THREE PHASE FLOW

Adding water to the previous oil-gas equations for a one-dimensional, horizontal system, we have the following three continuity equations:

$$-\frac{\partial}{\partial x}\left(\rho_{oL}u_o\right)=\frac{\partial}{\partial t}\left(\phi\rho_{oL}S_o\right)$$

$$-\frac{\partial}{\partial x}\left(\rho_g u_g+\rho_{oG}u_o\right)=\frac{\partial}{\partial t}\left[\phi\left(\rho_g S_g+\rho_{oG}S_o\right)\right]$$

$$-\frac{\partial}{\partial x}\left(\rho_w u_w\right)=\frac{\partial}{\partial t}\left(\phi\rho_w S_w\right)$$

and the corresponding Darcy equations for a horizontal system:

$$u_o=-\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}$$

$$u_g=-\frac{kk_{rg}}{\mu_g}\frac{\partial P_g}{\partial x}.$$

$$u_w=-\frac{kk_{rw}}{\mu_w}\frac{\partial P_w}{\partial x},$$

where

$$P_{cog}=P_g-P_o$$

$$P_{cow}=P_o-P_w$$

$$S_o+S_g+S_w=1.$$

Standard *Black Oil* PVT properties are as previously defined:

$$\rho_o=\frac{\rho_{oS}+\rho_{gS}R_{so}}{B_o}=\frac{\rho_{oS}}{B_o}+\frac{\rho_{gS}R_{so}}{B_o}=\rho_{oL}+\rho_{oG}$$

$$\rho_g=\frac{\rho_{gS}}{B_g}$$

$$\rho_w=\frac{\rho_{wS}}{B_w}$$

Undersaturated systems

We define an undersaturated system, as before, by:

$$P_o>P_{bp}$$

and

$$S_o = 0.$$

which implies that

and

$$S_g = 0 .$$

which implies that

$$B_o = f\left(P_o, P_{bp}\right)$$

and

$$R_{so} = f\left(P_{bp}\right).$$

The flow equations become:

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q'_o = \frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right)$$

and

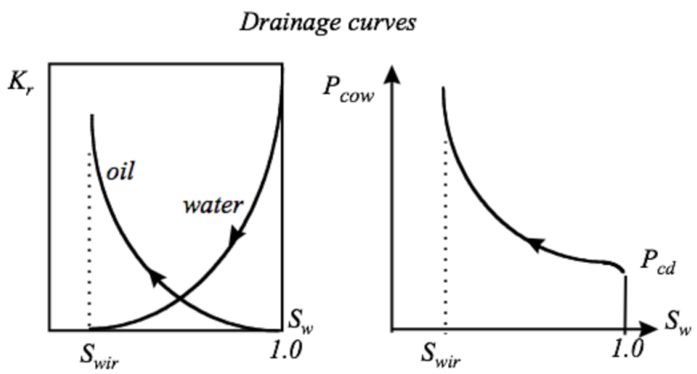
$$\frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q'_g - R_{so}q'_o = \frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right),$$

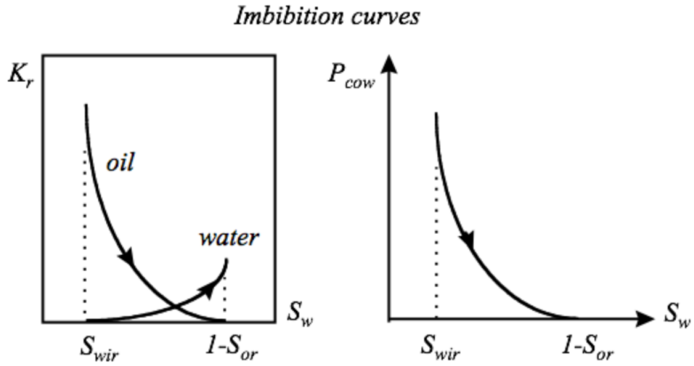
and

$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu B_w}\frac{\partial P_w}{\partial x}\right) - q'_w = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right).$$

Relative permeabilities and capillary pressures

For an undersaturated system, these relationships are just as for the oil-water system described before. Thus, the ideal drainage and imbibition curves are typically as follows:





Again, the above curves apply to a completely water-wet system. For less water-wet systems, the capillary pressure curve will have a negative part at high water saturation. The shape of the curves will depend on rock and wetting characteristics.

