NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Petroleum Engineering and Applied Geophysics

SOLUTION

Examination paper for TPG4150 Reservoir Recovery Techniques

Academic contact during examination: Jon Kleppe

Phone: 91897300/73594925

Examination date: December 19, 2015

Examination time (from-to): 0900-1300

Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

Language: English

Number of pages (front page excluded): 13

Number of pages enclosed: 0

Informasjon om trykking av eksamensoppgave
Originalen er:
1-sidig ☐ **2-sidig X**
sort/hvit X **farger** ☐

Checked by:

Date Signature

Symbols used are defined in the enclosed table

Question 1 (2+2+2+2=8 points)

This question relates to the group project work.

a) Discuss the main uncertainties in the Gulltopp group project?
b) What was the probable reason for pressure decline in the reservoir before oil production started?
c) Due to the pressure decline before production started, it is expected that oil moved into the aquifer due to fluid expansion. Was all of this oil lost, and did this migration of oil have a significant influence on the final reservoir oil recovery?
d) Which sensitivity calculations did your group make, and did you observe significant variations in reservoir behavior?

Solution

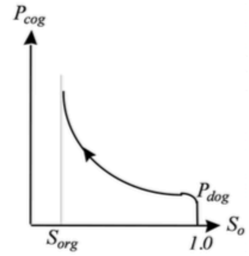
a) Key uncertainties are aquifer support, OIP, rock and fluid properties
b) Probably there is communication with surrounding reservoirs through the underlaying aquifer
c) When oil production starts the oil will be displaced back to the reservoir by aquifer water. However, the residual oil will be lost.
d) Should include the items under a). None have significant effect on the recovery.

Question 2 (2+10=12 points)

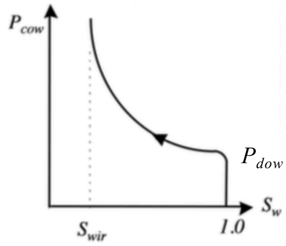
This question applies to an oil-gas-water reservoir at initial equilibrium conditions and with known WOC and GOC.

a) Make sketches of the capillary pressure curves required for computing the initial equilibrium saturation distributions in such an oil-water-gas reservoir.
b) Using the capillary pressure curves above, and known fluid densities, as well as a measured oil pressure somewhere in the oil column, outline using sketches the procedure for computing initial equilibrium saturation distributions.

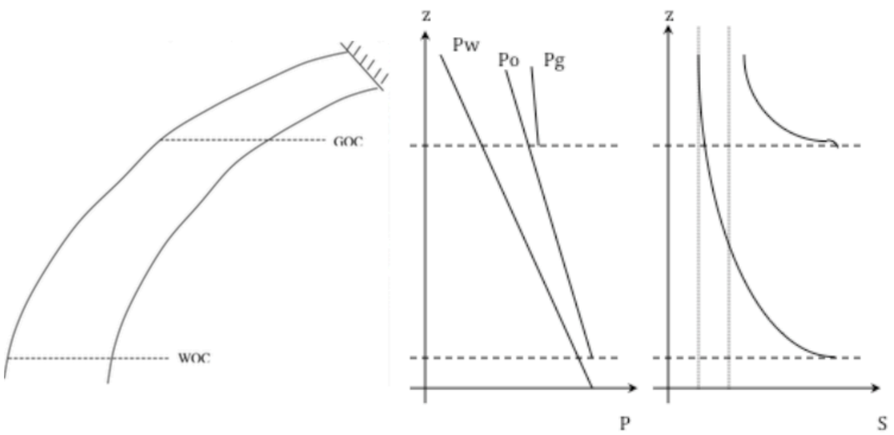
Solution



drainage o-g



drainage o-w



At the WOC Po-Pw=Pdow, and at GOC Pg-Po. Initial pressures are computed using densities and assuming equilibrium. At WOC Sw=1,0. At any z value, Pcow is computed from the difference in Po and Pw, and the corresponding Sw is found from the Pcow-curve. At GOC Sg=0. At any z-value above the corresponding Sg is found from the Pcog-curve

Question 3 (2+5+5=12 points)

Consider a reservoir with following initial conditions:

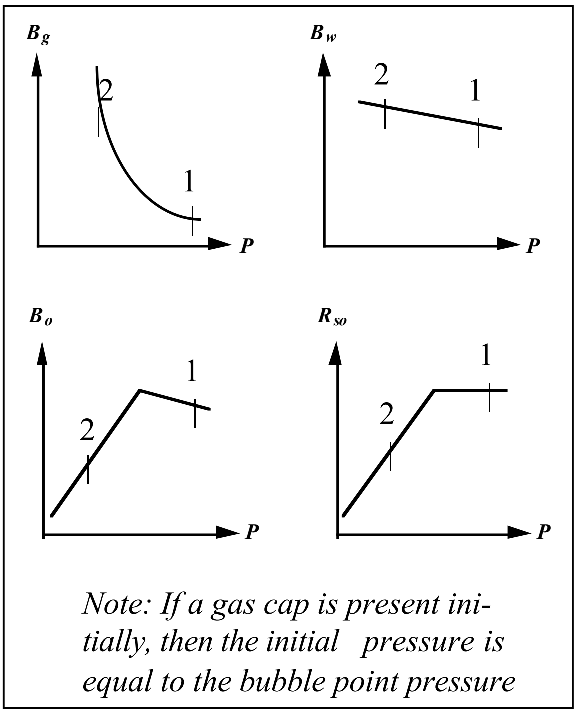
- Undersaturated oil
- Irreducible water saturation

The reservoir is being produced with water injection to a pressure **BELOW** the initial bubble point.

a) Sketch or write relationships for the relevant PVT properties for oil and gas vs. pressure, indicating initial and final conditions on the curves, and compressibilities for water and rock needed for material balance calculations.
b) **DERIVE** the complete material equations for oil, gas and water, including production terms for oil, gas and water and injection term for water.
c) Combine and simplify the equations for the following conditions, and find an expression for oil recovery factor, RF, with the following simplifications:
    - Neglect rock and water compressibilities
    - The standard volumes of cumulative water injected (Wi) and cumulative water produced (Wp) are equal.

Solution

a)



$$C_w = -\frac{1}{V_w}\frac{dV_w}{dP}$$
$$C_r = \frac{1}{\phi}\frac{d\phi}{dP}$$

b) Equation 1: Oil material balance

$$\left\{\begin{matrix} \textit{Oil present} \\ \textit{in the reservoir} \\ \textit{initially} \\ \textit{(st. vol.)} \end{matrix}\right\} - \left\{\begin{matrix} \textit{Oil} \\ \textit{produced} \\ \textit{(st. vol.)} \end{matrix}\right\} = \left\{\begin{matrix} \textit{Oil remaining} \\ \textit{in the reservoir} \\ \textit{finally} \\ \textit{(st. vol.)} \end{matrix}\right\}$$

or

$$N - N_p = V_{p2} S_{o2} / B_{o2}$$

yielding

$$S_{o2} = \frac{(N - N_p) B_{o2}}{V_{p2}}$$

Equation 2: Water material balance

$$\left\{\begin{matrix} \textit{Water present} \\ \textit{in the reservoir} \\ \textit{initially} \\ \textit{(st. vol.)} \end{matrix}\right\} - \left\{\begin{matrix} \textit{Water} \\ \textit{produced} \\ \textit{(st. vol.)} \end{matrix}\right\} + \left\{\begin{matrix} \textit{Water} \\ \textit{injected} \\ \textit{(st. vol.)} \end{matrix}\right\} + \left\{\begin{matrix} \textit{Aquifer} \\ \textit{influx} \\ \textit{(st. vol.)} \end{matrix}\right\} = \left\{\begin{matrix} \textit{Water remaining} \\ \textit{in the reservoir} \\ \textit{finally} \\ \textit{(st. vol.)} \end{matrix}\right\}$$

or

$$V_{p1} S_{w1} / B_{w1} - W_p + W_i + W_e = V_{p2} S_{w2} / B_{w2}$$

yielding

$$S_{w2} = \left[(1+m) N B_{o1} \left(\frac{S_{w1}}{1 - S_{w1}} \right) \left(\frac{1}{B_{w1}} \right) + \left(W_i + W_e - W_p \right) \right] \frac{B_{w2}}{V_{p2}}$$

