NTNU

Department of Petroleum Engineering and Applied Geophysics

Examination paper for TPG4150 Reservoir Recovery Techniques

Academic contact during examination: Jon Kleppe

Phone: 91897300/73594925

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Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

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Number of pages (front page excluded): 4

Number of pages enclosed: 0

Informasjon om trykking av eksamensoppgave
Originalen er:
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Date Signature

Symbols used are defined in the Attachment

Question 1 (10 points)

This question relates to the group project work.

a) Outline briefly the main objective of the Gullfaks I1 group project.
b) How did your group proceed to reach the objective?
c) Which sensitivity calculations did your group make, and did you observe significant variations in reservoir behavior?

Solution
The students should give sufficient info to prove that they actively participated in the group work.

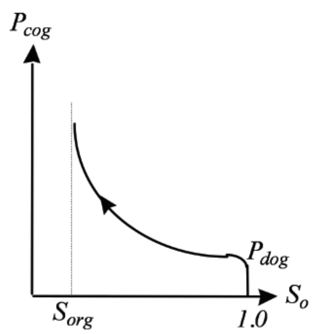
Question 2 (10 points)

Consider a cross-section for a homogeneous reservoir with defined WOC and GOC and a measured reference oil pressure at a reference depth:

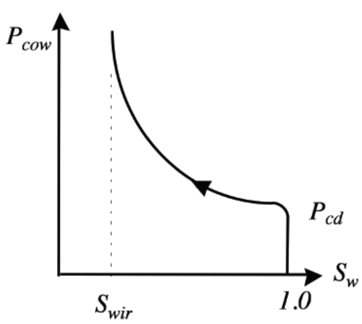
a) Sketch typical capillary pressure curves used for equilibrium calculations of initial saturations. Label important points.
b) Sketch typical initial water, oil and gas pressures vs. depth. Label important points used and explain briefly the procedure used.
c) Sketch the corresponding initial water, oil and gas saturation distributions determined by equilibrium calculations and capillary pressure curves. Label important points and explain briefly the procedure used.
d) Explain the concepts of WOC contact and free surface, using a sketch

Solution
a)

Solution

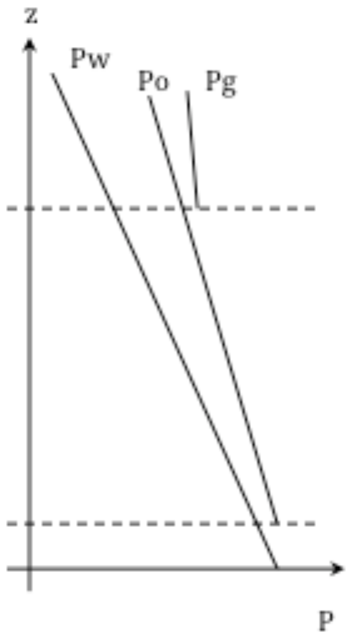


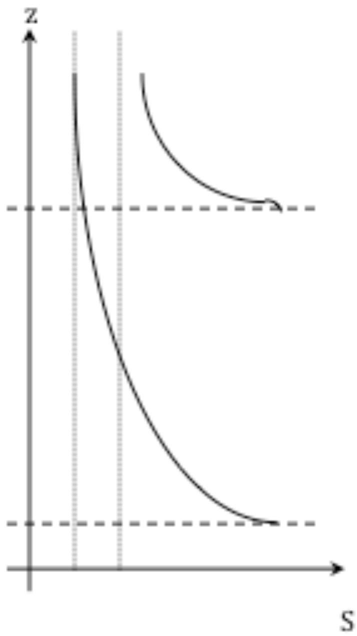
drainage o-g



drainage o-w

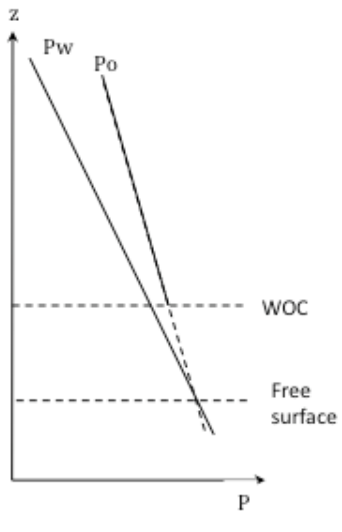
b) c)





