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Department of Petroleum Engineering and Applied Geophysics

SOLUTION

Examination paper for TPG4150 Reservoir Recovery Techniques

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Permitted examination support material: D/No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

Language: English

Number of pages: 15

Number of pages enclosed: 0

Checked by:

Date

Signature

Symbols used are defined in the enclosed table

Question 1 (5 points)

This question relates to the group project work.

a) State if you participated in a Gullfaks I1 or K1/K2 project
b) List the main production mechanisms in the field that you studied.
c) What was the value used for the gas cap parameter m in your calculations and explain how did you decide to use that value?
d) How did you obtain B_o and R_{so} for your calculations?
e) List at least two of the main conclusions of the project.
f) List at least three of the main uncertainties in the material balance calculations.

SOLUTION

Her må de bare vise at de har jobbet med prosjektet

Question 2 (16 points)

For displacement of oil by water in a reservoir cross-section, answer following questions:

a) What does the term "segregated flow" mean, and which factors determine this flow condition?
b) What does the term "diffuse flow" mean, and which factors determine this flow condition?
c) What does the term "vertical equilibrium" mean in reservoir analysis, and when is it a realistic assumption?
d) Sketch typical saturation profiles (in vertical direction) for “diffuse flow” conditions and “segregated flow” conditions.
e) What does the term "piston displacement" mean in reservoir analysis, and when is it a realistic assumption?
f) What is the Dykstra-Parsons method used for, and which assumptions are made for the method?
g) What is the Buckley-Leverett method used for, and which assumptions are made for the method?
h) What is the Dietz method used for, and which assumptions are made for the method?

SOLUTION

For displacement of oil by water in a reservoir cross-section, answer following questions:

a) What does the term "segregated flow" mean, and which factors determine this flow condition?
   - **Fluids separate according to density, and the flow is segregated flow if gravity gradients dominate the flow**

     ie. $g\Delta\rho >> \frac{\delta P}{\delta x}$

b) What does the term "diffuse flow" mean, and which factors determine this flow condition?
   - **Fluids do not separate according to density, and the flow is diffuse flow if dynamic pressure gradients dominate the flow**

ie. $\frac{\delta P}{\delta x} >> g\Delta\rho$ **(leads to uniform saturation distribution vertically)**

c) What does the term "vertical equilibrium" mean in reservoir analysis, and when is it a realistic assumption?

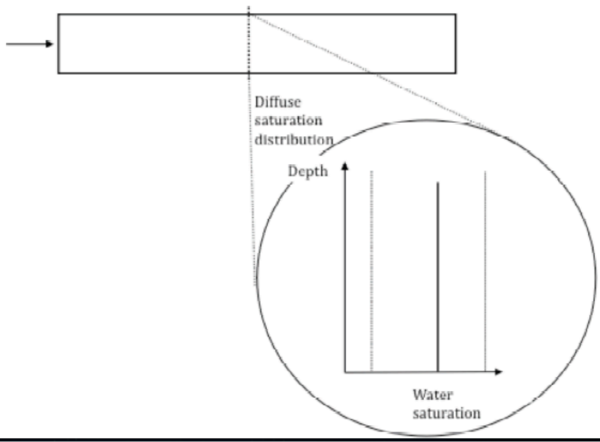
- **Fluids segregate vertically immediately (in accordance with capillary pressure), and may be realistic in high-permeability reservoirs with small dynamic gradients**

  ie. $g\Delta\rho >> \frac{\partial P}{\partial x}$ **(the "ultimate" segregated flow)**

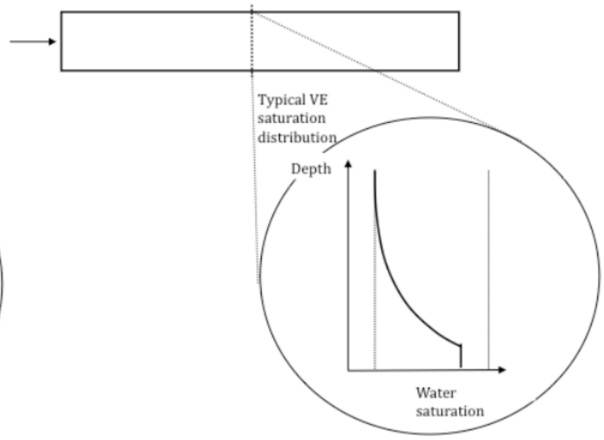
  May be a reasonable assumption in high permeability reservoirs where dynamic gradiens are small and vertical segregation takes place quickly

d) Sketch typical saturation profiles (in vertical direction) for "diffuse flow" conditions and "segregated flow" conditions.