Equation 3: Gas material balance

$$\left\{\begin{matrix}\text{Solution gas}\\ \text{present in the}\\ \text{reservoir initially}\\ \text{(st. vol.)}\end{matrix}\right\}+\left\{\begin{matrix}\text{Free gas}\\ \text{present in the}\\ \text{reservoir initially}\\ \text{(st. vol.)}\end{matrix}\right\}-\left\{\begin{matrix}\text{Gas}\\ \text{produced}\\ \text{(st. vol.)}\end{matrix}\right\}+\left\{\begin{matrix}\text{Gas}\\ \text{injected}\\ \text{(st. vol.)}\end{matrix}\right\}$$
$$=\left\{\begin{matrix}\text{Solution gas}\\ \text{present in the}\\ \text{reservoir finally}\\ \text{(st. vol.)}\end{matrix}\right\}+\left\{\begin{matrix}\text{Free gas}\\ \text{present in the}\\ \text{reservoir finally}\\ \text{(st. vol.)}\end{matrix}\right\}$$

or

$$NR_{so1} + mNB_{o1}/B_{g2} - R_p N_p = (N\text{-}N_p)R_{so2} + V_{p2}S_{g2}/B_{g2}$$

yielding

$$S_{g2}=\left\{N\left[(R_{so1}-R_{so2})+m(\frac{B_{o1}}{B_{g2}})\right]-N_p(R_p-R_{so2})+G_i\right\}(\frac{B_{g2}}{V_{p2}})$$

c) Combining the three equations by letting $S_{o2}+S_{g2}+S_{w2}=1$, and neglecting rock and water compressibilities, yields:

$$N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+W_pB_{w2}=N\left[(B_{o2}-B_{o1})+(R_{so1}-R_{so2})B_{g2}\right]+W_iB_{w2}$$

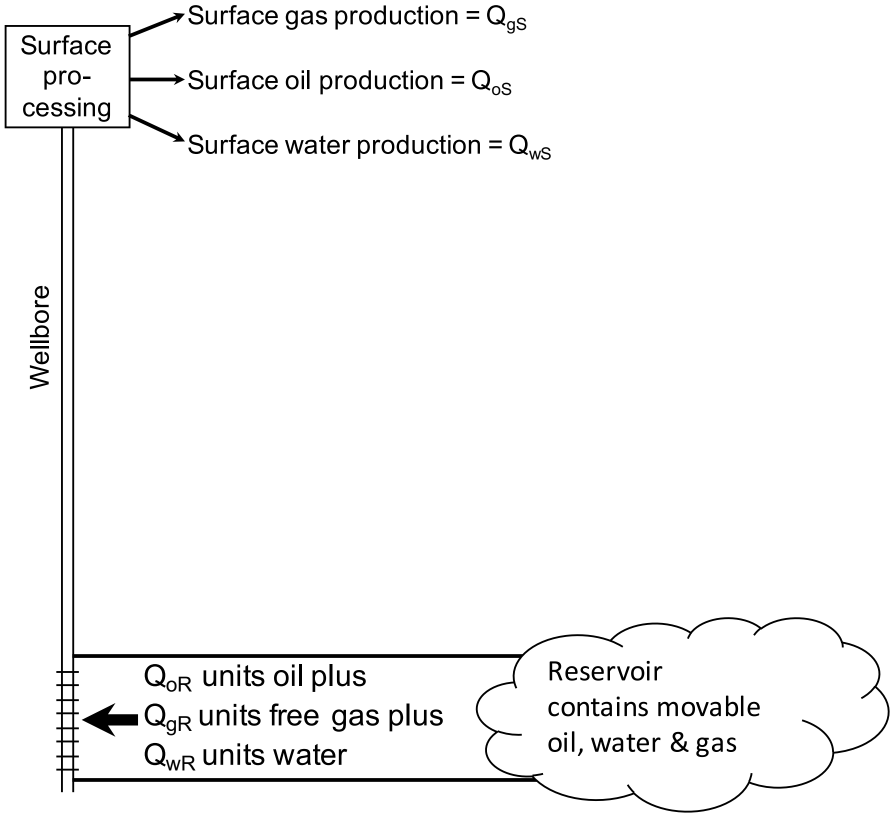
and, after simplifying and setting $W_i=W_p$:

$$RF=\frac{N_p}{N}=\frac{[(B_{o2}-B_{o1})+(R_{so1}-R_{SO2})B_{G2}]}{\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]}$$

Question 4 (6+6=12 points)

For the production system below where the oil production rate at surface conditions, Q_{oS}, is specified, and given that oil, gas and water mobilities are known and capillary pressures may be neglected:

a) Derive formulas for Q_{oR}, Q_{gR} and Q_{wR}
b) Derive formulas for Q_{gS} and Q_{wS}



