad i=1,N
\end{aligned}
$$

IMPES pressure solution

The pressure equation for the saturated oil-gas becomes:

$$
\begin{aligned}
&\Big\{Txo^t_{i+1/2}+\alpha_i\Big[Txg^t_{i+1/2}+\big(R_{so}T_{xo}\big)^t_{i+1/2}\Big]\Big\}\big(P_{o_{i+1}}-P_{o_i}\big)+\\
&\Big\{Txo^t_{i-1/2}+\alpha_i\Big[Txg^t_{i-1/2}+\big(R_{so}T_{xo}\big)^t_{i-1/2}\Big]\Big\}\big(P_{o_{i-1}}-P_{o_i}\big)\\
&+\alpha_i Txg^t_{i+1/2}\big(P_{cog_{i+1}}-P_{cog_i}\big)^t+\alpha_i Txg^t_{i-1/2}\big(P_{cog_{i-1}}-P_{cog_i}\big)^t\\
&-q'_{oi}-\alpha_i\Big(q'_g+R_{so}{}^t q'_{oi}\Big)_i=\\
&\Big(Cpoo^t_i+\alpha_i Cpog^t_i\Big)\Big(P_{o_i}-P^t_{o_i}\Big),\qquad i=1,N
\end{aligned}
$$

where

$$\alpha_i=-Csgo^t_i/Csgg^t_i.$$

The pressure equation may now be rewritten as:

$$a_iP_{o_{i-1}}+b_iP_{o_i}+c_iP_{o_{i+1}}=d_i,\quad i=1,N$$

and solved for pressures using a number of solution methods, ie. Gaussian elimination.

IMPES saturation solution

Having obtained the oil pressures above, we need to solve for gas saturations using either the oil equation or the gas equation. Using the oil equation yields:

$$S_{g_i}=S^t_{g_i}+\frac{1}{Csgo^t_i}\Big[Txo^t_{i+1/2}\big(P_{o_{i+1}}-P_{o_i}\big)+Txo^t_{i-1/2}\big(P_{o_{i-1}}-P_{o_i}\big)-q'_{oi}\quad-Cpoo^t_i\Big(P_{o_i}-P^t_{o_i}\Big)\Big],\qquad i=1,N$$

c)

Solution by Newtonian iteration

Let us express the oil equation as F_{o_i} and the gas equation as F_{g_i}. Each equation will depend on pressures and saturations in blocks *i-1, i* and *i+1*., as indicated below.

$$F_{o_i}(P_{o_{i-1}},P_{o_i},P_{o_{i+1}},S_{g_{i-1}},S_{g_i},S_{g_{i+1}})=0$$

$$F_{g_i}(P_{o_{i-1}},P_{o_i},P_{o_{i+1}},S_{g_{i-1}},S_{g_i},S_{g_{i+1}})=0\ ,\qquad i=1,N$$

By first-order Taylor series expansions, we obtain the following expressions, where iteration level is given by *k*:

$$F_{o_i}^{k+1} = F_{o_i}^{k} + \frac{\partial F_{o_i}}{\partial P_{o_{i-1}}}(P_{o_{i-1}}^{k+1} - P_{o_{i-1}}^{k}) + \frac{\partial F_{o_i}}{\partial P_{o_i}}(P_{o_i}^{k+1} - P_{o_i}^{k}) + \frac{\partial F_{o_i}}{\partial P_{o_{i+1}}}(P_{o_{i+1}}^{k+1} - P_{o_{i+1}}^{k})$$
$$+ \frac{\partial F_{o_i}}{\partial S_{g_{i-1}}}(S_{g_{i-1}}^{k+1} - S_{g_{i-1}}^{k}) + \frac{\partial F_{o_i}}{\partial S_{g_i}}(S_{g_i}^{k+1} - S_{g_i}^{k}) + \frac{\partial F_{o_i}}{\partial S_{g_{i+1}}}(S_{g_{i+1}}^{k+1} - S_{g_{i+1}}^{k})$$