Boundary conditions

The boundary and source/sink conditions for undersaturated oil-gas-water systems are similar to those for undersaturated oil-gas systems. In addition to injection of gas, we may also inject water. Production wells need to account for production of water in addition to oil and solution gas. The appropriate well equations for water and oil production are identical to the ones presented in the oil-water section.

Discrete equations

Developing the discrete equations along the same principles and using similar assumptions as in the previous cases, using P_o , P_{bp} and S_w as the primary variables, we get:

$$Tx_{o_{i+1/2}}\left(P_{o_{i+1}}-P_{o_i}\right)+Tx_{o_{i-1/2}}\left(P_{o_{i-1}}-P_{o_i}\right)-q'_{oi}$$
$$=C_{poo_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpo_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{swo_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

$$\left(R_{so}Tx_o\right)_{i+1/2}\left(P_{o_{i+1}}-P_{o_i}\right)+\left(R_{so}Tx_o\right)_{i-1/2}\left(P_{o_{i-1}}-P_{o_i}\right)-\left(R_{so}q'_o\right)_i-q'_{gi}$$
$$=C_{pog_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpg_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{swg_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

$$Tx_{w_{i+1/2}}\left[\left(P_{o_{i+1}}-P_{o_i}\right)-\left(P_{cow_{i+1}}-P_{cow_i}\right)\right]+Tx_{w_{i-1/2}}\left[\left(P_{o_{i-1}}-P_{o_i}\right)-\left(P_{cow_{i-1}}-P_{cow_i}\right)\right]-q'_{wi}$$
$$=C_{pow_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpw_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{sww_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

where

$$Tx_{o_{i+1/2}}=\frac{2\lambda_{o_{i+1/2}}}{\Delta x_i\left(\dfrac{\Delta x_{i+1}}{k_{i+1}}+\dfrac{\Delta x_i}{k_i}\right)}$$

$$\lambda_o=\frac{k_{ro}}{\mu_o B_o}$$

$$\lambda_{o_{i+1/2}}=\begin{cases}\lambda_{o_{i+1}} & if\ \ P_{o_{i+1}}\ge P_{o_i}\\ \lambda_{o_i} & if\ \ P_{o_{i+1}}<P_{o_i}\end{cases}$$

$$R_{so_{i+1/2}} = \begin{cases} R_{so_{i+1}} \;\; if \;\; P_{o_{i+1}} \ge P_{o_i} \\ R_{so_i} \;\; if \;\; P_{o_{i+1}} < P_{o_i} \end{cases}$$

etc.

and

$$C_{poo_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i$$

$$C_{bpo_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i$$

$$C_{swo_i} = -\frac{\phi_i}{B_{oi}\Delta t}$$

$$C_{pog_i} = \frac{\left(R_{so}\phi\right)_i\left(1-S_{w_i}\right)}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i$$

$$C_{bpg_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left[R_{so}\frac{\partial(1/B_o)}{\partial P_{bp}} + \frac{1}{B_o}\frac{dR_{so}}{dP_{bp}}\right]_i$$

$$C_{swg_i} = -\frac{\phi_i R_{so_i}}{B_{oi}\Delta t}$$

$$C_{pow_i} = \frac{\phi_i S_{w_i}}{\Delta t}\left(\frac{c_r}{B_w} + \frac{\partial(1/B_w)}{\partial P_w}\right)_i$$

$$C_{bpwi} = 0.$$

$$C_{sww_i} = \frac{\phi_i}{B_{wi}\Delta t} - \frac{\phi_i S_{w_i}}{\Delta t}\left(\frac{c_r}{B_w} + \frac{d(1/B_w)}{dP_w}\right)_i\left(\frac{dP_{cow}}{dS_w}\right)_i$$

