SATURATED OIL-GAS SIMULATION - IMPES SOLUTION

The major difference between two-phase oil-water flow and two-phase, saturated oil-gas flow is that the solution gas terms have to be included in the flow equations. Recall from the previous review of *Black Oil* PVT behavior that the oil density at reservoir conditions is defined as:

$$\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o}$$

First of all, for a saturated oil-gas system, the oil pressure is per definition equal to the bubble point pressure, or saturation pressure:

$$P_o = P_{bp}$$

and, in addition,

$$S_o \geq 0 .$$

The implication of these definitions, is that the formation volume factor and the solution gas-oil ratio are functions of oil pressure only,

$$B_o = f(P_o)$$

$$R_{so} = f(P_o)$$

Thus, for saturated oil, the solution gas term is no longer constant and will not be canceled out of the oil equation, as it did in single phase flow and in oil-water flow. We will, for mass balance purposes, separate the oil density into two parts, one that remains liquid at the surface and one that becomes gas:

$$\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o} = \frac{\rho_{oS}}{B_o} + \frac{\rho_{gS} R_{so}}{B_o} = \rho_{oL} + \rho_{oG}$$

We will write the oil mass balance so that its continuity equation includes the liquid part only, while the gas mass balance includes both free gas and solution gas in the reservoir, and thus all free gas at the surface:

$$-\frac{\partial}{\partial x}\left(\rho_{oL} u_o\right) = \frac{\partial}{\partial t}\left(\phi \rho_{oL} S_o\right)$$

$$-\frac{\partial}{\partial x}\left(\rho_g u_g + \rho_{oG} u_o\right) = \frac{\partial}{\partial t}\left[\phi\left(\rho_g S_g + \rho_{oG} S_o\right)\right].$$

In the gas equation, the solution gas will of course flow with the rest of the oil in the reservoir, at oil relative permeability, viscosity and pressure.

The Darcy equations for the two phases are:

$$u_o = -\frac{k k_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}$$

$$u_g = -\frac{k k_{rg}}{\mu_g}\frac{\partial P_g}{\partial x} .$$

Substituting Darcy's equations and the liquid oil density and the solution gas density definitions, together with the standard free gas density definition,

$$\rho_g = \frac{\rho_{gS}}{B_g}$$

into the continuity equations, and including production/injection terms in the equations, results in the following flow equations for the two phases:

$$\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_o' = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

and

$$\frac{\partial}{\partial x}\left(\frac{k k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x} + R_{so}\frac{k k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_g' - R_{so} q_o' = \frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g} + R_{so}\frac{\phi S_o}{B_o}\right),$$

where

$$P_{cog} = P_g - P_o$$

$$S_o + S_g = 1 .$$

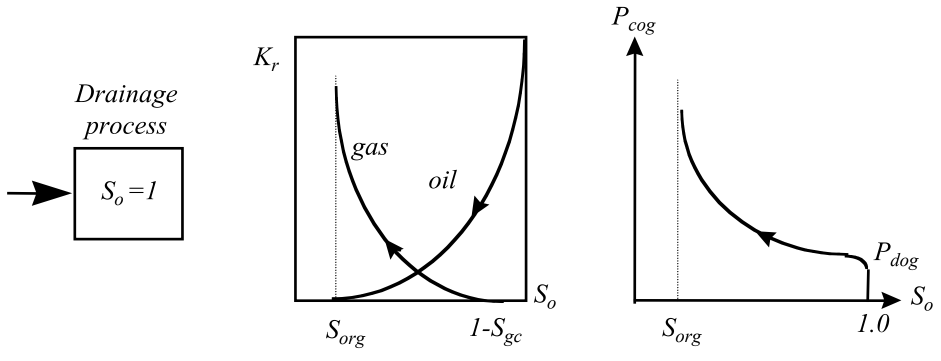
Relative permeabilities and capillary pressure are functions of gas saturation, while formation volume factors, viscosities and porosity are functions of pressures.

Fluid properties are defined by the standard *Black Oil* model for saturated oil, as we have reviewed previously. Before proceeding, we shall also review the relative permeabilities and capillary pressure relationships for oil-gas systems.