Diffuse flow



Segregated flow



e) What does the term "piston displacement" mean in reservoir analysis, and when is it a realistic assumption?

- **All movable oil is displaced immediately; require a very low mobility ratio**

f) What is the Dykstra-Parsons method used for, and which assumptions are made for the method?

- **Displacement in layered systems without communication**
- **Assumptions**
  - **Constant pressure drop for all layers**
  - **piston displacement**
  - **capillary pressure negligible**

g) What is the Buckley-Leverett method used for, and which assumptions are made for the method?

- **Displacement calculations under diffuce flow conditions**
- **Assumptions**
  - **diffuse flow conditions**
  - **no capillary dispursion at front**
  - **incompressible fluids**

h) What is the Dietz method used for, and which assumptions are made for the method?

- **stable displacement in inclined systems**
- **Assumptions**
  - **vertical equilibrium**

- **piston displacement**
- **negligible capillary pressure**

Question 3 (15 points)

Write or derive an expression (equation or text) that defines each of the following terms (see list of symbols at the back):

solution---15x1 point---

a) Formation volume factor $B = \frac{\text{(res.vol.)}}{\text{(st.vol.)}}$

b) Solution gas-oil ratio $R_{so} = \frac{\text{(st.vol. gas)}}{\text{(st.vol. oil)}}$

c) Fluid compressibility $c_f = -\frac{1}{V_f}(\frac{\partial V_f}{\partial P})_T$

d) Pore compressibility $c_r = \frac{1}{\phi}(\frac{\partial \phi}{\partial P})_T$

e) Total reservoir compressibility $c_T = c_r + \sum_{i=o,w,g} c_i S_i$

f) Expansion volume (approximate) due to compressibility

$$\Delta V = V_2 - V_1 \approx -V_1 c(P_2 - P_1)$$

g) Real gas law for hydrocarbon gas $PV = nZRT$

h) Reservoir oil density $\rho_{oR} = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o}$

i) Reservoir gas density $\rho_{gR} = \frac{\rho_{gS}}{B_g}$

j) Reservoir water density $\rho_{wR} = \frac{\rho_{wS}}{B_w}$

k) Relationship between oil compressibility (undersaturated) and formation volume factor

$$C_o = -\frac{1}{V_o}\frac{dV_o}{dP}, and\ V_o = V_{oS} B_o$$

$$Thus, C_o = -\frac{1}{B_o}\frac{dB_o}{dP}$$

l) An expression for gas compressibility using the real gas law

$$C_g = -\frac{1}{V_g}\frac{dV_g}{dP}, and\ PV_g = nZRT$$

$$Thus, C_g = \frac{1}{P} - \frac{1}{Z}\frac{dZ}{dP}$$

m) What do we mean with "microscopic" and "macroscopic" recovery factors?

- **microscopic is related to the end point residual saturation, as seen on relative permeability curves while microscopic is related to large-scale recovery factors mainly influenced by layering, heterogeneity, well coverage, etc.**

n) How can we improve the "microscopic" recovery of a reservoir?

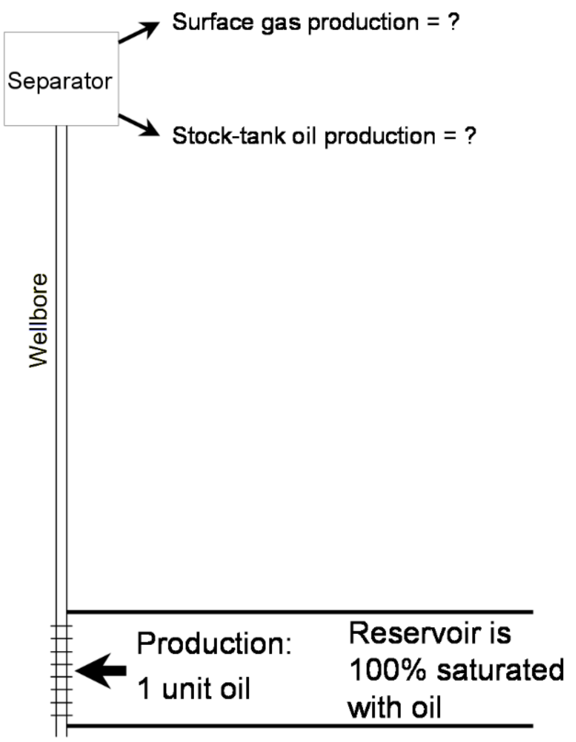
- **By reducing interfacial tension between rock and fluids, eg. by surfactant additions**

o) How can we improve the "macroscopic" recovery of a reservoir?
- **By better volumetric sweep, through better well coverage, blocking of thief zones, etc.**