At the WOC Po-Pw=Pdow, and at GOC Pg-Po=Pdog. Initial pressures are computed using densities and assuming equilibrium. At WOC Sw=1,0. At any z value, Pcow is computed from the difference in Po and Pw, and the corresponding Sw is found from the Pcow-curve. At GOC Sg=0. At any z-value above the corresponding Sg is found from the Pcog-curve

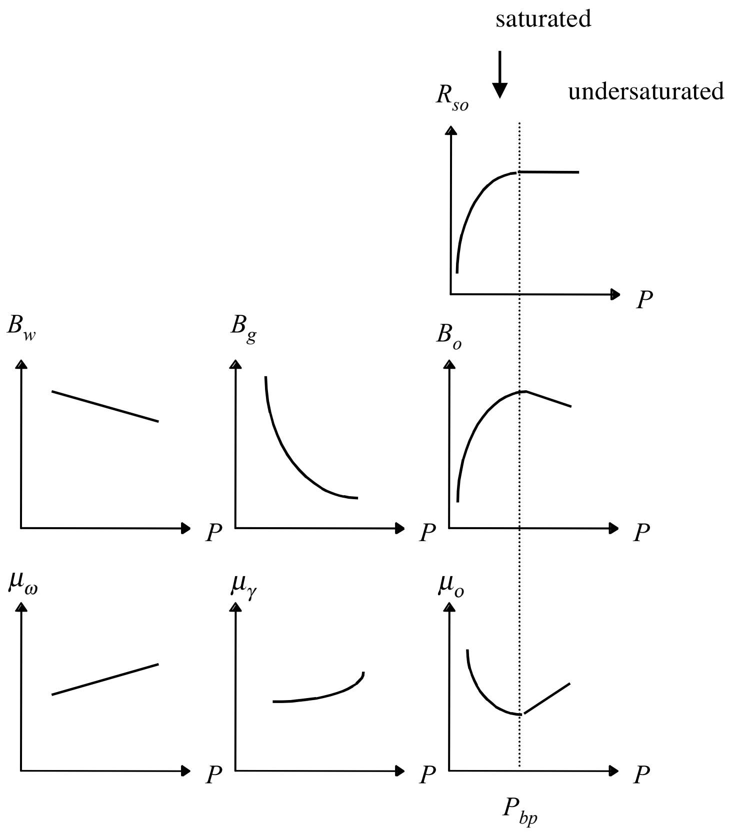
d)



<u>Question 3</u> (7 points)

Sketch typical B_o, B_w, B_g, μ_o, μ_w, μ_g, and R_{so} curves. Label axes, characteristic points and areas.

<u>Solution:</u>



Question 4 (14 points)

a) List all steps and formulas/equations/definitions used in the derivation of a one-phase fluid flow equation.
b) Derive the following equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)=(\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$

Show all steps in the derivation.
c) Which two main types of boundary conditions are normally used to represent reservoir fluid production and injection?
d) Write the steady-state form of equation b) above, and solve for pressure as a function of radius for boundary conditions $P(r=r_e)=P_e$ and $P(r=r_w)=P_w$

Solution

a)
--- 4 p ---
- Mass balance (continuity)
- Darcy´s
- PVT relationship
- Pore volume relationship

b)
--- 4 p ---
For a cylindrical geometry, the flow area is (for a full circle):

$$A=2\pi rh$$

The mass balance:

$$\{u\rho A\}_r-\{u\rho A\}_{r+\Delta r}=\frac{\partial}{\partial t}\{\phi A\Delta r\rho\}.$$

Substituting for area, assuming h to be constant, dividing by by $r\Delta r$, and taking the limit as Δr goes to zero, we get the continuity equation for cylindrical flow:

$$-\frac{1}{r}\frac{\partial}{\partial x}(r\rho u)=\frac{\partial}{\partial t}(\phi\rho).$$