$$F_{g_i}^{k+1} = F_{g_i}^{k} + \frac{\partial F_{g_i}}{\partial P_{o_{i-1}}}(P_{o_{i-1}}^{k+1} - P_{o_{i-1}}^{k}) + \frac{\partial F_{g_i}}{\partial P_{o_i}}(P_{o_i}^{k+1} - P_{o_i}^{k}) + \frac{\partial F_{g_i}}{\partial P_{o_{i+1}}}(P_{o_{i+1}}^{k+1} - P_{o_{i+1}}^{k})$$
$$+ \frac{\partial F_{g_i}}{\partial S_{g_{i-1}}}(S_{g_{i-1}}^{k+1} - S_{g_{i-1}}^{k}) + \frac{\partial F_{g_i}}{\partial S_{g_i}}(S_{g_i}^{k+1} - S_{g_i}^{k}) + \frac{\partial F_{g_i}}{\partial S_{g_{i+1}}}(S_{g_{i+1}}^{k+1} - S_{g_{i+1}}^{k})$$

$$i = 1, N$$

Thus, for a one-dimensional system we have *2N* equations and *2N* unknowns, and we can easily solve for estimates of oil pressures and gas and water saturations. By applying Newtonian iteration until we converge on a solution within some tolerance, we may obtain a solution to the equations. Our linear equations for iteration step *k+1* would then take the form:

$$a_{poo_i} P_{o_{i-1}}^{k+1} + b_{poo_i} P_{o_i}^{k+1} + c_{poo_i} P_{o_{i+1}}^{k+1} + a_{sgo_i} S_{g_{i-1}}^{k+1} + b_{sgo_i} S_{g_i}^{k+1} + c_{sgo_i} S_{g_{i+1}}^{k+1} = d_{o_i}$$

$$a_{pog_i} P_{o_{i-1}}^{k+1} + b_{pog_i} P_{o_i}^{k+1} + c_{pog_i} P_{o_{i+1}}^{k+1} + a_{sgg_i} S_{g_{i-1}}^{k+1} + b_{sgg_i} S_{g_i}^{k+1} + c_{sgg_i} S_{g_{i+1}}^{k+1} = d_{g_i}$$

$$i = 1, N$$

or, on a compact form: $\hat{a}_i \vec{X}_{i-1}^{k+1} + \hat{b}_i \vec{X}_i^{k+1} + \hat{c}_i \vec{X}_{i+1}^{k+1} = \vec{d}_i, \quad i = 1,N$

where

$$\hat{a}_i = \begin{Vmatrix} a_{poo_i} & a_{sgo_i} \\ a_{pog_i} & a_{sgg_i} \end{Vmatrix} \quad \hat{b}_i = \begin{Vmatrix} b_{poo_i} & b_{sgo_i} \\ b_{pog_i} & b_{sgg_i} \end{Vmatrix} \quad \vec{X}_i^{k+1} = \begin{vmatrix} P_{o_i}^{k+1} \\ S_{g_i}^{k+1} \end{vmatrix} \quad \vec{d}_i = \begin{vmatrix} d_{o_i} \\ d_{g_i} \end{vmatrix}$$

The equations are solved for pressures and saturations iteratively, updating coefficients after each iteration.

Question 5 (*27x0,5 points*)

Explain briefly the following terms as applied to reservoir simulation (short sentence and/or a formula for each):

a) Control volume
b) Mass balance
c) Taylor series
d) Numerical dispersion
e) Explicit
f) Implicit
g) Stability
h) Upstream weighting
i) Variable bubble point
j) Harmonic average
k) Transmissibility
l) Storage coefficient
m) Coefficient matrix
n) IMPES
o) Fully implicit
p) Cross section
q) Coning
r) PI
s) Stone´s relative permeability models
t) Discretization
u) History matching
v) Prediction
w) Black Oil
x) Compositional
y) Dual porosity
z) Dual permeability