The derivative terms to be computed numerically for each time step based on the input table to the model, now are:

$$\left(\frac{\partial(1/B_o)}{\partial P_o}\right)_i, \left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i, \left(\frac{d(1/B_w)}{dP_w}\right)_i, \left(\frac{dR_{so}}{dP_{bp}}\right)_i \; and \; \left(\frac{dP_{cow}}{dS_w}\right)_i$$

IMPES solution

For an IMPES solution of this system of equations, assumptions equivalent to the ones made in the previous cases are made, namely

$$T_{xo}{}^{t}, T_{xw}{}^{t}$$

$$R_{so}{}^{t}, P_{cow_i}^{t}$$

$$C_{poo}{}^{t}, C_{pog}{}^{t}, C_{pow}{}^{t}$$

$$C_{bpo}{}^{t}, C_{bpg}{}^{t}, C_{bpg}{}^{t}$$

$$C_{swo}{}^{t}, C_{swg}{}^{t}, C_{sww}{}^{t}$$

resulting in the following pressure equation

$$\begin{aligned}
&\left[Txo_{i+1/2}^{t} + \alpha_i\left(R_{so}T_{xo}\right)_{i+1/2}^{t} + \beta_i Txw_{i+1/2}^{t}\right]\left(Po_{i+1} - Po_i\right) + \\
&\left[Txo_{i-1/2}^{t} + \alpha_i\left(R_{so}T_{xo}\right)_{i-1/2}^{t} + \beta_i Txw_{i-1/2}^{t}\right]\left(Po_{i-1} - Po_i\right) \\
&-\beta_i Txw_{i+1/2}^{t}\left(Pcow_{i+1} - Pcow_i\right)^t - \beta_i Txw_{i-1/2}^{t}\left(Pcow_{i-1} - Pcow_i\right)^t \\
&-q'_{oi} - \alpha_i\left(q'_g + R_{so}{}^{t}q'_o\right)_i - \beta_i q'_{wi} = \\
&\left(Cpoo_i^t + \alpha_i Cpog_i^t + \beta_i Cpow_i^t\right)\left(Po_i - Po_i^t\right), \qquad i = 1, N
\end{aligned}$$

where

$$\alpha_i = -Cbpo_i^t / Cbpg_i^t$$

$$\beta_i = \frac{Cswo_i^t}{Csww_i^t}\left(\frac{Cswg_i^t Cbpo_i^t}{Cswo_i^t Cbpg_i^t} - 1\right).$$

Rewriting the pressure equation on the familiar form

$$a_i Po_{i-1} + b_i Po_i + c_i Po_{i+1} = d_i, \quad i = 1, N$$

we may solve for oil pressure by, for instance, as before, Gaussian elimination. Then, having obtained the oil pressures, we may combine the equations above to solve for bubble point pressures and water saturations. If the water equation are used for water saturation, since bubble point pressure does not enter this equation, and the oil equation for the bubble point pressures, we get the following explicit expressions:

$$\begin{aligned}
Sw_i = Sw_i^t + \frac{1}{Csww_i^t}\Big[\; & Txw_{i+1/2}^{t}\left[\left(Po_{i+1} - Po_i\right) - \left(Pcow_{i+1} - Pcow_i\right)^t\right] + Txw_{i-1/2}^{t}\left[\left(Po_{i-1} - Po_i\right) - \left(Pcow_{i-1} - Pcow_i\right)^t\right] \\
& - q'_{wi} - Cpow_i^t\left(Po_i - Po_i^t\right)\;\Big], \qquad i = 1, N
\end{aligned}$$

$$\begin{aligned}
Pbp_i = Pbp_i^t + \frac{1}{Cbpo_i^t}\Big[\; & Txo_{i+1/2}^{t}\left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^{t}\left(Po_{i-1} - Po_i\right) - q'_{oi} \\
& - Cpoo_i^t\left(Po_i - Po_i^t\right) - Cswo_i\left(Sw_i - Sw_i^t\right)\;\Big], \qquad i = 1, N
\end{aligned}$$