We use the compressibility definitions for rock and fluid (at constant temperature):

$$c_r=-\frac{1}{\phi}\frac{\partial\phi}{\partial P}$$

$$c_f=-\frac{1}{V}\frac{\partial V}{\partial P}\ or\ c_f=\frac{1}{\rho}\frac{\partial\rho}{\partial P}$$

to simplify the right hand side of the equation:

$$\frac{\partial}{\partial t}(\phi\rho)=\rho\phi c\frac{\partial P}{\partial t}$$

After substituting for Darcy´s equation on the left hand side and using the standard simplification in regard to the density term on the left side:

$$c_f\left(\frac{\partial P}{\partial x}\right)^2<<\frac{\partial^2 P}{\partial x^2}.$$

we get the following simple form of the cylindrical flow equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)=\frac{\phi\mu c}{k}\frac{\partial P}{\partial t}.$$

c)
--- 2 p ---

- Bottom hole pressure specified
- Production rate specified

d)
---2 p ---
The steady state form of the equation is:

$$\frac{d}{dr}\left(r\frac{dP}{dr}\right)=0$$

--- 2 p ---
Integrating twice, we get:

$$P(r)=C_1\ln(r)+C_2.$$

Application of the BC´s in order to find the integration constants yields

$$P(r)=P_w+\frac{(P_e-P_w)}{\ln(r_e/r_w)}\ln(r/r_w)$$

Question 5 (22 points)

Consider a production well and derive expressions for surface gas production rate (Q_{gs}), surface water production rate (Q_{ws}),, and surface oil production rate (Q_{os}), for the two cases below. You may neglect capillary pressures.

a) Undersaturated oil reservoir with 100% oil saturation and a reservoir flow rate of Q_{or}.

b) Saturated oil reservoir with oil, water and gas inflow and a reservoir flow rate of Q_{or}.

Solution

a)
Oil in stock-tank: $1/B_o$
Surface volume of gas: R_{so}/B_o
Surface volume of water: 0.

b)
Reservoir rates:

$$Q_{oR}=PI\cdot\lambda_o(P-P_{bh})$$
$$Q_{gR}=PI\cdot\lambda_g(P-P_{bh})$$
$$Q_{wR}=PI\cdot\lambda_w(P-P_{bh})$$

Combining the equations and eliminating $(P-P_{bh})$, we get:

$$Q_{gR}=Q_{oR}\cdot\lambda_g/\lambda_o$$
$$Q_{wR}=Q_{oR}\cdot\lambda_w/\lambda_o$$

The surface rates then become:

$$Q_{oS} = Q_{oR} / B_o$$
$$Q_{wS} = Q_{oR} \cdot \lambda_w / \lambda_o / B_w$$
$$Q_{gR} = Q_{oR} \cdot \lambda_g / \lambda_o / B_w + Q_{oS} R_{so}$$

Question 6 (14 points)

Solution

a) Discuss the terms "diffuse flow" and "segregated flow". Which factors determine these flow conditions?

--- 2 p ---

- Diffuse flow if dynamic pressure gradients dominate the flow
  ie. $\frac{\delta P}{\delta x} >> g\Delta\rho$ (leads to uniform saturation distribution vertically)
- Segregated flow if gravity gradients dominate the flow
  ie. $g\Delta\rho >> \frac{\delta P}{\delta x}$

b) What do we mean with the term "Vertical Equilibrium" in reservoir analysis and under what conditions is it a reasonable assumption?

--- 2 p ---

- Fluids segregate vertically immediately (in accordance with capillary pressure)
  ie. $g\Delta\rho >> \frac{\delta P}{\delta x}$ (the "ultimate" segregated flow)
  May be a reasonable assumption in high permeability reservoirs where dynamic gradiens are small and vertical segregation takes place quickly

c) What do we mean with the term "Piston Displacement" in reservoir analysis and under what conditions is it a reasonable assumption?

--- 2 p ---

Oil saturation behind displacement front is equal to residual oil saturation
May be a reasonable assumpion for very favourable mobility ratios, such as for water displacement of oil in many North Sea sandstone reservoirs

d) What assumptions are made in the application of Buckley-Leverett analysis?

--- 2 p ---

- Diffuse flow, no capillary dispersion at the displacement front

e) What assumptions are made in the application of the Dykstra-Parson's method?

--- 2 p ---

- Piston displacement, isolated layers, constant ΔP across layers

f) What assumptions are made in the application of the Vertical Equilibrium (VE) method?

