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Department of Petroleum Engineering and Applied Geophysics

Examination paper for TPG4160 Reservoir Simulation

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Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

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Number of pages enclosed: 0

Checked by:

Date Signature

Question 1 (26x0,5 points)

Explain briefly the following terms as applied to reservoir simulation (short sentence and/or a formula for each):

a) Control volume
b) Mass balance
c) Taylor series
d) Numerical dispersion
e) Explicit
f) Implicit
g) Stability
h) Upstream weighting
i) Variable bubble point
j) Harmonic average
k) Transmissibility
l) Storage coefficient
m) Coefficient matrix
n) IMPES
o) Fully implicit
p) Cross section
q) Coning
r) PI
s) Stone´s relative permeability models
t) Discretization
u) History matching
v) Prediction
w) Black Oil
x) Compositional
y) Dual porosity
z) Dual permeability

Question 2 (*1+2+2+2+4+4 points*)

Answer the following questions related to the derivation of reservoir fluid flow equations:

a) Write the mass balance equation (one-dimensional, one-phase)
b) List 3 commonly used expressions for relating fluid density to pressure
c) Write the most common relationship between velocity and pressure, and write an alternative relationship used for high fluid velocities.
d) Write the expression for the relationship between porosity and pressure.
e) Derive the following partial differential equation (show all steps):

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

f) Reduce the equation in e) to the simple diffusivity equation:

$$\frac{\partial^2 P}{\partial x^2}=\left(\frac{\phi\mu c}{k}\right)\frac{\partial P}{\partial t}$$

Question 3 (10 points)

Use Taylor series and show all steps in the discretization of the following equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)=\phi\left(\frac{c_r}{B}+\frac{d(1/B)}{dP}\right)\frac{\partial P}{\partial t}$$

Question 4 *(3+5+5 points)*

a) Show all steps in the derivation of the simple, one dimensional, radial, horizontal, one-phase diffusivity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r})=(\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$

b) Derive the numerical approximation for this equation using the transformation:

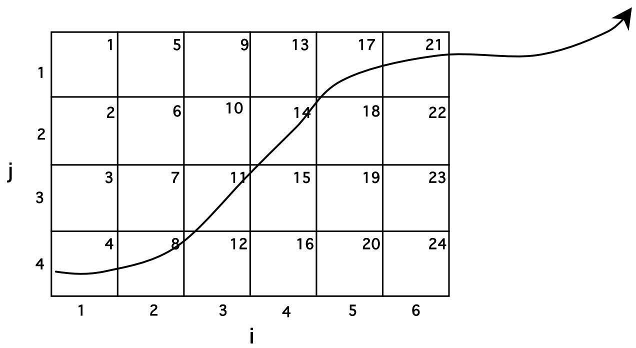
$$u=\ln(r)$$

c) Explain why the radial grid dimensions in cylindrical coordinates often are selected according to the formula:

$$\frac{r_{i+1/2}}{r_{i-1/2}}=(\frac{r_e}{r_w})^{1/N}$$

Question 5 *(4+4+6 points)*

In the following 2-dimensional cross-section of a reservoir (one fluid only), a well is producing at a constant rate Q (st. vol. oil/unit time) and perforates the grid blocks 4, 8, 11, 14, 17 and 21 in the x-z grid system shown:



The (unknown) bottom hole pressure P_{bh} is specified at a reference depth d_{ref}. Assume that hydrostatic pressure equilibrium exists inside the well tubing.

a) Write the expression for oil rate from each perforated block (in terms of productivity indices, mobility terms, pressure differences and hydrostatic pressure differences)

b) Write the expression for the total oil flow rate for the well (group the constants into parameters A, B, C, D, F, G, H, representing a constant term and the contribution to flow from the 6 grid block pressures involved)

c) The standard pressure equation for this grid system, without the well terms, is:

$$e_{i,j}P_{i,j-1} + a_{i,j}P_{i-1,j} + b_{i,j}P_{i,j} + c_{i,j}P_{i+1,j} + f_{i,j}P_{i,j+1} = d_{i,j} \quad i = 1,...,N_1, \quad j = 1,...,N_2$$

Sketch the coefficient matrix for this system, including the well. Indicate how the coefficient matrix is altered by the well (approximately, with x´s and lines labeled with the appropriate coefficient name).

<u>Question 6</u> *(3+2+3+5 points)*

The discretized form of the oil equation may be written as

$$Tx_{o_{i+1/2}}(P_{oi+1} - P_{oi}) + Tx_{o_{i-1/2}}(P_{oi-1} - P_{oi}) - q'_{oi} = C_{poi}(P_{oi} - P^t_{oi}) + C_{soi}(S_{wi} - S^t_{wi})$$

a) What is the physical significance of each of the 5 terms in the equation?

Using the following transmissibility as example,

$$Tx_{o_{i-1/2}} = \frac{2k_{i-1/2}\lambda_{oi-1/2}}{\Delta x_i(\Delta x_i + \Delta x_{i-1})}$$

b) What type of averaging method is normally applied to absolute permeability between grid blocks? Why? Write the expression for average permeability between grid blocks (*i*-1) and (*i*).

c) Write an expression for the selection of the conventional <u>*upstream mobility term*</u> for use in the transmissibility term of the oil equation above for flow between the grid blocks (*i*-1) and (*i*).

d) Make a sketch of a typical Buckley-Leverett saturation profile resulting from the displacement of oil by water (ie. analytical solution). Then, show how the corresponding profile, if calculated in a numerical simulation model, typically is influenced by the choice of mobilities between the grid blocks (sketch curves for saturations computed with upstream or average mobility terms, respectively).

<u>Question 7</u> (5x2 points)

Normally, we use either a *Black Oil* fluid description or a *compositional* fluid description in reservoir simulation.

a) What are the *components* and the *phases* used in *Black Oil* modeling?
b) What are the *components* and the *phases* used in *compositional* modeling?
c) Write the standard flow equations for the components required for *Black Oil* modeling (one dimensional, horizontal, constant flow area).
d) Write the standard flow equations the components required for *compositional* modeling (one dimensional, horizontal, constant flow area). Let

   C_{kg} = *mass fraction of component k present in the gas phase*

   C_{ko} = *mass fraction of component k present in the oil phase*.

e) A *Black Oil* fluid description may be regarded as a subset of a *compositional* fluid description. Define the pseudo-components required in order to reduce the *compositional* equations to *Black Oil* equations (one dimensional, horizontal, constant flow area)

<u>Question 8</u> (6x2 points)

Normally, we use either a *conventional* model or a *fractured* model in simulation of a reservoir.

a) Describe the main differences between a *conventional* reservoir and a *fractured* reservoir, in terms of the physics of the systems.
b) How can we identify a fractured reservoir from standard reservoir data?
c) Explain **briefly** the primary concept used in deriving the flow equations for a dual-porosity model.
d) Write the basic equations (one-phase, one-dimension) for
    - a two-porosity, two-permeability system
    - a two-porosity, one-permeability system
e) In terms of the physics of reservoir flow, what is the key difference between the two formulations in question d)?
f) How is the fluid exchange term in the flow equations in question d) represented? What are the shortcomings of this representation?