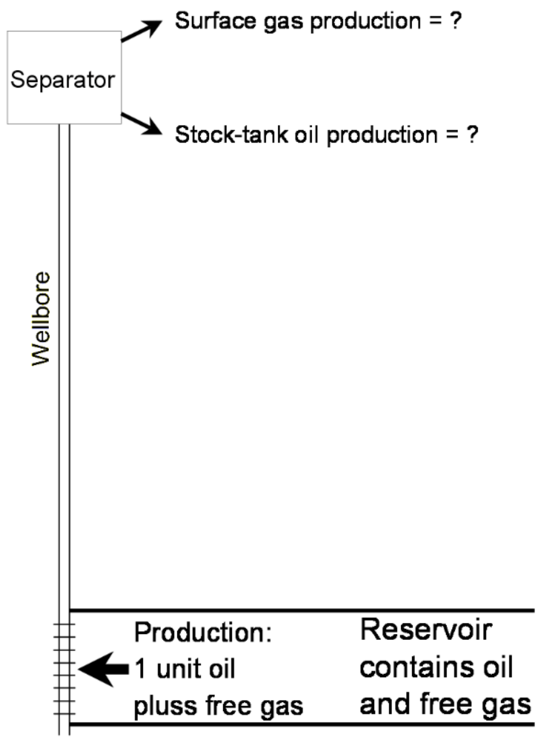
Question 4 (8 points)

For the two situations below (*i* and *ii*) please derive expressions for surface gas production, surface water production, and surface oil production. You may neglect capillary pressures.

i)



ii)



SOLUTION

i) (4 points) Oil in stock-tank: $1/B_o$

Surface volume of gas: R_{so}/B_o

Surface volume of water: 0.

ii) (6 points) Oil in stock-tank: $1/B_o$

Surface volume of gas: solution gas + free gas = $\mathrm{R_{so}/B_o} + \dfrac{k_{rg}\mu_o}{\mu_g k_{ro}\mathrm{B_g}}$

Surface volume of water: = 0.

Question 5 (8 points)

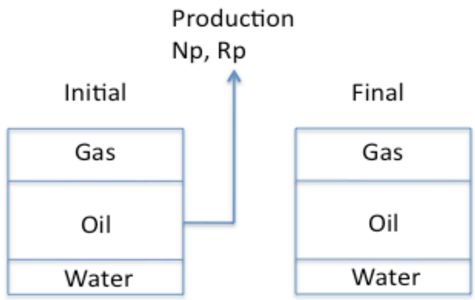
An oil reservoir with an initial gas cap and immobile water is being produced from initial conditions and down to a final closing pressure. The production stream consists of oil and gas.

Write (derive) the material balance equations needed, and find an expression for the oil recovery factor. Neglect water and rock compressibilities.

SOLUTION

* Det er også OK hvis de har begynt med den komplette MBE-ligningen (forutsatt at de husker den riktig) og reduserer den til denne oppgaven.

Utledning:



Pore volume:	Vp	Vp
Oil present (st):	N	VpSo2/Bo2
Oil present (res):	NBo1	VpSo2
Free gas (st):	mNBo1/Bg1	VpSg2/Bg2
Free gas (res):	mNBo1	VpSg2
Solution gas (st):	NRso1	VpSo2Rso2/Bo2
Water present (st):	VpSw1/Bw1	VpSw2/Bw
Water present (res):	VpSw1	VpSw2

General mass balance equation:

$$\begin{pmatrix}\text{Amount of fluids present}\\ \text{in the reservoir initially}\\ \text{(st. vol.)}\end{pmatrix} - \begin{pmatrix}\text{Amount of}\\ \text{fluids produced}\\ \text{(st. vol.)}\end{pmatrix} = \begin{pmatrix}\text{Amount of fluids remaining}\\ \text{in the reservoir finally}\\ \text{(st. vol.)}\end{pmatrix}$$