--- 2 p ---

- Instantaneous segregation of fluids

g) What assumptions are made in the application of Dietz' method for stability analysis?

--- 2 p ---

- Vertical equilibrium, piston displacement, no capillary pressure

<u>Question 7</u> (13 points)

<u>Solution</u>

a) Start with Darcy's equations for displacement of oil by water in an inclined layer at an angle α (positive upwards):

$$q_o = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha\right)$$

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial (P_o - P_c)}{\partial x} + \rho_w g \sin\alpha\right)$$

and derive the expression for water fraction flowing, f_w, inclusive capillary pressure and gravity.

Rewriting the equations as

$$-q_o \frac{\mu_o}{kk_{ro}A} = \frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha$$

$$-q_w \frac{\mu_w}{kk_{rw}A} = \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} + \rho_w g \sin\alpha$$

and then subtracting the first equation from the second one, we get

$$-\frac{1}{kA}\left(q_w \frac{\mu_w}{k_{rw}} - q_o \frac{\mu_o}{k_{ro}}\right) = -\frac{\partial P_c}{\partial x} + \Delta\rho g \sin\alpha$$

Substituting for

$$q = q_w + q_o$$

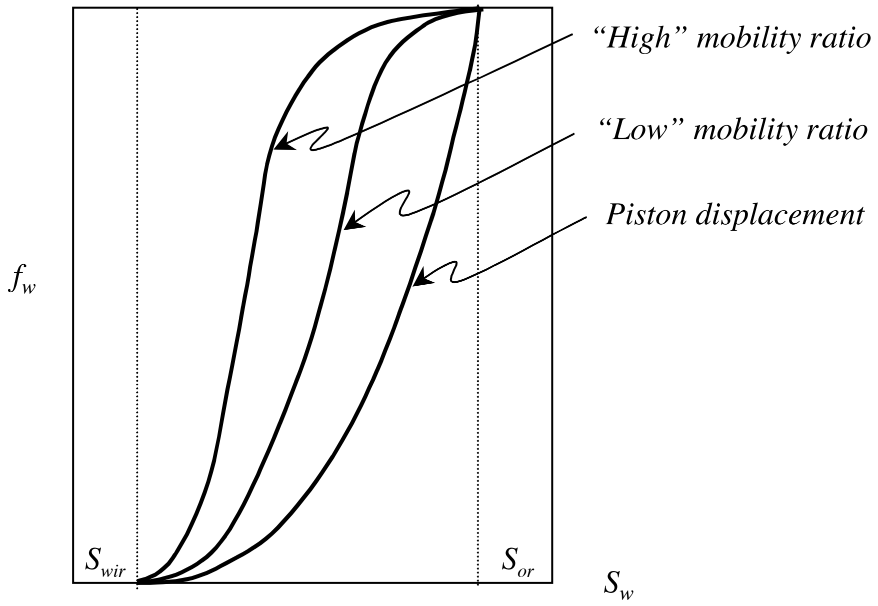
$$f_w = \frac{q_w}{q}$$

and solving for the fraction of water flowing, we get the following expression:

$$f_w = \frac{1 + \dfrac{kk_{ro}A}{q\mu_o}\left(\dfrac{\partial P_c}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \dfrac{k_{ro}}{\mu_o}\dfrac{\mu_w}{k_{rw}}}$$

b) Make typical sketches for water fraction flowing, f_w, vs. water saturation, assuming capillary pressure and gravity may be neglected, for the following cases:

- a high mobility ratio
- a low mobility ratio
- for piston displacement

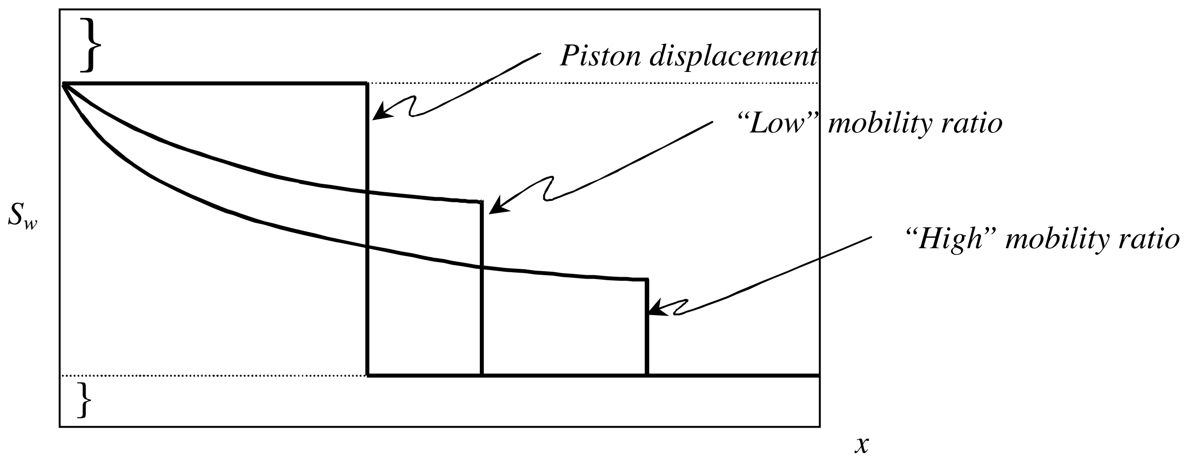

"High" mobility ratio
"Low" mobility ratio
Piston displacement
f_w
S_{wir}
S_{or}
S_w

c) Make a typical sketch for water saturation vs. x for water displacement of oil in a horizontal system (Buckley-Leverett), assuming capillary pressure and gravity may be neglected, for the following cases:
   - a high mobility ratio
   - a low mobility ratio
   - for piston displacement

   Explain the physical meaning behind these curves in terms of break-through time, water-cut at break-through and recovery factor.

--- 3 p ---

The higher the mobility ratio, the lower will be the water saturation at the front, and the break-through of water will happen earlier. The water-cut at break-through and also the oil recovery factor will thus be lower. The lower the mobility ratio the break-through time will be longer and water-cut at break-through as well as oil recovery factor will be higher. Piston displacement gives a perfect displacement so that water-cut at break-through is 100% and all the movable oil will have been recovered.



<u>Question 8</u> (10 points)

The general form of the Material Balance Equation may be written as (se attached definitions of the symbols used):

$$N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+W_pB_{w2}=$$
$$N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}+mB_{o1}\left(\frac{B_{g2}}{B_{g1}}-1\right)-\left(1+m\right)B_{o1}\frac{C_r+C_wS_{w1}}{1-S_{w1}}\left(P_2-P_1\right)\right]$$
$$+\left(W_i+W_e\right)B_{w2}+G_iB_{g2}$$

a) What is the primary assumption behind the use of the Material Balance Equation, and which "driving mechanisms" or "energies" are included in the equation?

solution

Primary assumption: Zero-dimensional system
(homogeneous system/no flow inside reservoir)

Driving mechanisms:
- Expansion/contraction of reservoir fluids (including gas cap)
- Expansion/contraction of reservoir rock
- Aquifer influx
- Gas/water injection

b) Reduce the equation and find the expression for oil recovery factor (N_p/N) for the following reservoir system:
- The reservoir is originally 100% saturated with oil at a pressure higher than the bubble point pressure
- The production stream consists of oil and gas
- No injection of fluids
- No aquifer

solution

$$N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+\cancel{W_pB_{w2}}=$$
$$N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}+\cancel{m}B_{o1}\left(\frac{B_{g2}}{B_{g1}}-1\right)-\left(1+\cancel{m}\right)B_{o1}\frac{C_r+\cancel{C_wS_{w1}}}{1-\cancel{S_{w1}}}\left(P_2-P_1\right)\right]$$
$$+\left(\cancel{W_i+W_e}\right)B_{w2}+\cancel{G_iB_{g2}}$$

$$\Rightarrow N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]=N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}-B_{o1}C_r\left(P_2-P_1\right)\right]$$

$$RF=\frac{N_p}{N}=\frac{\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}-B_{o1}C_r\left(P_2-P_1\right)}{B_{o2}+\left(R_p-R_{so2}\right)B_{g2}}$$

c) Simplify the expression in b) for the following situations:

i) $P_2 \geq P_{bp}$

ii) $P_2 < P_{bp}$, c_r and c_w may be neglected

solution

$P_2 \geq P_{bp}$

$$N_p\left[B_{o2} + \left(R_p \cancel{-} R_{so2}\right)B_{g2}\right] = N\left[\left(B_{o2} - B_{o1}\right) + \left(R_{so1} \cancel{-} R_{so2}\right)B_{g2} - B_{o1}C_r\left(P_2 - P_1\right)\right]$$

$$\Rightarrow RF = \frac{N_p}{N} = \frac{B_{o1}}{B_{o2}}\left[\left(\frac{B_{o2}}{B_{o1}} - 1\right) - C_r\left(P_2 - P_1\right)\right]$$

solution

$P_2 < P_{bp}$, c_r and c_w may be neglected

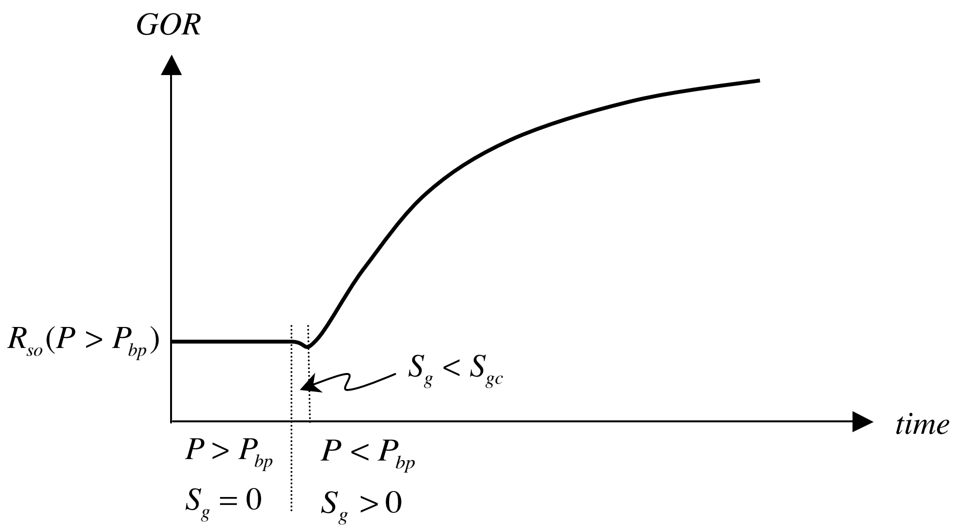
$$N_p\left[B_{o2} + \left(R_p - R_{so2}\right)B_{g2}\right] = N\left[\left(B_{o2} - B_{o1}\right) + \left(R_{so1} - R_{so2}\right)B_{g2} - B_{o1}C_r\left(\cancel{P_2 - P_1}\right)\right]$$

$$\Rightarrow RF = \frac{N_p}{N} = \frac{\left(B_{o2} - B_{o1}\right) + \left(R_{so1} - R_{so2}\right)B_{g2}}{\left[B_{o2} + \left(R_p - R_{so2}\right)B_{g2}\right]}$$

d) Make the following sketches for the reservoir in b):

- A typical curve for GOR vs. time for the reservoir . Explain details of the curve.

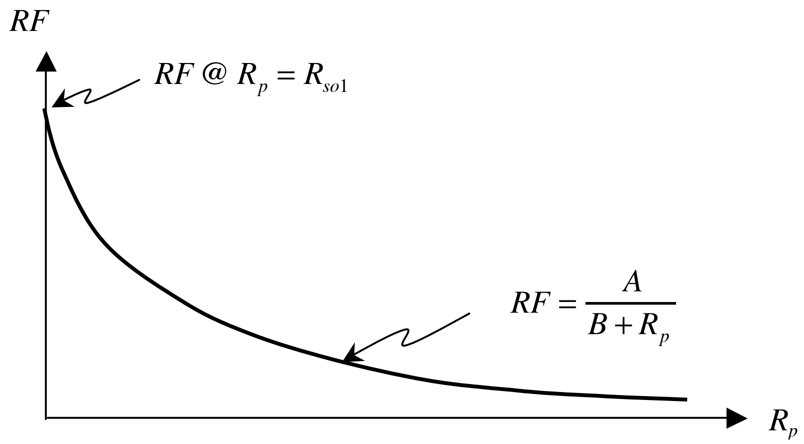
solution



- A typical curve for oil recovery factor, N_p/N, vs. cumulative gas-oil ratio, R_p. Explain details of the curve.