Solution

a) $Q_{oR} = Q_{oS}B_o$

$$Q_{gR} = Q_{oR}\frac{\lambda_g}{\lambda_o}$$

$$Q_{wR} = Q_{oR}\frac{\lambda_w}{\lambda_o}$$

b) $Q_{gS} = \frac{Q_{gR}}{B_g} + Q_{oS}R_{so}$

$$Q_{wS} = \frac{Q_{oR}}{B_w} = Q_{oR}\frac{\lambda_w}{\lambda_o B_w}$$

Question 5 (3+6+3=12 points)

a) List all assumptions made in the derivation of the following equation:

$$\frac{\partial^2 P}{\partial x^2} = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$

b) Derive the equation, showing all steps and formulas/equations/definitions.

c) Sketch typical pressure profiles vs. x in a one-dimensional horizontal porous medium at several time levels. Label steady and unsteady parts.

Solution

a)

- One dimensional flow
- Linear flow
- Horizontal flow
- One phase flow

- Darcy´s equation applies
- Small fluid compressibility (liquid)
- Permeability and viscosity are constants

b) Derive the equation, showing all steps and formulas/equations/definitions.

Continuity equation:

$$-\frac{\partial}{\partial x}(A\rho u)=\frac{\partial}{\partial t}(A\phi\rho).$$

For constant cross sectional area, the continuity equation simplifies to:

$$-\frac{\partial}{\partial x}(\rho u)=\frac{\partial}{\partial t}(\phi\rho).$$

Darcy's equation

$$u=-\frac{k}{\mu}\frac{\partial P}{\partial x}.$$

Rock compressibility

$$c_r=\frac{1}{\phi}\frac{d\phi}{dP}.$$

Fluid compressibility

$$c_f=-(\frac{1}{V})(\frac{\partial V}{\partial P})_T.$$

Substituting Darcy´s equation

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\partial}{\partial t}(\rho\phi)$$

The right hand side (RHS) of the equation may be expanded as:

$$\frac{\partial}{\partial t}(\rho\phi)=\rho\frac{\partial}{\partial t}(\phi)+\phi\frac{\partial}{\partial t}(\rho)$$

Since porosity and density both are functions of pressure, we may write:

$$\frac{\partial}{\partial t}(\phi)=\frac{d\phi}{dP}\frac{\partial P}{\partial t}$$

and

$$\frac{\partial}{\partial t}(\rho)=\frac{d\rho}{dP}\frac{\partial P}{\partial t}.$$

From the compressibility expressions we may obtain the following relationships:

$$\frac{d\rho}{dP}=\rho c_f \text{ and } \frac{d\phi}{dP}=\phi c_r.$$

By substituting these expressions into the equation, we obtain the following form of the right hand side of the flow equation:

$$\frac{\partial}{\partial t}(\rho\phi)=\phi\rho(c_f+c_r)\frac{\partial P}{\partial t}.$$

The left hand side of the flow equation may be expanded as follows:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right)+\frac{k}{\mu}\frac{\partial P}{\partial x}\frac{\partial}{\partial x}(\rho)=\rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right)+\frac{k}{\mu}\frac{\partial P}{\partial x}\frac{d\rho}{dP}\frac{\partial P}{\partial x}$$

Assume that k=constant and μ=constant. Let us also substitute for $\frac{d\rho}{dP}=\rho c_f$. The LHS may now be written as:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\rho k}{\mu}\left[\frac{\partial^2 P}{\partial x^2}+c_f\left(\frac{\partial P}{\partial x}\right)^2\right].$$

Since c_f is small, at least for liquids, and the pressure gradient is small for the low velocity flow we normally have in reservoirs, we make the following assumption:

$$c_f\left(\frac{\partial P}{\partial x}\right)^2 << \frac{\partial^2 P}{\partial x^2}$$

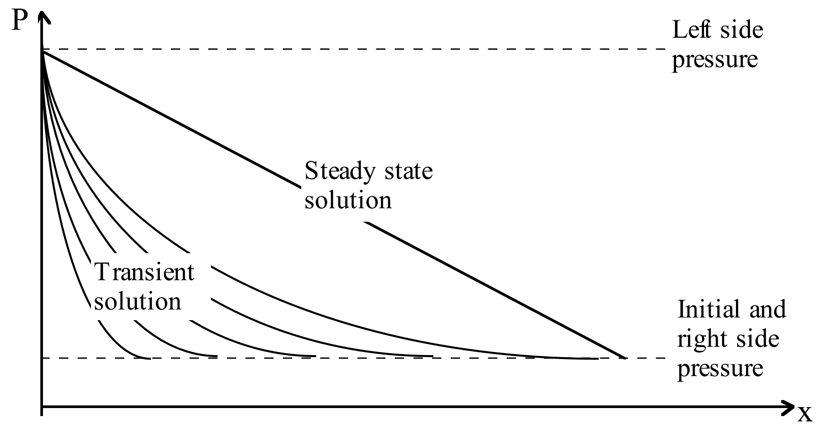
Then, our LHS simplifies to:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\rho k}{\mu}\frac{\partial^2 P}{\partial x^2}.$$