Solution

a) Control volume **small volume used in derivation of continuity equation**
b) Mass balance **principle applied to control volume in derivation of continuity equation**
c) Taylor series **expansion formula used for derivation of difference approximations (or formula:** $f(x+h)=f(x)+\frac{h}{1!}f'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+....$**)**
d) Numerical dispersion **error term associated with finite difference approximations derived by use of Taylor series**
e) Explicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t**
f) Implicit **as applied to discretization of diffusivity equation: time level used in Taylor series approximation is t+Δt**

g) Stability **as applied to implicit and explicit discretization of diffusivity equation: explicit form is conditional stable for** $\Delta t \leq \frac{1}{2}\frac{\phi\mu c}{k}(\Delta x)^2$**, while implicit form is unconditionally stable**
h) Upstream weighting **descriptive term for the choice of mobility terms in transmissibilities**
i) Variable bubble point **term that indicates that the discretization og undersaturated flow equation includes the possibility for bubble point to change, such as for the case of gas injection in undersaturated oil**
j) Harmonic average **averaging method used for permeabilities when flow is in series**
k) Transmissibility **flow coefficient in discrete equations that when muliplied with pressure difference between grid blocks yields flow rate.**
l) Storage coefficient **flow coefficient in discrete equations that when muliplied with pressure change or saturation change in a time step yields mass change in grid block**
m) Coefficient matrix **the matrix of coefficient in the set of linear equations**
n) IMPES **an approximate solution method for two or three phase equations where all coefficients and capillary pressures are computed at time level of previous time step when generating the coefficient matrix**
o) Fully implicit **an solution method for two or three phase equations where all coefficients and capillary pressures are computed at the current time level generating the coefficient matrix. Thus, iterations are required on the solution.**
p) Cross section **an x-z section of a reservoir**
q) Coning **the tendency of gas and water to form a cone shaped flow channel into the well due to pressure drawdown in the close neigborhood.**
r) PI **the productivity index of a well**
s) Stone´s relative permeability models **methods for generating 3-phase relative permeabilities for oil based on 2-phase data**
t) Discretization **converting of a contineous PDE to discrete form**
u) History matching **in simulation the adjustment of reservoir parameters so that the computed results match observed data.**
v) Prediction **computing future performance of reservoir, normally following a history matching.**
w) Black Oil **simplified hydrocarbon description model which includes two phases (oil, gas) and only two components (oil, gas), with mass transfer between the components through the solution gas-oil ratio parameter.**
x) Compositional **detailed hydrocarbon description model which includes two phases but N components (methane, ethane, propane, ...).**
y) Dual porosity **denotes a reservoir with two porosity systems, normally a fractured reservoir**
z) Dual permeability **denotes a reservoir with two permeabilities (block-to-block contact) in addition to two porosities, normally a fractured reservoir**

Question 6 *(3+3+3 points)*

The discretized form of the left hand side of the oil equation may be written in terms of transmissibilities and pressure differences, as

$$Tx_{o_{i+1/2}}(P_{oi+1} - P_{oi}) + Tx_{o_{i-1/2}}(P_{oi-1} - P_{oi})$$

Using the following transmissibility as example,

$$Tx_{o_{i-1/2}} = \frac{2k_{i-1/2}\lambda_{oi-1/2}}{\Delta x_i\left(\Delta x_i + \Delta x_{i-1}\right)}$$

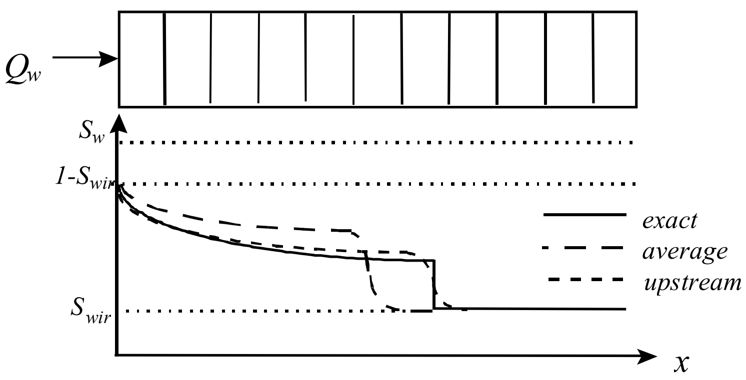
a) What type of averaging method is normally applied to absolute permeability between grid blocks? Why? Write the expression for average permeability between grid blocks (*i*-1) and (*i*).
b) Write an expression for the selection of the conventional *upstream mobility term* for use in the transmissibility term of the oil equation above for flow between the grid blocks (*i*-1) and (*i*).
c) Make a sketch of a typical Buckley-Leverett saturation profile resulting from the displacement of oil by water (ie. analytical solution). Then, show how the corresponding profile, if calculated in a numerical simulation model, typically is influenced by the choice of mobilities between the grid blocks (sketch typical curves for saturation profiles computed with upstream or average mobility terms, respectively).