Oil eq. N - Np = VpSo2/Bo2
Gas eq. mNBo1/Bg1 +NRso1 - Gp = VpSg2/Bg2 + VpSo2 Rso2/Bo2
Water eq. Water and rock are imcompressible, thus Sw1=Sw2=constant
Pores Rock is incompressible, thus Vp is constant
Rearrange equations and add saturations: So2+Sg2+Sw2=1.0
Rearranging yelds:

$$\text{Recovery factor: } RF = \frac{\left[\frac{mB_{o1}}{B_{g1}} + R_{so1} - (1+m)\frac{B_{o1}}{B_{g2}} - (R_{so2} - \frac{B_{o2}}{B_{g2}})\right]}{\left[R_p - (R_{so2} - \frac{B_{o2}}{B_{g2}})\right]}$$

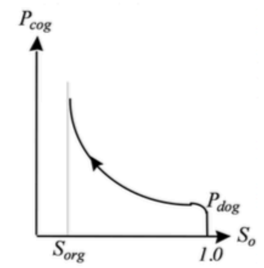
or $$RF = \frac{\left[(B_{o2} - B_{o1}) + B_{g2}(R_{so1} - R_{so2}) + mB_{o1}(\frac{B_{g2}}{B_{g1}} - 1) \right]}{\left[B_{g2}(R_p - R_{so2}) + B_{o2} \right]}$$

Question 6 (8 points)

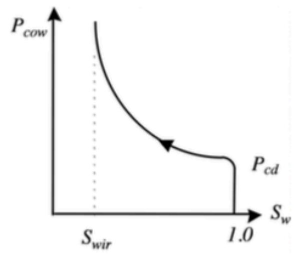
For the homogeneous reservoir section below

a) Sketch typical capillary pressure curves used for equilibrium calculations of initial saturations. Label important points.
b) Sketch typical initial water, oil and gas pressures vs. depth. Label important points used and explain briefly the procedure used.
c) Sketch the corresponding initial water, oil and gas saturation distributions determined by equilibrium calculations and capillary pressure curves. Label important points and explain briefly the procedure used.
d) Explain the concepts of WOC contact and free surface, using a sketch

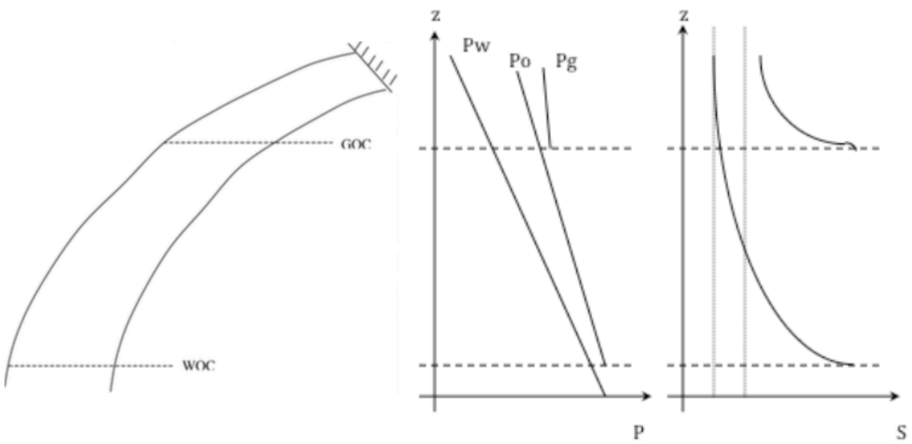
SOLUTION



drainage o-g

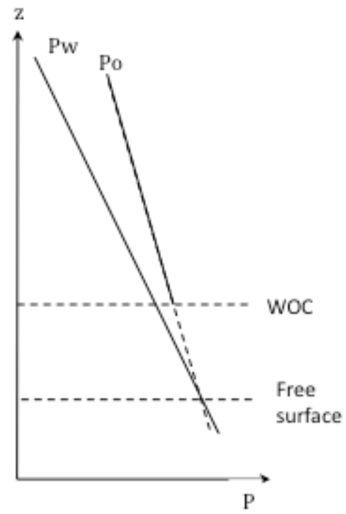


drainage o-w



At the WOC Po-Pw=Pdow, and at GOC Pg-Po. Initial pressures are computed using densities and assuming equilibrium. At WOC Sw=1,0. At any z value, Pcow is computed from the difference in Po and Pw, and the corresponding Sw is found from the Pcow-curve. At GOC Sg=0. At any z-value above the corresponding Sg is found from the Pcog-curve