$$RF = \frac{N_p}{N} = \frac{\left(B_{o2} - B_{o1}\right) + \left(R_{so1} - R_{so2}\right)B_{g2} - B_{o1}C_r\left(P_2 - P_1\right)}{B_{o2} + \left(R_p - R_{so2}\right)B_{g2}}$$

$$\Rightarrow RF = \frac{A}{B + R_p} \quad \text{(for a given set of } P_1 \text{ and } P_2\text{)}$$



e) Reduce the equation for the following reservoir system:
- The reservoir is originally at bubble point pressure and has a gas cap
- The production stream consists of oil and gas
- No injection of fluids
- No aquifer

solution

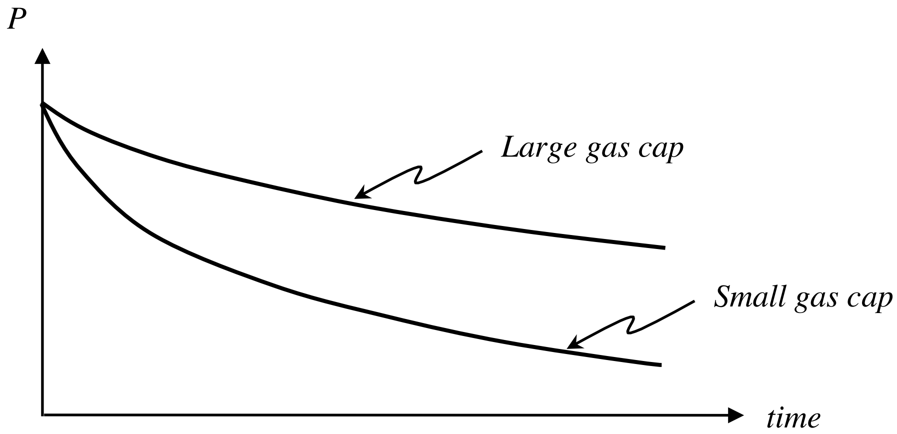
$$N_p\left[B_{o2} + \left(R_p - R_{so2}\right)B_{g2}\right] + \cancel{W_p B_{w2}} =$$
$$N\left[\left(B_{o2} - B_{o1}\right) + \left(R_{so1} - R_{so2}\right)B_{g2} + mB_{o1}\left(\frac{B_{g2}}{B_{g1}} - 1\right) - \left(1 + m\right)B_{o1}\frac{C_r + C_w S_{w1}}{1 - S_{w1}}\left(P_2 - P_1\right)\right]$$
$$+\left(\cancel{W_i + W_e}\right)B_{w2} + \cancel{G_i B_{g2}}$$

$$\Rightarrow N_p\left[B_{o2} + \left(R_p - R_{so2}\right)B_{g2}\right] =$$
$$N\left[\left(B_{o2} - B_{o1}\right) + \left(R_{so1} - R_{so2}\right)B_{g2} + mB_{o1}\left(\frac{B_{g2}}{B_{g1}} - 1\right) - \left(1 + m\right)B_{o1}\frac{C_r + C_w S_{w1}}{1 - S_{w1}}\left(P_2 - P_1\right)\right]$$

f) Make the following sketches:

- A typical curve for reservoir pressure vs. time for a large gas cap.
- A typical curve for reservoir pressure vs. time for a small gas cap.

solution



g) Reduce the equation for the following reservoir system:

- The reservoir is originally at a pressure higher than the bubble point pressure and contains oil and water
- The production stream consists of oil, water and gas
- No injection of fluids
- Water flows into the reservoir from an aquifer.