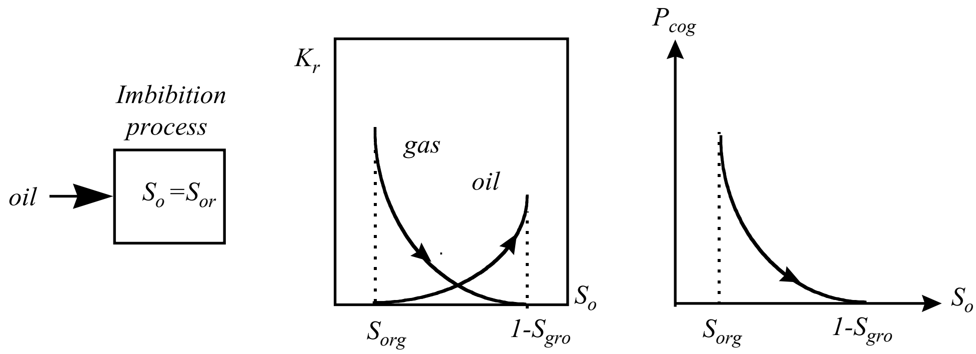
Review of oil-gas relative permeabilities and capillary pressure

Normally, only drainage curves are required in gas-oil systems, since gas displaces oil. However, sometimes reimbibition of oil into areas previously drained by gas displacement may happen. Reimbibition phenomena may be particularly important in gravity drainage processes in fractured reservoirs.

Starting with the porous rock completely filled with oil, and displacing by gas, the drainage relative permeability and capillary pressure curves will be defined:



If the process is reversed when all mobile oil has been displaced, by injecting oil to displace the gas, imbibition curves are defined as:



The shape of the gas-oil curves will of course depend on the surface tension properties of the system, as well as on the rock characteristics.

Discretization of flow equations

The discretization procedure for oil-gas equations is very much similar to the one for oil-water equations. In fact, for the oil equation, it is identical, with a small exception for the saturation, which now is for gas and not water. Thus, the *discretized oil equation* may be written:

$$T_{xo_{i+1/2}}\left(P_{o_{i+1}}-P_{o_i}\right)+T_{xo_{i-1/2}}\left(P_{o_{i-1}}-P_{o_i}\right)-q'_{oi}$$
$$=C_{poo_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{sgo_i}\left(S_{g_i}-S_{g_i}^t\right),\qquad i=1,N$$

Definitions of the terms in the equation are given below:

$$T_{xo_{i+1/2}}=\frac{2\lambda_{o_{i+1/2}}}{\Delta x_i\left(\frac{\Delta x_{i+1}}{k_{i+1}}+\frac{\Delta x_i}{k_i}\right)}$$

$$T_{xo_{i-1/2}}=\frac{2\lambda_{o_{i-1/2}}}{\Delta x_i\left(\frac{\Delta x_{i-1}}{k_{i-1}}+\frac{\Delta x_i}{k_i}\right)}$$

where

$$\lambda_o=\frac{k_{ro}}{\mu_o B_o}$$

and the upstream mobilities are selected as:

$$\lambda_{o_{i+1/2}}=\begin{cases}\lambda_{o_{i+1}} & if\ \ P_{o_{i+1}}\ge P_{o_i}\\ \lambda_{o_i} & if\ \ P_{o_{i+1}}<P_{o_i}\end{cases}$$

$$\lambda_{o_{i-1/2}}=\begin{cases}\lambda_{o_{i-1}} & if\ \ P_{o_{i-1}}\ge P_{o_i}\\ \lambda_{o_i} & if\ \ P_{o_{i-1}}<P_{o_i}\end{cases}$$

The right side coefficients are:

$$C_{poo_i}=\frac{\phi_i(1-S_{g_i})}{\Delta t}\left[\frac{c_r}{B_o}+\frac{d(1/B_o)}{dP_o}\right]_i$$

$$C_{sgo_i} = -\frac{\phi_i}{B_{oi}\Delta t_i}$$

Left hand side of gas equation

For the gas equation, we will partly use similar approximations as for the oil equation, and also introduce new approximations for the solution gas terms. First,