- Explain the concepts of WOC contact and free surface, using a sketch



Question 7 (12 points)

a) Start with Darcy's equations for displacement of oil by water in an inclined layer at an angle α (positive upwards):

$$q_o = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha\right)$$

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial (P_o - P_c)}{\partial x} + \rho_w g \sin\alpha\right)$$

and derive the expression for water fraction flowing, f_w, inclusive capillary pressure and gravity.

b) Make typical sketches for water fraction flowing, f_w, vs. water saturation, assuming capillary pressure and gravity may be neglected, for the following cases:
- a high mobility ratio
- a low mobility ratio
- for piston displacement

c) Make a typical sketch for water saturation vs. x for water displacement of oil in a horizontal system (Buckley-Leverett), assuming capillary pressure and gravity may be neglected, for the following cases:
- a high mobility ratio
- a low mobility ratio
- for piston displacement

SOLUTION

a) Rewriting the equations as

$$-q_o \frac{\mu_o}{kk_{ro}A} = \frac{\partial P_o}{\partial x} + \rho_o g \sin\alpha$$

$$-q_w \frac{\mu_w}{kk_{rw}A} = \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} + \rho_w g \sin\alpha$$

and then subtracting the first equation from the second one, we get

$$-\frac{1}{kA}\left(q_w\frac{\mu_w}{k_{rw}}-q_o\frac{\mu_o}{k_{ro}}\right)=-\frac{\partial P_c}{\partial x}+\Delta\rho g\sin\alpha$$