The complete partial differential flow equation (PDE) for this simple rock-fluid system then becomes:

$$\frac{\partial^2 P}{\partial x^2}=\left(\frac{\phi\mu c_t}{k}\right)\frac{\partial P}{\partial t}$$

c) Sketch typical pressure profiles vs. x in a one-dimensional horizontal porous medium at several time levels. Label steady and unsteady parts.



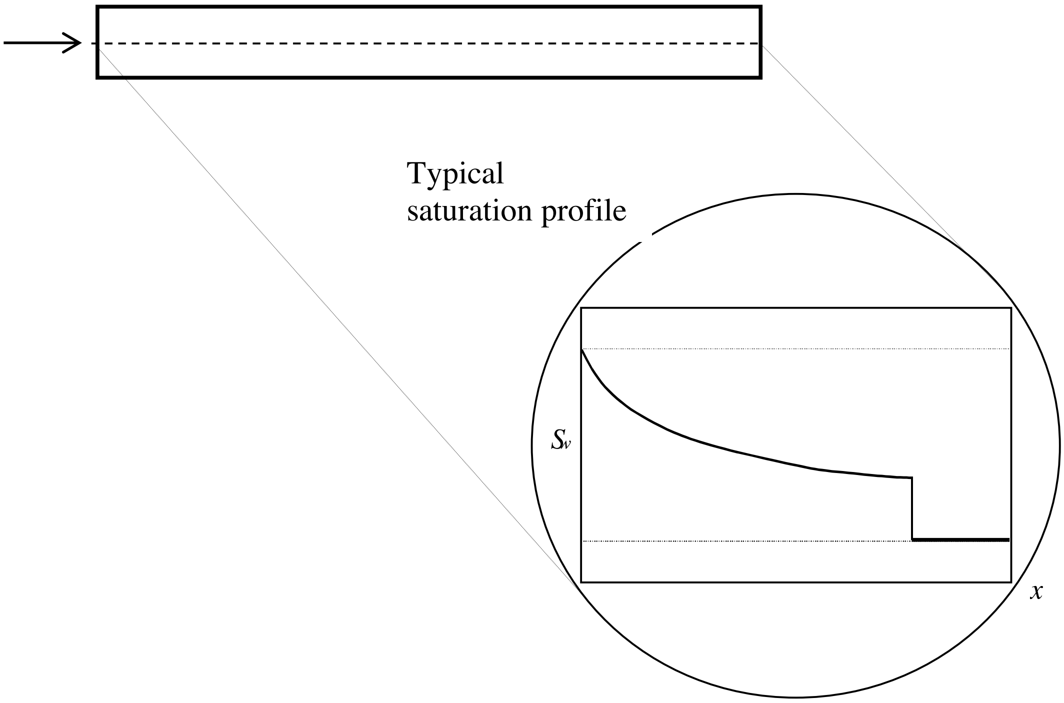
<u>Question 6</u> (2+2+2=6 points)

Explain the Buckley-Leverett method. Include:

a) The basic assumptions behind the method
b) A sketch of a typical Buckley-Leverett displacement situation
c) A <u>short</u> discussion of when the method is applicable

<u>Solution</u>

a. The basic assumptions behind the method
   - is derived for displacement in a single layer
   - diffuse flow
   - no capillary dispersion at the displacement front
b. A sketch of a typical Buckley-Leverett displacement situation



c. A short discussion of when the method is applicable

- applies to situations where dynamic pressure gradients dominate the flow ie. $\frac{\delta P}{\delta x} >> g\Delta\rho$ (leads to uniform saturation distribution vertically)

Question 7 (2+2+2=6 points)

Explain the Dietz method. Include:

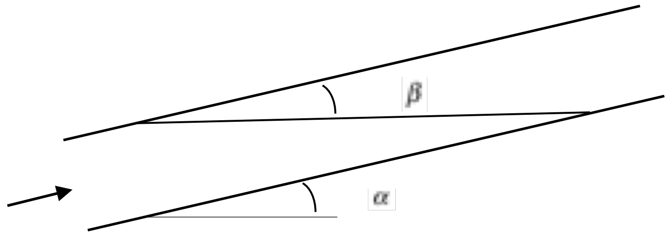
a) The basic assumptions behind the method
b) A sketch of a typical Dietz displacement situation
c) A short discussion of when the method is applicable

Solution

Explain the Dietz method. Include:

a) The basic assumptions behind the method
    - is developed for stable displacement in an inclined layer
    - vertical equilibrium
    - piston displacement
    - no capillary pressure

b) A sketch of a typical Dietz displacement situation



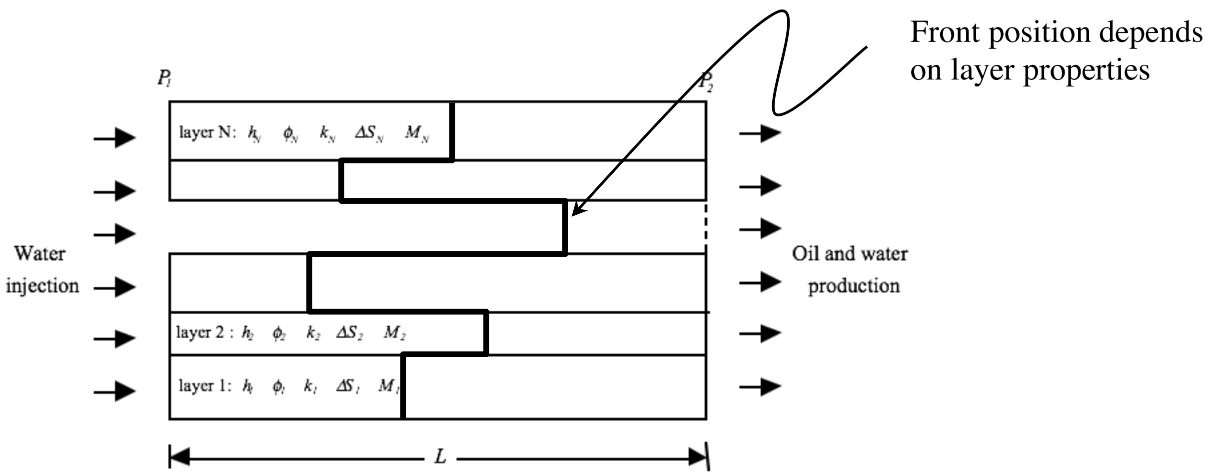
c) A short discussion of when the method is applicable
- applies to displacement situations in inclined layers where gravity is controlling the displacement, ie. where $g\Delta\rho >> \frac{\delta P}{\delta x}$

Question 8 (2+2+2=6 points)

Explain the Dykstra-Parson's method. Include:
a) The basic assumptions behind the method
b) A sketch of a typical Dykstra-Parson displacement situation
c) A short discussion of when the method is applicable

Solution
a) The basic assumptions behind the method
- is developed for displacement in a layered system
- piston displacement
- isolated layers
- constant ΔP across layers

b) A sketch of a typical Dykstra-Parson displacement situation



c) A short discussion of when the method is applicable
- applies to situations where layers are isolated and where mobility ratio is favourable so that the displacement is very efficient

Question 9 (2+2+2=6 points)