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x} + R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x}\right) + \frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)$$

Then, we use similar approximations for the *free gas term* as we did for oil and for water:

$$\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x}\right)_i \approx T_{xgi+1/2}(P_{gi+1} - P_{gi}) + T_{xgi-1/2}(P_{gi-1} - P_{gi})$$

where the gas transmissibilities are defined as:

$$T_{xg_{i+1/2}} = \frac{2\lambda_{g_{i+1/2}}}{\Delta x_i\left(\frac{\Delta x_{i+1}}{k_{i+1}} + \frac{\Delta x_i}{k_i}\right)}$$

$$T_{xg_{i-1/2}} = \frac{2\lambda_{g_{i-1/2}}}{\Delta x_i\left(\frac{\Delta x_{i-1}}{k_{i-1}} + \frac{\Delta x_i}{k_i}\right)}$$

where

$$\lambda_g = \frac{k_{rg}}{\mu_g B_g}$$

and the upstream mobilities are selected as:

$$\lambda_{g_{i+1/2}} = \begin{cases} \lambda_{g_{i+1}} & if \;\; P_{g_{i+1}} \ge P_{g_i} \\ \lambda_{g_i} & if \;\; P_{g_{i+1}} < P_{g_i} \end{cases}$$

$$\lambda_{g_{i-1/2}} = \begin{cases} \lambda_{g_{i-1}} & if \;\; P_{g_{i-1}} \ge P_{g_i} \\ \lambda_{g_i} & if \;\; P_{g_{i-1}} < P_{g_i} \end{cases}$$

The solution gas term may be approximated as the oil flow term, with the exception that the solution term has to be included as follows:

$$\frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)_i \approx \left(R_{so}T_{xo}\right)_{i+1/2}(P_{oi+1} - P_{oi}) + \left(R_{so}T_{xo}\right)_{i-1/2}(P_{oi-1} - P_{oi})$$

For the solution gas-oil ratios, we will again use the upstream principle, just as for the mobilities:

$$R_{so_{i+1/2}} = \begin{cases} R_{so_{i+1}} \;\; if \;\; P_{o_{i+1}} \geq P_{o_i} \\ R_{so_i} \;\; if \;\; P_{o_{i+1}} < P_{o_i} \end{cases}$$

$$R_{so_{i-1/2}} = \begin{cases} R_{so_{i-1}} \;\; if \;\; P_{o_{i-1}} \geq P_{o_i} \\ R_{so_i} \;\; if \;\; P_{o_{i-1}} < P_{o_i} \end{cases}$$

Right hand side of gas equation

The right hand side of the gas equation consists of a *free gas* term and a *solution gas* term:

$$\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g} + \frac{\phi R_{so} S_o}{B_o}\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}\right) + \frac{\partial}{\partial t}\left(\frac{\phi R_{so} S_o}{B_o}\right)$$

Using similar approximations as for water for the *free gas* term, we may write:

$$\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}\right) \approx \frac{\phi_i S_{g_i}}{\Delta t}\left(\frac{c_r}{B_g} + \frac{d(1/B_g)}{dP_g}\right)_i \left[(P_{o_i} - P_{o_i}^t) + \left(\frac{dP_{cog}}{dS_g}\right)_i (S_{g_i} - S_{g_i}^t)\right] + \frac{\phi_i}{B_{g_i}\Delta t}(S_{g_i} - S_{g_i}^t)\,.$$

The *solution gas* term may be expanded into:

$$\frac{\partial}{\partial t}\left(\frac{\phi R_{so} S_o}{B_o}\right) = R_{so}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) + \frac{\phi S_o}{B_o}\frac{\partial R_{so}}{\partial t}$$

The first term is identical to the right hand side of the oil equation, multiplied by R_{so} . Thus,

$$\left[R_{so}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)\right]_i \approx R_{so_i} C_{poo_i} + R_{so_i} C_{sgo_i}$$