Substituting for

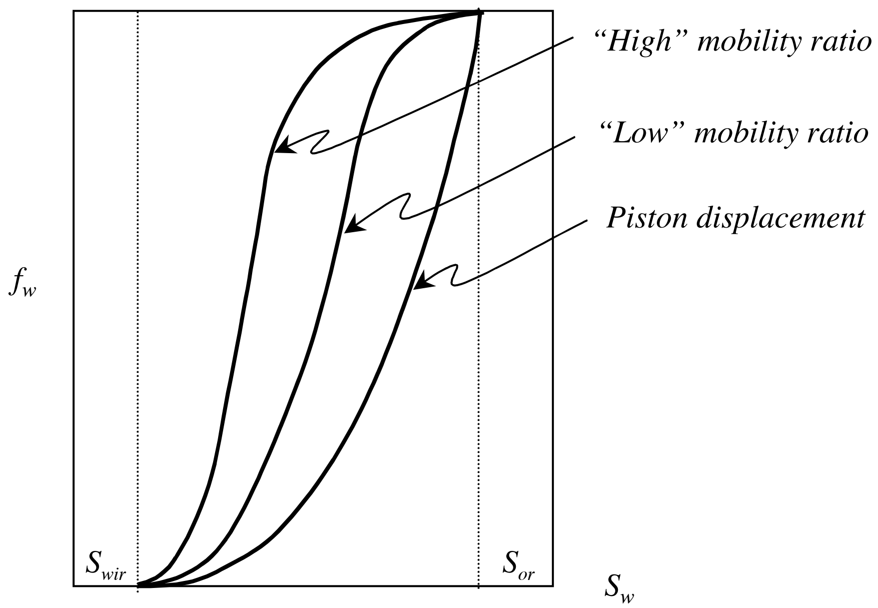
$q=q_w+q_o$

$f_w=\frac{q_w}{q}$

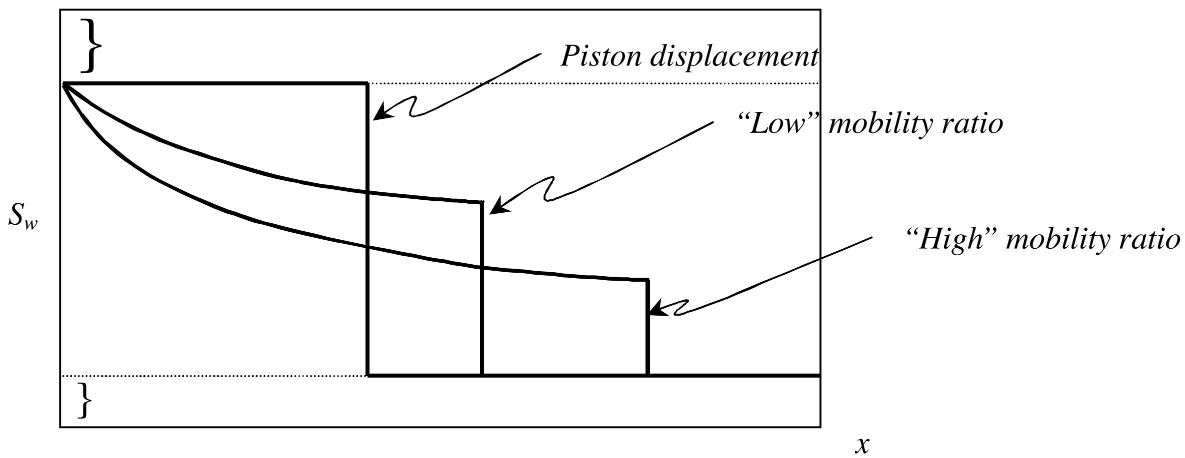
and solving for the fraction of water flowing, we get the following expression:

$$f_w=\frac{1+\frac{kk_{ro}A}{q\mu_o}\left(\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right)}{1+\frac{k_{ro}}{\mu_o}\frac{\mu_w}{k_{rw}}}$$

b)



c)



Question 8 (10 points)

For water displacement of oil in a fractured reservoir the wetting conditions of the reservoir rock may greatly influence the recovery process.

a) Sketch the following capillary pressure curves:
   - A typical imbibition curve for a 100% water wetted system
   - A typical imbibition curve for a system that is partly oil wet.

   Mark the following items on the curves:
   - End point saturations
   - The area for spontaneous imbibition
   - The area for forced imbibition

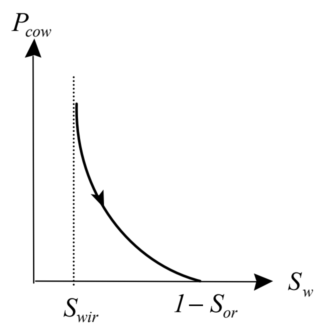
b) What is the final (theoretical) oil recovery factor for a 100% water-wet fractured reservoir under water flooding? Write the appropriate expression.

c) The vertical continuity (contact) between matrix blocks in the reservoir may in some cases influence significantly the oil recovery. Explain shortly for which situations this is true for the following processes:
   - Water displacement
   - Gas displacement

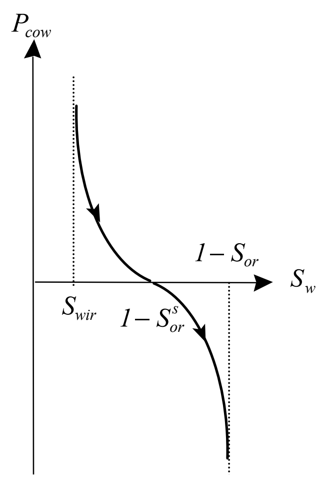
SOLUTION

a)

- A typical imbibition curve for a 100% water wetted system



- A typical imbibition curve for a system that is partly oil wet.



b)

$$RF = \frac{OIP_{initially} - OIP_{finally}}{OIP_{initially}} = \frac{V_p\left[(1-S_{wir})-(S_{or})\right]}{V_p(1-S_{wir})} = \frac{1-S_{wir}-S_{or}}{1-S_{wir}}$$

c)

- Water displacement

  a. *For a 100% water-wet reservoir, there is no influence of vertical contact on oil recovery, since all movable oil is recovered by spontaneous imbibition*
  b. *For mixed-wet reservoir, spontaneous imbibition recovers oil only until* $P_{cow} = 0$ *ie. until* $S_o = S_{or}^s$. *Thereafter, oil is recovered by forced imbibition by gravity for* $P_{cow} < 0$. *The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller blocks*

- Gas displacement

  Since the process is a drainage process, for gas to enter the matrix blocks, and thus replace oil, the difference in phase pressures must exceed the displacement capillary pressure, ie. $P_g - P_o > P_{cogd}$. *Thus, oil is recovered by means of gravity forces. The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller block.s*

<u>Question 9</u> (8 points)

Start with Darcy´s equations for oil and gas (neglect capillary pressure), and

a) Derive an expression for *GOR* (gas-oil ratio) at surface conditions for a well that perforates one layer in a horizontal, <u>undersaturated</u> reservoir.
b) Derive an expression for *GOR* (gas-oil ratio) at surface for a well that perforates one layer in a horizontal, <u>saturated</u> reservoir (Neglect capillary pressure).
c) Sketch a typical curve of GOR vs. time for an initially undersaturated oil reservoir that is produced through pressure depletion. Explain all details.