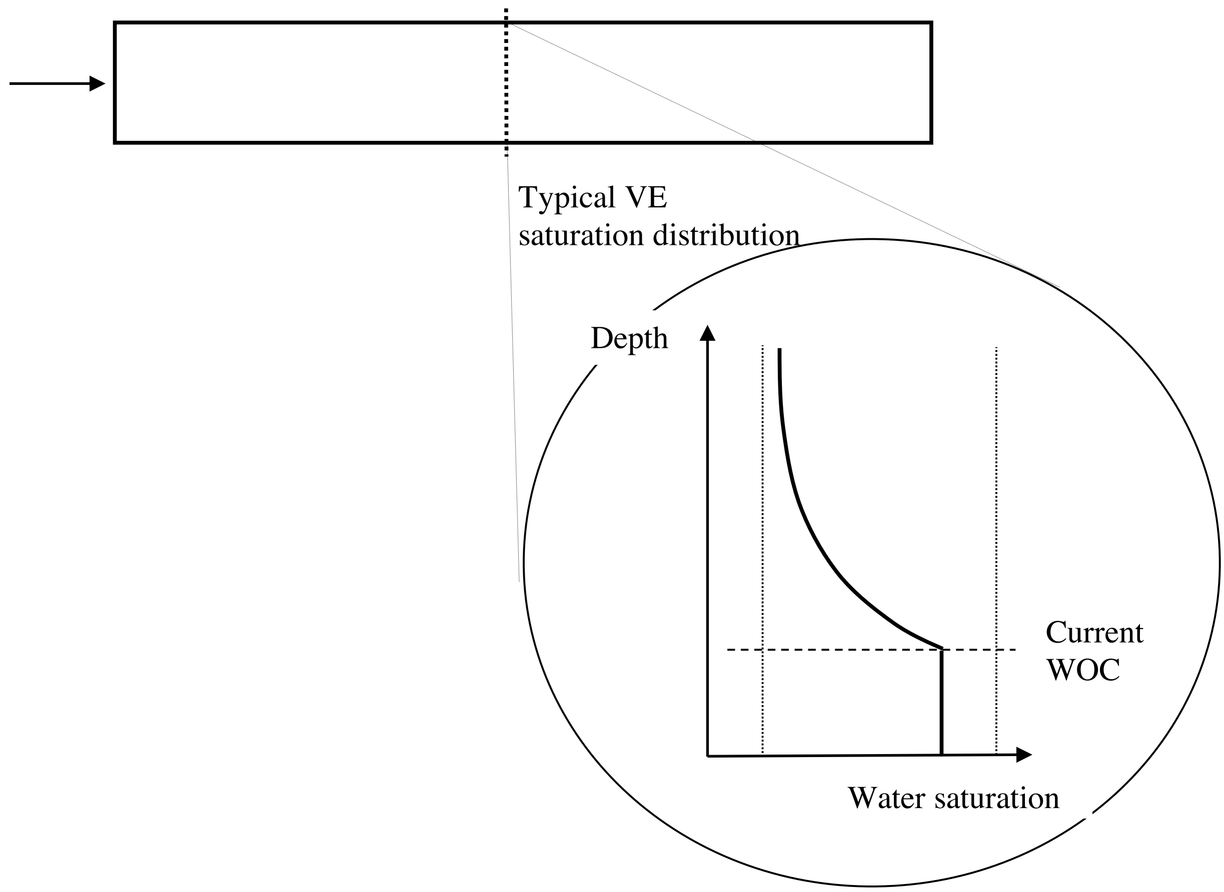
Explain the Vertical Equilibrium (VE) method. Include:

a) The basic assumptions behind the method
b) A sketch of a typical Vertical Equilibrium (VE) displacement situation
c) A short discussion of when the method is applicable

Solution

a) The basic assumptions behind the method
   - is developed for displacement under gravity dominance, ie. where $g\Delta\rho >> \frac{\delta P}{\delta x}$
   - instantaneous vertical equilibrium

b) A sketch of a typical Vertical Equilibrium (VE) displacement situation



c) A short discussion of when the method is applicable
   - applies to high permeability reservoir with excellent vertical communication, where gravity forces are strongly dominant

Question 10 (2+2+2=6 points)

a) What do we mean with "microscopic" and "macroscopic" recovery factors?
b) How can we improve the "microscopic" recovery of a reservoir?
c) How can we improve the "macroscopic" recovery of a reservoir?

Solution

a) What do we mean with "microscopic" and "macroscopic" recovery factors?
   - microscopic is related to the end point residual saturation, as seen on imbibition relative permeability curves and capillary pressure curve, while microscopic is related to large-scale recovery factors mainly influenced by layering, heterogeneity, well coverage, etc.

b) How can we improve the "microscopic" recovery of a reservoir?
   - By reducing interfacial tension between rock and fluids, eg. by surfactant additions to the injection water

c) How can we improve the "macroscopic" recovery of a reservoir?
   - By better volumetric sweep, through better well coverage, blocking of thief zones, etc.

Question 11 (2+4+8=14 points)

For water displacement of oil in a fractured reservoir the wetting conditions of the reservoir rock may greatly influence the recovery process.

a) Sketch the following capillary pressure curves:
   - A typical imbibition curve for a 100% water wetted system
   - A typical imbibition curve for a system that is partly oil wet.