Saturated systems

We define a saturated system by:

$$P_o = P_{bp}$$

and

$$S_o \geq 0.$$

and thus

$$B_o = f(P_o)$$

$$R_{so} = f(P_o)$$

The flow equations become:

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q'_o = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

and

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x} + R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q'_g - R_{so} q'_o = \frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g} + R_{so}\frac{\phi S_o}{B_o}\right),$$

and

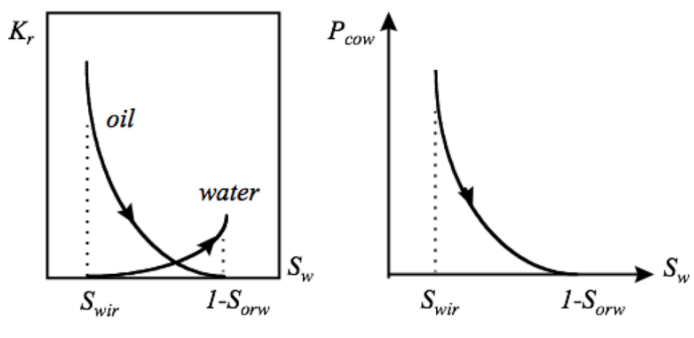
$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial x}\right) - q'_w = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right)$$

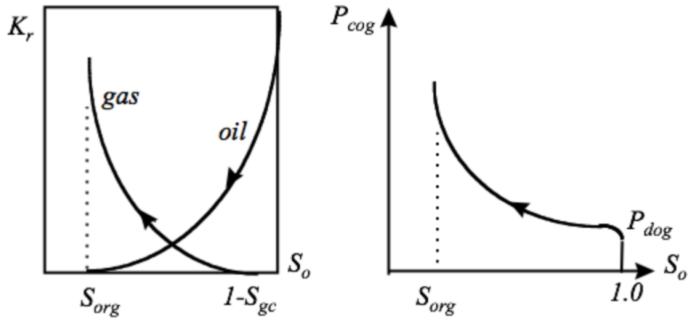
Three phase relative permeabilities and capillary pressures

Since we now have three phases flowing, we need to define the relative permeabilities and capillary pressures anew. Although the following functional relationship not always are valid in practice, we will here use the conventional definitions for a completely water wet system with no contact between gas and water phases. Thus, the parameters below are functions only of the variables indicated:

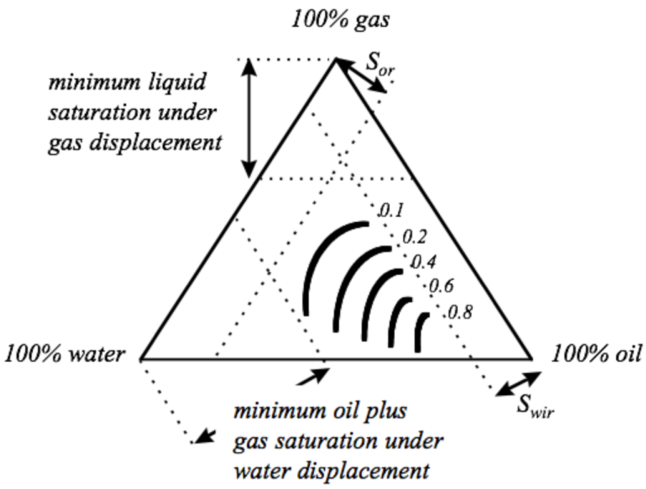
$$k_{rw}(S_w)$$
$$k_{rg}(S_g)$$
$$k_{ro}(S_w, S_g)$$
$$P_{cow}(S_w)$$
$$P_{cog}(S_g)$$