For the second term, the following approximation may be used:

$$\left(\frac{\phi S_o}{B_o}\frac{\partial R_{so}}{\partial t}\right)_i = \left(\frac{\phi S_o}{B_o}\frac{dR_{so}}{dP_o}\frac{\partial P_o}{\partial t}\right)_i \approx \frac{1}{\Delta t}\left(\frac{\phi S_o}{B_o}\frac{dR_{so}}{dP_o}\right)_i \left(P_{o_i} - P_{o_i}^t\right)$$

Then, combining the terms, the approximation of the gas equation becomes:

$$\begin{aligned} &T_{xg_{i+1/2}}\left[\left(P_{o_{i+1}} - P_{o_i}\right) + \left(P_{cog_{i+1}} - P_{cog_i}\right)\right] + T_{xg_{i-1/2}}\left[\left(P_{o_{i-1}} - P_{o_i}\right) + \left(P_{cog_{i-1}} - P_{cog_i}\right)\right] - q'_{gi} \\ &+ \left(R_{so}T_{xo}\right)_{i+1/2}\left(P_{o_{i+1}} - P_{o_i}\right) + \left(R_{so}T_{xo}\right)_{i-1/2}\left(P_{o_{i-1}} - P_{o_i}\right) - \left(R_{so}q'_o\right)_i \\ &\qquad = C_{pog_i}\left(P_{o_i} - P_{o_i}^t\right) + C_{sgg_i}\left(S_{g_i} - S_{g_i}^t\right), \qquad i = 1, N \end{aligned}$$

where

$$C_{pog_i} = \frac{\phi_i}{\Delta t}\left[S_g\left(\frac{c_r}{B_g} + \frac{d(1/B_g)}{dP_g}\right) + R_{so}(1 - S_g)\left(\frac{c_r}{B_o} + \frac{d(1/B_o)}{dP_o}\right) + \frac{(1 - S_g)}{B_o}\frac{dR_{so}}{dP_o}\right]_i$$

and

$$C_{sgg_i} = \frac{\phi_i}{\Delta t}\left[S_g\left(\frac{c_r}{B_g} + \frac{d(1/B_g)}{dP_g}\right)\frac{dP_{cog}}{dS_g} - \frac{R_{so}}{B_o} + \frac{1}{B_g}\right]_i$$

The derivative terms appearing in the expressions above:

$$\left(\frac{d(1/B_o)}{dP_o}\right)_i, \left(\frac{d(1/B_g)}{dP_g}\right)_i, \left(\frac{dR_{so}}{dP_o}\right)_i \; and \left(\frac{dP_{cog}}{dS_g}\right)_i$$

are all computed numerically for each time step based on the input table to the model.

Boundary conditions

The boundary conditions for oil-gas systems are similar to those of oil-water systems. Normally, we inject gas in a grid block at constant surface rate or at constant bottom hole pressure, and produce oil and gas from a grid block at constant bottom hole pressure, or at constant surface oil rate. As for oil-water flow, we may sometimes want to specify constant reservoir voidage rate, where either the rate of injection of gas is to match a specified rate of oil and gas production at reservoir conditions, so that average reservoir pressure remains constant, or the reservoir production rate is to match a specified gas injection rate.

Constant gas injection rate

Again, as for water injection, a gas rate term is already included in the gas equation. Thus, for a constant surface gas injection rate of Q_{gi} (negative) in a well in grid block i :

$$q'_{gi} = Q_{gi} / (A\Delta x_i)\,.$$

Then, at the end of a time step, after having solved the equations, the bottom hole injection pressure for the well may be calculated using the well equation:

$$Q_{gi} = WC_i \lambda_{gi}(P_{g_i} - P_{bh_i})\,.$$

The well constant in the equation above is defined just as for oil-water flow:

$$WC_i = \frac{2\pi k_i h}{ln(\frac{r_e}{r_w})},$$

where r_w is the well radius and the drainage radius is theoretically defined as:

$$r_e = \sqrt{\frac{\Delta y \Delta x_i}{\pi}}\,.$$

As for water injection, we will use the sum of the mobilities of the fluids present in the injection block in the well equation. Thus, the following well equation is used for the injection of gas in an oil-gas system:

$$Q_{gi}B_{gi} = WC_i\left(\frac{kr_{o_i}}{\mu_{oi}} + \frac{kr_{g_i}}{\mu_{gi}}\right)(P_{g_i} - P_{bh_i}),$$

or

$$Q_{gi} = WC_i\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(P_{g_i} - P_{bh_i})$$