Solution

a) Harmonic average is used, based on a derivation of average permeability of series flow, assuming steady flow and using Darcy´s equation

b) $$\lambda_{o_{i-1/2}} = \begin{cases} \lambda_{o_{i-1}} & if \quad P_{o_{i-1}} \geq P_{o_i} \\ \lambda_{o_i} & if \quad P_{o_{i-1}} < P_{o_i} \end{cases}$$

c)



Question 7 (3+1+1+1+1+3 points)

For a one-dimensional, horizontal, 3-phase oil, water, gas system, the general flow equations are (including well terms):

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_o' = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right),$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x} + R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_g' - R_{so}q_o' = \frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g} + R_{so}\frac{\phi S_o}{B_o}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial x}\right) - q_w' = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right)$$

a) Explain briefly the physical meaning of each term in all three equations.
b) What are the criteria for **saturated** flow? What are the functional dependencies of R_{so} and B_o?
c) What are the primary unknowns when solving the **saturated** equations?
d) What are the criteria for **undersaturated** flow? What are the functional dependencies of R_{so} and B_o?
e) What are the primary unknowns when solving the **undersaturated** equations?
f) Rewrite the equations above for **undersaturated** flow conditions.

Solution

a)

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)}_{\text{transport of oil}} - \underbrace{q_o'}_{\text{well potential}} = \underbrace{\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)}_{\text{accumulation of oil}}$$

$$\frac{\partial}{\partial x}\left(\underbrace{\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x}}_{\text{transport of free gas}} + \underbrace{R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}}_{\text{transport of sol. gas}}\right) - \underbrace{q_g'}_{\text{gas well potential}} - \underbrace{R_{so}q_o'}_{\text{oil well pot. (solution gas)}} = \frac{\partial}{\partial t}\left(\underbrace{\frac{\phi S_g}{B_g}}_{\text{accumulation. of free gas}} + \underbrace{R_{so}\frac{\phi S_o}{B_o}}_{\text{accumulation of solution gas}}\right)$$

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial x}\right)}_{\text{transport of water}} - \underbrace{q_w'}_{\text{well potential}} = \underbrace{\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right)}_{\text{accumulation of water}}$$

b) Saturated flow: criteria $P_o = P_{bp}$ and $S_g > 0$.
dependencies $B_o = f(P_o)$ and $R_{so} = f(P_o)$.

c) P_o, S_w *and* S_g

d) Undersaturated flow: criteria $P_o > P_{bp}$ and $S_g = 0$.
dependencies $B_o = f(P_o, P_{bp})$ and $R_{so} = f(P_{bp})$.

e) P_o, S_w *and* P_{bp}

f)

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_o' = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

$$\frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)-q'_g-R_{so}q'_o=\frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right),$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial x}\right)-q'_w=\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right)$$

Question 8 (2+5+2 points)

For a one-dimensional, vertical (z), 3 phase oil, water, gas system, outline how initial pressures and saturations may be computed in a simulation model, assuming that equilibrium conditions apply:

a) Sketch the reservoir, with a grid superimposed, including gas-oil-contact (GOC) and water-oil-contact (WOC).
b) Sketch the oil-gas and oil-water capillary pressure curves, and show the how the initial equilibrium pressures and saturations are determined in the continuous system.
c) Sketch the initial saturations as they are applied to the grid blocks.

Solution