Using curves for imbibition oil-water processes and drainage gas-oil processes, typical relationships are as follows:





However, the two oil relative permeability curves above are two phase curves. As indicated, the three phase oil relative permeability would be a function of both water and gas saturations. Plotting it in a ternary diagram, so that each saturation is represented by one of the sides, we can define an area of mobile oil limited by the system's maximum and minimum saturations (which not necessarily are constants). Inside this area, $iso-k_{ro}$ curves may be drawn, as illustrated below:



However, due to the experimental difficulties of measuring three phase k_{ro}, we most of the time construct it from two phase oil-water k_{row} and two phase oil-gas k_{rog}. The simplest approach is to just multiply the to

$$k_{ro} = k_{rog} k_{rog} .$$

However, since some of the limiting saturations in three phase flow not necessarily are the same as for two phase flow, this model is not representative. For instance, the minimum oil saturation, S_{or}, for three phase flow is process dependent and a very difficult parameter to estimate.

The so-called Stone-models are simple, but have been the most commonly used models, although a variety of models exist. For the purpose of illustration, we will describe Stone's model 1 and model 2. For ***Stone's model 1***, we define normalized saturations as

$$S_{oD} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or}}$$

$$S_{wD} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or}}$$

$$S_{gD} = \frac{S_g}{1 - S_{wir} - S_{or}}$$

Then we define the functions

$$\beta_w = \frac{k_{row}}{1 - S_{wD}}$$

$$\beta_g = \frac{k_{rog}}{1 - S_{gD}} .$$

The three phase oil relative permeability is defined as

$$k_{ro} = S_{oD}\beta_w\beta_g$$

Please note that the above formulas assume that end pont relative permeabilities are 1. If this is not the case, the relative permeability formula must be modified accordingly.

Stone's model 2 does not require the estimation of S_{or}, as it attempts to estimate it implicitly by its formulation. The model simply is

$$k_{ro} = (k_{rog} + k_{rg})(k_{row} + k_{rw}) - (k_{rw} + k_{rg}) .$$

In this model, S_{or} is defined by k_{ro} becoming negative. The two models of Stone predict quite different k_{ro}'s in many cases, and one should be very careful in selecting which model to use in each situation. Several other methods exist.

Boundary conditions

The boundary conditions for saturated oil-gas-water systems are similar to the boundary conditions for saturated oil-gas systems, with the addition of water similarly to the procedures presented in the oil-water section. Thus, we may have injection of gas and water, and production wells need to account for production of water in addition to oil, solution gas and free gas. The appropriate well equations for water, gas and oil production are identical to the ones presented in the oil-water, and int the saturated oil-gas sections.

Discrete equations

Again, developing the discrete equations as before, but now using P_o, S_g and S_w as the primary variables, we get:

$$Tx_{o_{i+1/2}}\left(P_{o_{i+1}}-P_{o_i}\right)+Tx_{o_{i-1/2}}\left(P_{o_{i-1}}-P_{o_i}\right)-q'_{oi}$$
$$=C_{poo_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgo_i}\left(S_{g_i}-S_{g_i}^t\right)+C_{swo_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

$$Tx_{g_{i+1/2}}\left[\left(P_{o_{i+1}}-P_{o_i}\right)+\left(P_{cog_{i+1}}-P_{cog_i}\right)\right]+Tx_{g_{i-1/2}}\left[\left(P_{o_{i-1}}-P_{o_i}\right)+\left(P_{cog_{i-1}}-P_{cog_i}\right)\right]-q'_{gi}$$
$$\left(R_{so}Tx_o\right)_{i+1/2}\left(P_{o_{i+1}}-P_{o_i}\right)+\left(R_{so}Tx_o\right)_{i-1/2}\left(P_{o_{i-1}}-P_{o_i}\right)-\left(R_{so}q'_o\right)_i$$
$$=C_{pog_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgg_i}\left(S_{g_i}-S_{g_i}^t\right)+C_{swg_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