If the injection wells are constrained by a maximum bottom hole pressure, to avoid fracturing of the formation, this should be checked at the end of each time step, and, if necessary, be followed by a reduction of the injection rate, or by conversion of the well to a constant bottom hole pressure injection well.

Just as for the water injection case, capillary pressure is normally neglected in the well equation, particularly in the case of field scale simulation, so that the well equation becomes:

$$Q_{gi} = WC_i\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(P_{o_i} - P_{bh_i}).$$

For gas-oil flow, the capillary pressure is normally small, so that even in simulation of cores used in laboratory experiments, the errors resulting from neglecting capillary pressure in the well equation will be small.

Injection at constant bottom hole pressure

The well equation for injection at constant bottom hole pressure is the same as the one above:

$$Q_{gi} = WC_i\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(P_{g_i} - P_{bh_i})$$

or, if the capillary pressure of the injection block is neglected:

$$Q_{gi} = WC_i\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(P_{o_i} - P_{bh_i}).$$

Again, the terms of the equation must be included in the appropriate coefficients in the pressure solution. At the end of the time step, the above equation may be used to compute the actual gas injection rate for the step.

Constant oil production rate

For the oil equation, this condition is handled as for the constant water injection rate. Thus, for a constant surface oil production rate of Q_{oi} (positive) in a well in grid block i :

$$q'_{oi} = Q_{oi} / (A\Delta x_i).$$

However, oil production will always be accompanied by solution gas production, and in addition, the well may produce free gas. The gas equation will thus have gas production terms given by:

$$q'_{gsi} = q'_{oi} R_{so_i} \qquad \text{(solution gas)}$$

and

$$q'_{gfi} = q'_{oi}\frac{\lambda_{gi}}{\lambda_{oi}} + \lambda_{gi}P_{cogi} \qquad \text{(free gas)}$$

In case the gas-oil capillary pressure is neglected around the production well, the total gas production becomes:

$$q'_{gti} = q'_{gsi} + q'_{gfi} = q'_{oi}\left(\frac{\lambda_{gi}}{\lambda_{oi}} + R_{so_i}\right).$$

At the end of a time step, after having solved the equations, the bottom hole production pressure for the well may be calculated using the well equation for oil:

$$Q_{oi} = WC_i\lambda_{oi}(P_{o_i} - P_{bh_i}).$$

As for oil-water systems, production wells in oil-gas systems are normally constrained by a minimum bottom hole pressure. If this is reached, the well should be converted to a constant bottom hole pressure well.

The gas-oil ratio at the surface is:

$$\mathrm{GOR}_i = \frac{q'_{gti}}{q'_{oi}},$$

which for negligible capillary pressure in the producing grid block reduces to the familiar expression:

$$\mathrm{GOR}_i = \frac{\lambda_{gi}}{\lambda_{oi}} + R_{so_i}.$$

Frequently, well rates are constrained by maximum GOR levels, due to limitations in process equipment. If a maximum gas-oil ratio level is exceeded for a well, the highest GOR grid block may be shut in, in case more than one gridblocks are perforated, or the production rate may have to be reduced.