solution

$$N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+W_pB_{w2}=$$

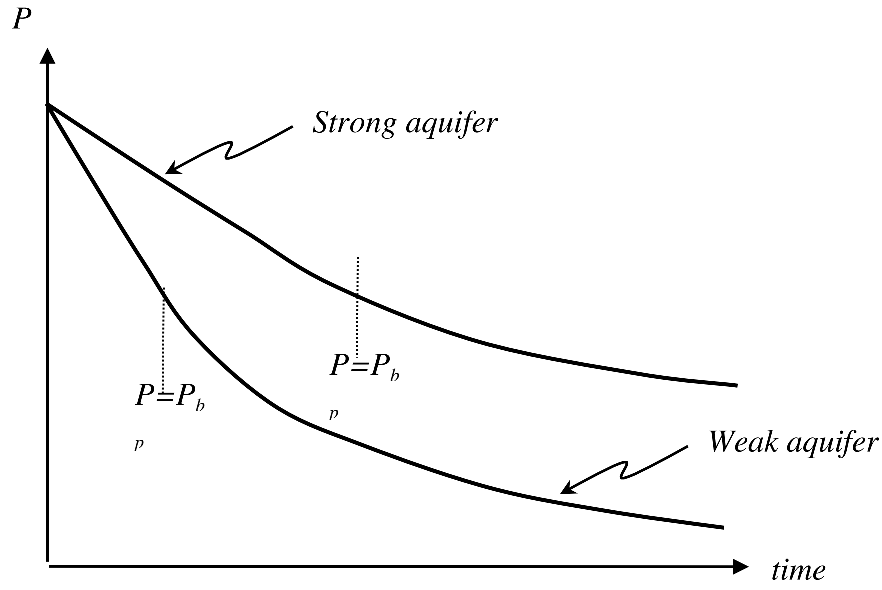
$$N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}+\cancel{mB_{o1}}\left(\frac{B_{g2}}{B_{g1}}-1\right)-\left(1+\cancel{m}\right)B_{o1}\frac{C_r+C_wS_{w1}}{1-S_{w1}}\left(P_2-P_1\right)\right]$$

$$+\left(\cancel{W_i}+W_e\right)B_{w2}+\cancel{G_iB_{g2}}$$

$$\Rightarrow N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+W_pB_{w2}=N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}-B_{o1}\frac{C_r+C_wS_{w1}}{1-S_{w1}}\left(P_2-P_1\right)+W_eB\right.$$

h) Make the following sketches:
- A typical curve for reservoir pressure vs. time for a reservoir with a strong aquifer.
- A typical curve for reservoir pressure vs. time for a reservoir with a weak aquifer.

solution



Attachment - Definition of symbols

B_g	Formation volume factor for gas (res.vol./st.vol.)
B_o	Formation volume factor for oil (res.vol./st.vol.)
B_w	Formation volume factor for water (res.vol./st.vol.)
C_r	Pore compressibility (pressure^{-1})
C_w	Water compressibility (pressure^{-1})
ΔP	$P_2 - P_1$
G_i	Cumulative gas injected (st.vol.)
GOR	Producing gas-oil ratio (st.vol./st.vol.)
G_p	Cumulative gas produced (st.vol.)
k	Absolute permeability
k_{ro}	Relative permeability to oil
k_{rw}	Relative permeability to oil
k_{rg}	Relative permeability to oil
m	Initial gas cap size (res.vol. of gas cap)/(res.vol. of oil zone)
M_e	End point mobility ratio
N	Original oil in place (st.vol.)
N_{ge}	Gravity number
N_p	Cumulative oil produced (st.vol.)
P	Pressure
P_{cow}	Capillary pressure between oil and water
P_{cog}	Capillary pressure between oil and gas
q_{inj}	Injection rate (res.vol./time)
R_p	Cumulative producing gas-oil ratio (st.vol./st.vol) = G_p / N_p
R_{so}	Solution gas-oil ratio (st.vol. gas/st.vol. oil)
S_g	Gas saturation
S_o	Oil saturation
S_w	Water saturation
T	Temperature
V_b	Bulk volume (res.vol.)
V_p	Pore volume (res.vol.)
WC	Producing water cut (st.vol./st.vol.)
W_e	Cumulative aquifer influx (st.vol.)
W_i	Cumulative water injected (st.vol.)
W_p	Cumulative water produced (st.vol.)
ρ	Density (mass/vol.)
ϕ	Porosity
μ_g	Gas viscosity
μ_o	Oil viscosity
μ_w	Water viscosity
γ	Hydrostatic pressure gradient (pressure/distance)