$$Tx_{w_{i+1/2}}\left[\left(P_{o_{i+1}}-P_{o_i}\right)-\left(P_{cow_{i+1}}-P_{cow_i}\right)\right]+Tx_{o_{i-1/2}}\left[\left(P_{o_{i-1}}-P_{o_i}\right)-\left(P_{cow_{i-1}}-P_{cow_i}\right)\right]-q'_{wi}$$
$$=C_{pow_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgw_i}\left(S_{g_i}-S_{g_i}^t\right)+C_{sww_i}\left(S_{w_i}-S_{w_i}^t\right),\qquad i=1,N$$

where, as before

$$Tx_{o_{i+1/2}}=\frac{2\lambda_{o_{i+1/2}}}{\Delta x_i\left(\dfrac{\Delta x_{i+1}}{k_{i+1}}+\dfrac{\Delta x_i}{k_i}\right)}$$

$$\lambda_o=\frac{k_{ro}}{\mu_o B_o}$$

$$\lambda_{o_{i+1/2}}=\begin{cases}\lambda_{o_{i+1}} & if\ \ P_{o_{i+1}}\ge P_{o_i}\\ \lambda_{o_i} & if\ \ P_{o_{i+1}}<P_{o_i}\end{cases}$$

$$R_{so_{i+1/2}}=\begin{cases}R_{so_{i+1}} & if\ \ P_{o_{i+1}}\ge P_{o_i}\\ R_{so_i} & if\ \ P_{o_{i+1}}<P_{o_i}\end{cases}$$

etc.

and

$$C_{poo_i}=\frac{\phi_i(1-S_{w_i}-S_{g_i})}{\Delta t}\left(\frac{c_r}{B_o}+\frac{d(1/B_o)}{dP_o}\right)_i$$

$$C_{sgo_i}=-\frac{\phi_i}{B_{oi}\Delta t}$$

$$C_{swo_i}=-\frac{\phi_i}{B_{oi}\Delta t}$$

$$Cpog_i = \frac{\phi_i}{\Delta t}\left[Sg_i\left(\frac{c_r}{B_g}+\frac{d(1/B_g)}{dP_g}\right)_i + R_{soi}\left(1-Sw_i-Sg_i\right)\left(\frac{c_r}{B_o}+\frac{d(1/B_o)}{dP_o}\right)_i + \frac{\left(1-Sw_i-Sg_i\right)}{B_{oi}}\left(\frac{dR_{so}}{dP_o}\right)_i\right]$$

$$Csgg_i = \frac{\phi_i}{\Delta t}\left[S_g\left(\frac{c_r}{B_g}+\frac{d(1/B_g)}{dP_g}\right)\frac{dP_{cog}}{dS_g} - \frac{R_{so}}{B_o}+\frac{1}{B_g}\right]_i$$

$$Cswg_i = -\frac{\phi_i Rso_i}{B_{oi}\Delta t}$$

$$Cpow_i = \frac{\phi_i Sw_i}{\Delta t}\left(\frac{c_r}{B_w}+\frac{\partial(1/B_w)}{\partial P_w}\right)_i$$

$$Csgw_i = 0$$

$$Csww_i = \frac{\phi_i}{B_{wi}\Delta t} - \frac{\phi_i Sw_i}{\Delta t}\left(\frac{c_r}{B_w}+\frac{d(1/B_w)}{dP_w}\right)_i\left(\frac{dPcow}{dS_w}\right)_i$$

The derivative terms to be computed numerically for each time step based on the input table to the model, now are:

$$\left(\frac{d(1/B_o)}{dP_o}\right)_i, \left(\frac{d(1/B_g)}{dP_g}\right)_i, \left(\frac{d(1/B_w)}{dP_w}\right)_i, \left(\frac{dR_{so}}{dP_o}\right)_i, \left(\frac{dP_{cog}}{dS_g}\right)_i \text{ and } \left(\frac{dP_{cow}}{dS_w}\right)_i$$