   Mark the following items on the curves:
   - End point saturations
   - The area for spontaneous imbibition
   - The area for forced imbibition

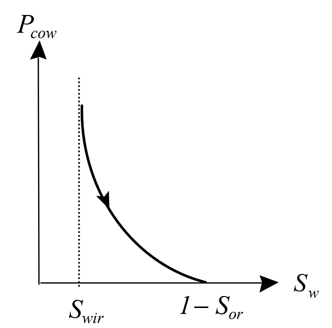
b) What is the final (theoretical) oil recovery factor for a 100% water-wet fractured reservoir under water flooding? Write the appropriate expression.

c) The vertical continuity (contact) between matrix blocks in the reservoir may in some cases influence significantly the oil recovery. Explain shortly for which situations this is true for the following processes:
   - Water displacement
   - Gas displacement

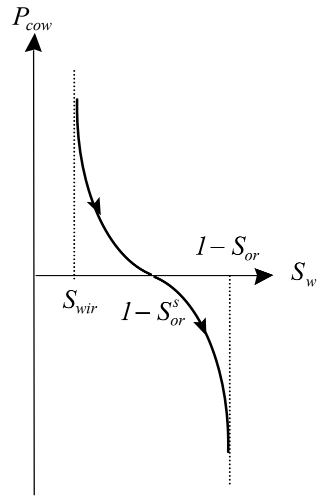
Solution

a)

- A typical imbibition curve for a 100% water wetted system



- A typical imbibition curve for a system that is partly oil wet.



b)

$$RF = \frac{OIP_{initially} - OIP_{finally}}{OIP_{initially}} = \frac{V_p\left[(1-S_{wir})-(S_{or})\right]}{V_p(1-S_{wir})} = \frac{1-S_{wir}-S_{or}}{1-S_{wir}}$$

c)

- Water displacement

  a. *For a 100% water-wet reservoir, there is no influence of vertical contact on oil recovery, since all movable oil is recovered by spontaneous imbibition*
  b. *For mixed-wet reservoir, spontaneous imbibition recovers oil only until* $P_{cow} = 0$ *ie. until* $S_o = S_{or}^s$. *Thereafter, oil is recovered by forced imbibition by gravity for* $P_{cow} < 0$. *The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller blocks*

- Gas displacement

  Since the process is a drainage process, for gas to enter the matrix blocks, and thus replace oil, the difference in phase pressures must exceed the displacement capillary pressure, ie. $P_g - P_o > P_{cogd}$. *Thus, oil is recovered by means of gravity forces. The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller block.s*

Attachment - Definition of symbols

B_g	Formation volume factor for gas (res.vol./st.vol.)
B_o	Formation volume factor for oil (res.vol./st.vol.)
B_w	Formation volume factor for water (res.vol./st.vol.)
C_r	Pore compressibility (pressure^{-1})
C_w	Water compressibility (pressure^{-1})
ΔP	$P_2 - P_1$
G_i	Cumulative gas injected (st.vol.)
GOR	Producing gas-oil ratio (st.vol./st.vol.)
G_p	Cumulative gas produced (st.vol.)
k	Absolute permeability
k_{ro}	Relative permeability to oil
k_{rw}	Relative permeability to oil
k_{rg}	Relative permeability to oil
m	Initial gas cap size (res.vol. of gas cap)/(res.vol. of oil zone)
M_e	End point mobility ratio
N	Original oil in place (st.vol.)
N_{ge}	Gravity number
N_p	Cumulative oil produced (st.vol.)
P	Pressure
P_{cow}	Capillary pressure between oil and water
P_{cog}	Capillary pressure between oil and gas
q_{inj}	Injection rate (res.vol./time)
R_p	Cumulative producing gas-oil ratio (st.vol./st.vol) = G_p / N_p
R_{so}	Solution gas-oil ratio (st.vol. gas/st.vol. oil)
S_g	Gas saturation
S_o	Oil saturation
S_w	Water saturation
T	Temperature
V_b	Bulk volume (res.vol.)
V_p	Pore volume (res.vol.)
WC	Producing water cut (st.vol./st.vol.)
W_e	Cumulative aquifer influx (st.vol.)
W_i	Cumulative water injected (st.vol.)
W_p	Cumulative water produced (st.vol.)
ρ	Density (mass/vol.)
ϕ	Porosity
μ_g	Gas viscosity
μ_o	Oil viscosity
μ_w	Water viscosity
γ	Hydrostatic pressure gradient (pressure/distance)