Producing GOR of a well is 1100 (sm^3 gas/sm^3 oil) and the solution gas-oil ratio (R_{so}) is 100 (sm^3 gas/sm^3 oil). Formation-volume factors for oil and gas are: $B_o = 2$ and $B_g = 0,005$.

d) What is GOR at reservoir conditions (rm^3 gas/rm^3 oil)?
e) What is the fraction of the surface-GOR (sm^3 gas/sm^3 oil) coming from the free gas in the reservoir?
f) What is the gas-oil mobility ratio in the reservoir?

SOLUTION

a)

$$q_o = -\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$q_g = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g}\frac{\partial P}{\partial r} = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$GOR = \frac{q_g}{q_o} = R_{so}$$

b)

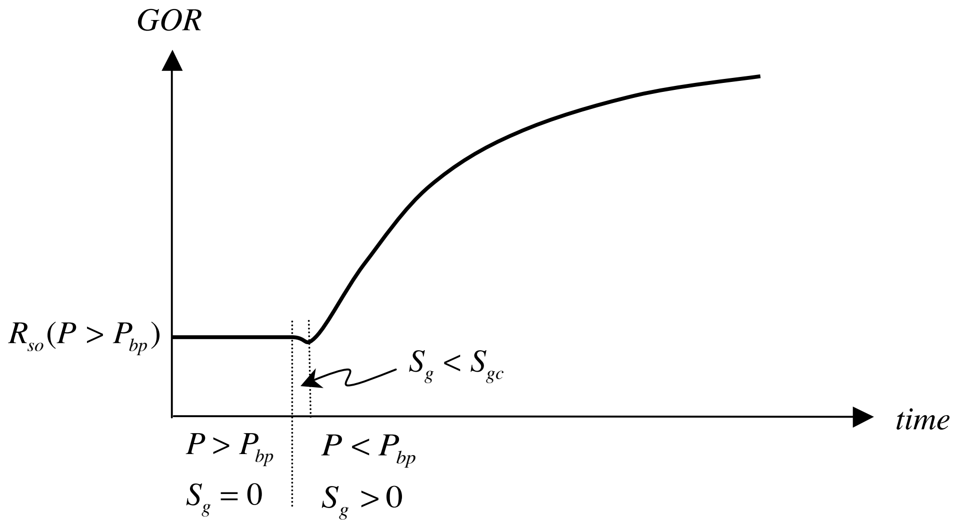
$$q_o = -\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$q_g = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g B_g}\frac{\partial P}{\partial r}$$

$$GOR = \frac{q_g}{q_o} = \frac{-R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g B_g}\frac{\partial P}{\partial r}}{-\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}}$$

$$GOR = R_{so} + \frac{k_g \mu_o B_o}{\mu_g k_o B_g}$$

c)



d)

$$GOR_s = \left(\frac{q_g}{q_o}\right)_s$$

$$GOR_r = \left(\frac{q_g}{q_o}\right)_r = \frac{(q_{gs} - q_{os}R_{so})B_g}{q_{os}B_o} = (GOR_s - R_{so})\frac{B_g}{B_o} = 1000\frac{0,005}{2} = 2,5$$

e)

$$GOR_{total} = R_{so} + \frac{k_g \mu_o B_o}{\mu_g k_o B_g} = 1100$$

$$GOR_{free\ gas} = 1100 - R_{so} = 1000\ sm^3\ gas / sm^3\ oil$$

f)

$$GOR_{total} = GOR_{solution\ gas} + GOR_{free\ gas} = R_{so} + \frac{k_g \mu_o B_o}{\mu_g k_o B_g} = 1100$$

$$GOR_{free\ gas} = \frac{k_g \mu_o B_o}{\mu_g k_o B_g} = 1000 \Rightarrow M_{go} = \frac{k_g \mu_o}{\mu_g k_o} = 1000 \frac{B_g}{B_o} = 1000 \frac{0,005}{2} = 2,5$$

<u>Question 10</u> (10 points)

a) List all steps and show formulas/equations/definitions used in the derivation of a one-dimensional (*x*) one-phase (oil), horizontal fluid flow equation.

b) Show all details in the derivation of the following equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$$

c) Which two main types of boundary conditions are normally used to represent reservoir fluid production and injection?