<u>Production at constant reservoir voidage rate</u>

As for the oil-water system, the total production of fluids from a well in block □, at reservoir conditions, is to match the reservoir injection volume so that the reservoir pressure remains approximately constant. Thus,

$$Q_{oi}B_{oi} + Q_{gi}B_{gi} = -Q_{g_{inj}}B_{g_{inj}},$$

which, again assuming that capillary pressure is negligible, leads to:

$$q'_{oi} = \frac{\lambda_{oi}}{\lambda_{oi}B_{oi} + \lambda_{gi}B_{gi}}(-Q_{gi}B_{g_{inj}})/(A\Delta x_i)$$

and

$$q'_{gi} = \frac{\lambda_{gi}}{\lambda_{oi}B_{oi} + \lambda_{gi}B_{gi}}(-Q_{gi}B_{g_{inj}})/(A\Delta x_i).$$

The solution gas rate term thus becomes:

$$R_{so_i}q'_{oi} = \frac{R_{so_i}\lambda_{oi}}{\lambda_{oi}B_{oi} + \lambda_{gi}B_{gi}}(-Q_{gi}B_{g_{inj}})/(A\Delta x_i)$$

<u>Production at constant bottom hole pressure</u>

Using a production well in grid block i with constant bottom hole pressure, P_{bh_i}, as an example, we have an oil rate of:

$$Q_{gi} = WC_i\lambda_{oi}(P_{o_i} - P_{bh_i})$$

and a free gas rate of:

$$Q_{gi} = WC_i\lambda_{gi}(P_{g_i} - P_{bh_i}).$$

Substituting into the flow terms in the flow equations, the oil rate becomes:

$$q'_{oi} = \frac{WC_i}{A\Delta x_i}\lambda_{oi}(P_{o_i} - P_{bh_i}),$$

and the free gas rate:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\lambda_{gi}(P_{g_i} - P_{bh_i})$$

and finally the solution gas rate:

$$R_{so_i}q'_{oi} = R_{so_i}\frac{WC_i}{A\Delta x_i}\lambda_{oi}(P_{o_i} - P_{bh_i})$$

Again, the flow rate terms have to be included in the appropriate matrix coefficients when solving for pressures. At the end of each time step, actual rates are computed by the equations above, and GOR is computed as in the previous cases.

Solution by IMPES method

The procedure for IMPES solution is similar to the oil-water case. Thus, we make the same assumptions in regard to the coefficients:

$$\begin{aligned}&T_{xo}{}^t, T_{xg}{}^t\\ &C_{poo}{}^t, C_{pog}{}^t\\ &C_{sgo}{}^t, C_{sgg}{}^t\\ &P_{cog}{}^t\end{aligned}.$$

Having made these approximations, the discretized flow equations become:

$$\begin{aligned}&T_{xo}{}^t_{i+1/2}\left(P_{o_{i+1}} - P_{o_i}\right) + T_{xo}{}^t_{i-1/2}\left(P_{o_{i-1}} - P_{o_i}\right) - q'_{oi}\\ &\qquad = C_{poo}{}^t_i\left(P_{o_i} - P_{o_i}^t\right) + C_{sgo}{}^t_i\left(S_{g_i} - S_{g_i}^t\right), \qquad i = 1, N\end{aligned}$$

$$\begin{aligned}&T_{xg}{}^t_{i+1/2}\left[\left(P_{o_{i+1}} - P_{o_i}\right) + \left(P_{cog_{i+1}} - P_{cog_i}\right)^t\right]\\ &+T_{xg}{}^t_{i-1/2}\left[\left(P_{o_{i-1}} - P_{o_i}\right) + \left(P_{cog_{i-1}} - P_{cog_i}\right)^t\right] - q'_{gi}\\ &+\left(R_{so}T_{xo}\right)^t_{i+1/2}\left(P_{o_{i+1}} - P_{o_i}\right) + \left(R_{so}T_{xo}\right)^t_{i-1/2}\left(P_{o_{i-1}} - P_{o_i}\right) - \left(R_{so}q'_o\right)_i\\ &\qquad = C_{pog}{}^t_i\left(P_{o_i} - P_{o_i}^t\right) + C_{sgg}{}^t_i\left(S_{g_i} - S_{g_i}^t\right), \qquad i = 1, N\end{aligned}$$

