Interpolation in tabular data

(Reference: Chapter 3 in W. H. Preuss, *et al.*, Numerical Recipes in Fortran, 2nd ed., Cambridge University Press, 1992)

Learning objectives

1. Review of methods for computer-aided interpolation
2. Develop problem solution skills using computers and numerical methods
3. Develop programming skills using FORTRAN

   FORTRAN elements in this module
   - DO-loops
   - use of subroutines
   - IF-sequences

Introduction

Often, we have functional data where values of the function *f(x)* are known at a set of points $x_1, x_2, x_3, ..., x_N,$ but we do not have an analytical expression for *f(x)* that lets us calculate the value of the function at any point. Examples in petroleum are laboratory measurments of relative permeability to oil at a series of oil saturations in a core sample. Measurements are normally carried out at a few points, perhaps as few as 5, but in application of the data in a reservoir simulation model, data at close intervals are required.

The Lagrange's interpolation formula

Lagrange's formula for interpolation (of order *N-1*) may be written as:

$$f(x) = \frac{(x-x_2)(x-x_3)...(x-x_N)}{(x_1-x_2)(x_1-x_3)...(x_1-x_N)} f(x_1) + \frac{(x-x_1)(x-x_3)...(x-x_N)}{(x_2-x_1)(x_2-x_3)...(x_2-x_N)} f(x_2)$$

$$+ ..... + \frac{(x-x_1)(x-x_2)...(x-x_{N-1})}{(x_N-x_1)(x_N-x_2)...(x_N-x_{N-1})} f(x_N)$$

In order to determeine the functional value For å bestemme funksjonsverdien *f(x)* at the value of the argument *x* employing an order of interpolation of *(N-1)*, we need *N* pairs of values of *f(x)* and *x*. The most common formulas are the first-order (linear), second-order (parabolic) and third-order interpolation.

First-order interpolation (linear):

Linear interpolation (straight line) is obtained by setting *N* equal to 2 in the formula above. We then get the following expression:

$$f(x) = \frac{(x-x_2)}{(x_1-x_2)} f(x_1) + \frac{(x-x_1)}{(x_2-x_1)} f(x_2)$$

Second-order interpolationg (quadratic):

By setting *N* equal to 3 in the formula above, we get an expression for second-order interpolation (parabolic):

$$f(x)=\frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}f(x_1)+\frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}f(x_2)+\frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}f(x_3)$$

Third-order interpolation

As the final example; by setting *N* equal to 4, the formula for third-order interpolation is the result:

$$f(x)=\frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)}f(x_1)+\frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}f(x_2)$$
$$+\frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}f(x_3)+\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}f(x_4)$$

Programming exercise

Make a FORTRAN program that uses Lagrange's formula for interpolation in order to find a value of the function *f(x)* corresponding to a value *x* in a table $(f(x_i), x_i,\ i=1,N)$. The program should be made so that the order of interpolation, *M*, is an input parameter. The program shall consist of a main program that først reads the table values. These include the number of table entries, N, and values FXT(I) og XT(I), I=1,...,N. The program should check if the table is in ascending order – if not, the sorting shold be performed by calling the sorting subroutine (previous exercise). Then, it should read single values of X and M, and carry out interpolation of order M in order to determine the FX-value.

Then, the subroutine LAGRANGE(X,FX,M,N,XT,FXT) is called for interpolation. In order to apply the Lagrange formula, it first needs to locate the X-value in the input table. It may be convenient to make a separate subroutine for this, LOCATE(X,I1,N,XT), that returns position I1 (ie. X is larger than XT(I1) and less thanXT(I1+1)). The LAGRANGE routine then carries out the interpolation and returns the result to the main program.

In this exercise, we use first-, second- and third-order interpolation. After carrying out the inperpolation, the main program will write the interpolated values as well as the input values in a table. The main program should check that *M<N*. If not, a message should be written and the runs should be stopped.

Data set 1

Here, the function $f(x)=x^3$ has been used to create the following table:

x	*f(x)*
0,2	0,008
1	1
0,4	0,064
0,6	0,216
0	0
0,8	0,512

Read the input table and use the FORTRAN program to find values for *f(x)* at *x*=(0,1;0,3;0,5;0,7;0,9) using 1.-, 2.- and 3.-order Lagrange interpolation. The program should print the following table with interpolated values:

x	*1. order*	*2. order*	*3. order*
0,1			
0,3			
0,5			
0,7			
0,9			

Data set 2

The following values for relative permeability (k_{ro}) vs. oil saturation (S_o) for oil has been measured on a core sample in the laboratory:

S_o	k_{ro}
0,2	0
0,33	0,03
0,53	0,2
0,82	0,7

Read the table and use the FORTRAN program to generate a new k_{ro} table with 5% intervals in oil saturation (S_o=0,2;0,25;0,3...). Make these new tables using 1.-, 2.- and 3.-order Lagrange interpolation. Plot the new tables using Excel.

References to the textbook :

Lagrange interpolation: page 99
Finding table entries: page 110