IMPES solution

We again assume that all the coefficients are at old time level:

$$Txo^t, Txg^t, Txw^t$$
$$Rso^t, Pcog_i^t, Pcow_i^t$$
$$Cpoo^t, Cpog^t, Cpow^t$$
$$Csgo^t, Csgg^t, Csgw^t$$
$$Cswo^t, Cswg^t, Csww^t$$

resulting in the following pressure equation

$$\left[Txo_{i+1/2}^t + \alpha_i\left(T_{xg}+R_{so}T_{xo}\right)_{i+1/2}^t + \beta_i Txw_{i+1/2}^t\right](Po_{i+1}-Po_i) +$$
$$\left[Txo_{i-1/2}^t + \alpha_i\left(T_{xg}+R_{so}T_{xo}\right)_{i-1/2}^t + \beta_i Txw_{i-1/2}^t\right](Po_{i-1}-Po_i)$$
$$+\alpha_i Txg_{i+1/2}^t\left(Pcog_{i+1}-Pcog_i\right)^t + \alpha_i Txg_{i-1/2}^t\left(Pcog_{i-1}-Pcog_i\right)^t$$
$$-\beta_i Txw_{i+1/2}^t\left(Pcow_{i+1}-Pcow_i\right)^t - \beta_i Txw_{i-1/2}^t\left(Pcow_{i-1}-Pcow_i\right)^t$$
$$-q'_{oi} - \alpha_i\left(q'_g + R_{so}{}^t q'_o\right)_i - \beta_i q'_{wi} =$$
$$\left(Cpoo_i^t + \alpha_i Cpog_i^t + \beta_i Cpow_i^t\right)\left(Po_i - Po_i^t\right), \qquad i = 1,N$$

where

$$\alpha_i = -Cs_{go_i}^{\;t} / Cs_{gg_i}^{\;t}$$

$$\beta_i = \frac{Cs_{wo_i}^{\;t}}{Cs_{ww_i}^{\;t}} \left(\frac{Cs_{wg_i}^{\;t} Cs_{go_i}^{\;t}}{Cs_{wo_i}^{\;t} Cs_{gg_i}^{\;t}} - 1 \right).$$

Again rewriting the pressure equation on the familiar form

$$a_i Po_{i-1} + b_i Po_i + c_i Po_{i+1} = d_i, \quad i = 1, N$$

we may solve for oil pressure using Gaussian elimination or some other method. Then, by combining the equations above, we obtain the following explicit expressions for the two saturations:

$$Sw_i = Sw_i^{\;t} + \frac{1}{Csww_i^{\;t}} \Big[\; Txw_{i+1/2}^{\;t} \Big[\big(Po_{i+1} - Po_i\big) - \big(Pcow_{i+1} - Pcow_i\big)^t \Big] + Txw_{i-1/2}^{\;t} \Big[\big(Po_{i-1} - Po_i\big) - \big(Pcow_{i-1} - Pcow_i\big)^t \Big]$$

$$- q'_{wi} - Cpow_i^{\;t} \big(Po_i - Po_i^{\;t}\big) \; \Big], \qquad i = 1, N$$

$$Sg_i = Sg_i^{\;t} + \frac{1}{Csgg_i^{\;t}} \Big[\; Txo_{i+1/2}^{\;t} \big(Po_{i+1} - Po_i\big) + Txo_{i-1/2}^{\;t} \big(Po_{i-1} - Po_i\big) - q'_{oi}$$

$$- Cpoo_i^{\;t} \big(Po_i - Po_i^{\;t}\big) - Cswo_i^{\;t} \big(Sw_i - Sw_i^{\;t}\big) \; \Big], \qquad i = 1, N$$