IMPES pressure solution

The pressure equation for the saturated oil-gas becomes:

$$\left\{Txo_{i+1/2}^{t}+\alpha_i\left[Txg_{i+1/2}^{t}+\left(R_{so}T_{xo}\right)_{i+1/2}^{t}\right]\right\}\left(Po_{i+1}-Po_i\right)+$$
$$\left\{Txo_{i-1/2}^{t}+\alpha_i\left[Txg_{i-1/2}^{t}+\left(R_{so}T_{xo}\right)_{i-1/2}^{t}\right]\right\}\left(Po_{i-1}-Po_i\right)$$
$$+\alpha_i Txg_{i+1/2}^{t}\left(Pcog_{i+1}-Pcog_i\right)^t+\alpha_i Txg_{i-1/2}^{t}\left(Pcog_{i-1}-Pcog_i\right)^t$$
$$-q'_{oi}-\alpha_i\left(q'_g+R_{so}{}^t q'_{oi}\right)_i=$$
$$\left(Cpoo_i^t+\alpha_i Cpog_i^t\right)\left(Po_i-Po_i^t\right),\qquad i=1,N$$

where

$$\alpha_i=-Csgo_i^t\,/\,Csgg_i^t\,.$$

The pressure equation may now be rewritten as:

$$a_i Po_{i-1}+b_i Po_i+c_i Po_{i+1}=d_i\,,\quad i=1,N$$

where

$$a_i=Txo_{i-1/2}^{t}+\alpha_i\left(Tsg+R_{so}Txo\right)_{i-1/2}^{t}$$

$$c_i=Txo_{i+1/2}^{t}+\alpha_i\left(Tsg+R_{so}Txo\right)_{i+1/2}^{t}$$

$$b_i=-Txo_{i-1/2}^{t}-Txo_{i+1/2}^{t}-Cpoo_i^t-\alpha_i\left[\left(Txg+R_{so}Txo\right)_{i-1/2}^{t}+\left(Txg+R_{so}Txo\right)_{i+1/2}^{t}+Cpog_i^t\right]$$

$$\begin{aligned}d_i=&-(Cpoo_i^t+\alpha_i Cpog_i^t)Po_i^t+q'_{oi}+\alpha_i\left(q'_g+R_{so}q'_o\right)_i\\&-\alpha_i Txg_{i+1/2}^{t}(Pcog_{i+1}-Pcog_i)^t-\alpha_i Txg_{i-1/2}^{t}(Pcog_{i-1}-Pcog_i)^t\end{aligned}$$

Modifications for boundary conditions

Again, all rate specified well conditions are included in the rate terms q'_{oi}, q'_{gi} and $Rso_i q'_{oi}$. With the coefficients involved at old time level, coefficients, these rate terms are already appropriately included in the d_i term above.

For _injection of gas at bottom hole pressure specified well conditions_, the following expression applies (again using the case of neglected capillary pressure as example; however, capillary pressure can easily be included):

$$Q_{gi}=WC_i\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi}+\lambda_{gi}\right)^t(Po_i-Pbh_i)\,.$$

In a block with a well of this type, the following matrix coefficients are modified (assuming that there is not a production well in the injection block):

$$\begin{aligned}b_i=&-Txo_{i-1/2}^{t}-Txo_{i+1/2}^{t}-Cpoo_i^t\\&-\alpha_i\left[\frac{WC_i}{A\Delta x_i}\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi}+\lambda_{gi}\right)^t+\left(Txg+R_{so}Txo\right)_{i-1/2}^{t}+\left(Txg+R_{so}Txo\right)_{i+1/2}^{t}+Cpog_i^t\right]\end{aligned}$$

$$d_i = -(Cpoo_i^t + \alpha_i Cpog_i^t)Po_i^t - \alpha_i \frac{WC_i}{\Delta x_i}\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)^t Pbh_i$$
$$- \alpha_i Txg_{i+1/2}^t (Pcog_{i+1} - Pcog_i)^t - \alpha_i Txg_{i-1/2}^t (Pcog_{i-1} - Pcog_i)^t$$