<u>SOLUTION</u>

a)

List all steps and formulas/equations/definitions used in the derivation of a one-dimensional (*x*) one-phase (oil) with Black Oil fluid description, horizontal fluid flow equation.

- **Continuity equation:** $-\frac{\partial}{\partial x}(A\rho_o u_o) = \frac{\partial}{\partial t}(A\phi\rho_o)$
- **Darcy's equation:** $u_o = -\frac{k}{\mu_o}\left(\frac{\partial P_o}{\partial x}\right)$
- **Fluid description** $\rho_o = \frac{\rho_{os} + \rho_{gs} R_{so}}{B_o} = \frac{\text{constant}}{B_o}$
- **Pore description:** $c_r = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial P_o}\right)_T$

b)

<u>Continuity equation:</u>

$$-\frac{\partial}{\partial x}(A\rho_o u_o) = \frac{\partial}{\partial t}(A\phi\rho_o)$$

For constant cross sectional area, the continuity equation simplifies to:

$$-\frac{\partial}{\partial x}(\rho_o u_o) = \frac{\partial}{\partial t}(\phi\rho_o)$$

Darcy's equation

$$u_o = -\frac{k}{\mu_o}\left(\frac{\partial P_o}{\partial x}\right)$$

Rock compressibility

$$c_r = \frac{1}{\phi}\frac{d\phi}{dP_o}$$

Fluid compressibility

$$\rho_o = \frac{\text{constant}}{B_o}$$

Substituting Darcy´s equation and oil density into the continuity equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) = \frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right)$$

The right hand side (RHS) of the equation may be expanded as:

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right) = \phi\frac{d}{dP_o}\left(\frac{1}{B_o}\right)\frac{\partial P_o}{\partial t} + \frac{1}{B_o}\frac{d\phi}{dP_o}\frac{\partial P_o}{\partial t}$$

The final equation then becomes:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) = \phi\left[\frac{c_r}{B_o} + \frac{\partial}{\partial P_o}(1/B_o)\right]\frac{\partial P_o}{\partial t}$$

c)

- **Bottom hole pressure specified**
- **Production rate specified**

Attachment - Definition of symbols

B_g	Formation volume factor for gas (res.vol./st.vol.)
B_o	Formation volume factor for oil (res.vol./st.vol.)
B_w	Formation volume factor for water (res.vol./st.vol.)
C_r	Pore compressibility (pressure^{-1})
C_w	Water compressibility (pressure^{-1})
ΔP	$P_2 - P_1$
G_i	Cumulative gas injected (st.vol.)
GOR	Producing gas-oil ratio (st.vol./st.vol.)
G_p	Cumulative gas produced (st.vol.)
k	Absolute permeability
k_{ro}	Relative permeability to oil
k_{rw}	Relative permeability to oil
k_{rg}	Relative permeability to oil
m	Initial gas cap size (res.vol. of gas cap)/(res.vol. of oil zone)
M_e	End point mobility ratio
N	Original oil in place (st.vol.)
N_{ge}	Gravity number
N_p	Cumulative oil produced (st.vol.)
P	Pressure
P_{cow}	Capillary pressure between oil and water
P_{cog}	Capillary pressure between oil and gas
q_{inj}	Injection rate (res.vol./time)
R_p	Cumulative producing gas-oil ratio (st.vol./st.vol) = G_p / N_p
R_{so}	Solution gas-oil ratio (st.vol. gas/st.vol. oil)
S_g	Gas saturation
S_o	Oil saturation
S_w	Water saturation
T	Temperature
V_b	Bulk volume (res.vol.)
V_p	Pore volume (res.vol.)
WC	Producing water cut (st.vol./st.vol.)
W_e	Cumulative aquifer influx (st.vol.)
W_i	Cumulative water injected (st.vol.)
W_p	Cumulative water produced (st.vol.)
ρ	Density (mass/vol.)
ϕ	Porosity
μ_g	Gas viscosity
μ_o	Oil viscosity
μ_w	Water viscosity
γ	Hydrostatic pressure gradient (pressure/distance)