For <u>*production at bottom hole pressure specified well conditions*</u>, we have the following expressions:

$$q'_{oi} = \frac{WC_i}{A\Delta x_i}\lambda_{oi}(Po_i - Pbh_i),$$

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\lambda_{gi}(Pg_i - Pbh_i)$$

and

$$Rso_i q'_{oi} = \frac{WC_i}{A\Delta x_i} Rso_i \lambda_{oi}(Po_i - Pbh_i).$$

In a block with a well of this type, the following matrix coefficients are modified:

$$b_i = -Txo_{i-1/2}^t - Txo_{i+1/2}^t - Cpoo_i^t - \frac{WC_i}{A\Delta x_i}\lambda_{oi}^t$$
$$- \alpha_i \left[\frac{WC_i}{A\Delta x_i}\left(Rso_i \lambda_{oi} + \lambda_{gi}\right)^t + \left(Txg + R_{so}Txo\right)_{i-1/2}^t + \left(Txg + R_{so}Txo\right)_{i+1/2}^t + Cpog_i^t \right]$$

$$d_i = -(Cpoo_i^t + \alpha_i Cpog_i^t)Po_i^t - \frac{WC_i}{\Delta x_i}\lambda_{oi}^t Pbh_i - \alpha_i \frac{WC_i}{\Delta x_i}\left(\lambda_{gi} + R_{so}q'_o\right)_i^t Pbh_i$$
$$- \alpha_i Txg_{i+1/2}^t (Pcog_{i+1} - Pcog_i)^t + \alpha_i Txg_{i-1/2}^t (Pcog_{i-1} - Pcog_i)^t$$

As for oil-water, the pressure equation may now be solved for oil pressures by using Gaussian elimination.

IMPES saturation solution

Having obtained the oil pressures above, we need to solve for gas saturations using either the oil equation or the gas equation. In the following we will use the oil equation for this purpose:

$$Txo_{i+1/2}^t\left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^t\left(Po_{i-1} - Po_i\right) - q'_{oi}$$
$$= Cpoo_i^t\left(Po_i - Po_i^t\right) + Csgo_i^t\left(Sg_i - Sg_i^t\right), \qquad i = 1, N$$

Again, since gas saturation only appears as an unknown in the last term on the right side of the oil equation, we may solve for it explicitly:

$$Sg_i = Sg_i^t + \frac{1}{Csgo_i^t}\left[Txo_{i+1/2}^t\left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^t\left(Po_{i-1} - Po_i\right) - q'_{oi} \quad - Cpoo_i^t\left(Po_i - Po_i^t\right)\right], \qquad i = 1, N$$

For grid blocks having pressure specified production wells, we make appropriate modifications, as discussed previously:

$$S_{g_i} = S_{g_i}^t + \frac{1}{C_{sgq_i}^t}\left[Txo_{i+1/2}^t\left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^t\left(Po_{i-1} - Po_i\right) - \frac{WC_i}{A\Delta x_i}\lambda_{oi}^t\left(Po_i - Pbh_i^t\right) - Cpoo_i^t\left(Po_i - Po_i^t\right)\right]$$

$$i = 1, N$$

Having obtained oil pressures and water saturations for a given time step, well rates or bottom hole pressures may be computed, if needed, from, from the following expression for an injection well:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(Po_i - Pbh_i),$$

and for a production well:

$$q'_{oi} = \frac{WC_i}{A\Delta x_i}\lambda_{oi}(Po_i - Pbh_i),$$

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\lambda_{gi}(Pg_i - Pbh_i)$$

and

$$Rso_i q'_{oi} = \frac{WC_i}{A\Delta x_i} Rso_i \lambda_{oi}(Po_i - Pbh_i).$$

The surface gas-oil ratio is computed as:

$$GOR_i = \frac{q'_{gi} + Rso_i q'_{oi}}{q'_{oi}}.$$

Required adjustments in well rates and well pressures, if constrained by upper or lower limits are made at the end of each time step, before all coefficients are updated and we can proceed to the next time step.

Applicability of IMPES method

Applicability of the IMPES method for oil-gas systems is fairly much as for oil-water systems. However, since saturation changes in gas-oil systems generally are more rapid than for oil-water systems, due to the fact that the gas viscosity is much smaller than for liquids, smaller time step sizes may be required.

Textbook: Chapter 6, p. 57-62, Appendix B, p. 132-139, Appendix C, p. 140-144