NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Petroleum Engineering and Applied Geophysics

SOLUTION

Examination paper for TPG4150 Reservoir Recovery Techniques

Academic contact during examination: Jon Kleppe

Phone: 91897300/73594925

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Other information:

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Number of pages: 7

Number of pages enclosed: 0

Checked by:

Date

Signature

Symbols used are defined in the enclosed table

Question 1 (8 points)

This question relates to the group project work.

a) State if you participated in a Norne or Gullfaks K1/K2 project
b) List the main production mechanisms in the field that you studied.
c) What was the value used for the gas cap parameter m in your calculations and explain how did you decide to use that value?
d) How did you obtain B_o and R_{so} for your calculations?
e) List at least two of the main conclusions of the project.
f) List at least three of the main uncertainties in the material balance calculations.

Solution

Question 2 (10 points)

Write an expression (equation or text) that defines each of the following terms (see list of symbols at the back):
a) Formation volume factor
b) Solution gas-oil ratio
c) Fluid compressibility
d) Pore compressibility
e) Total reservoir compressibility
f) Expansion volume due to compressibility and pressure change
g) Real gas law for hydrocarbon gas
h) Reservoir oil density
i) Reservoir gas density
j) Reservoir water density

Solution

a) Formation volume factor $$B = \frac{\text{(res.vol.)}}{\text{(st.vol.)}}$$

b) Solution gas-oil ratio $$R_{so} = \frac{\text{(st.vol. gas)}}{\text{(st.vol. oil)}}$$

c) Fluid compressibility $$c_f = -\frac{1}{V_f}\left(\frac{\partial V_f}{\partial P}\right)_T$$

d) Pore compressibility $$c_r = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial P}\right)_T$$

e) Total reservoir compressibility $$c_T = c_r + \sum_{i=o,w,g} c_i S_i$$

f) Expansion volume due to compressibility $$\Delta V = V_2 - V_1 \approx -V_1 c (P_2 - P_1)$$

g) Real gas law for hydrocarbon gas $PV = nZRT$

h) Reservoir oil density $\rho_{oR} = \dfrac{\rho_{oS} + \rho_{gS} R_{so}}{B_o}$

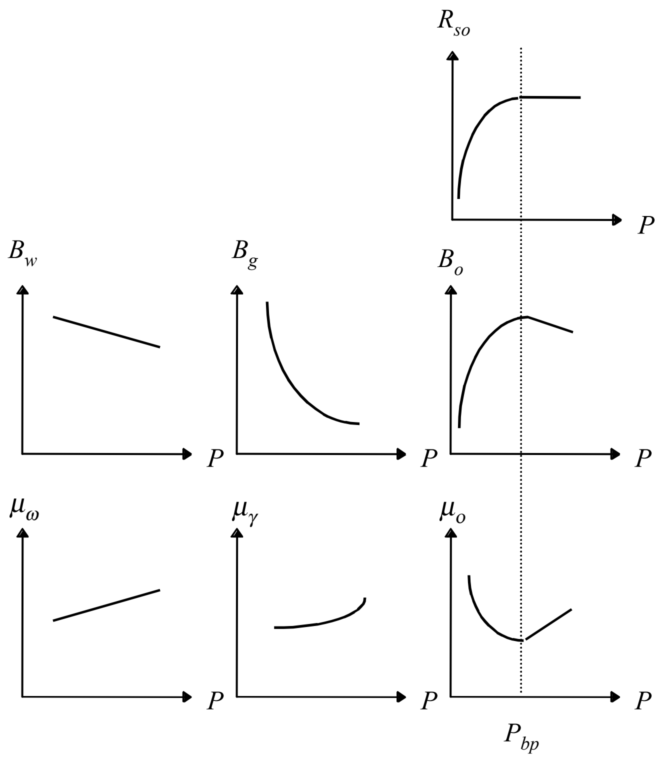
i) Reservoir gas density $\rho_{gR} = \dfrac{\rho_{gS}}{B_g}$

j) Reservoir water density $\rho_{wR} = \dfrac{\rho_{wS}}{B_w}$

Question 3 (5 points)

Sketch typical B_o, B_w, B_g, μ_o, μ_w, μ_g, and R_{so} curves. Label axes, characteristic points and areas.

Solution

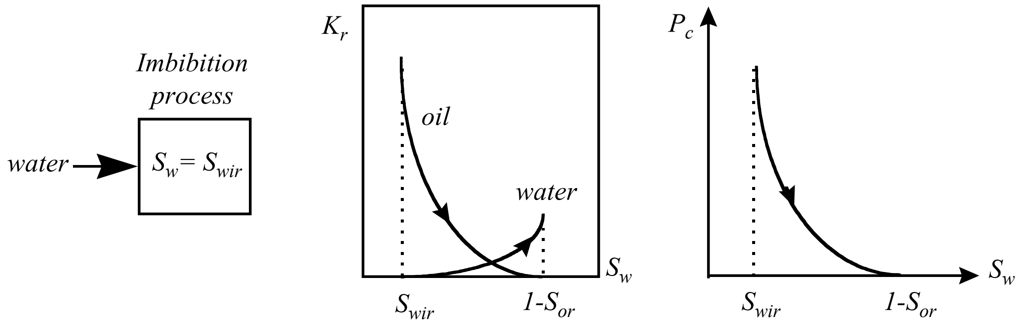


Question 4 (6 points)

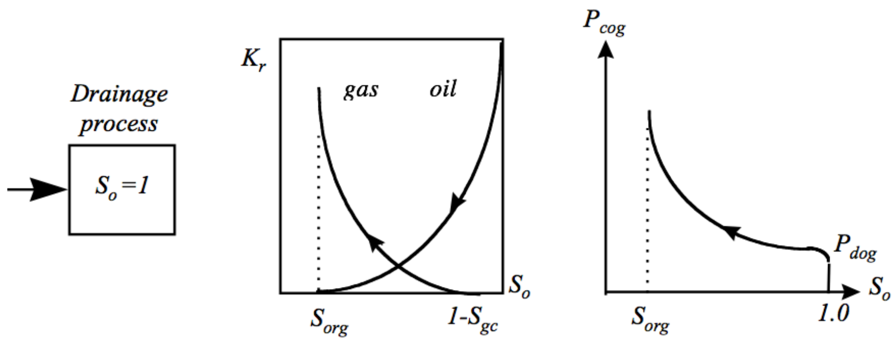
a) Sketch typical imbibition and drainage k_{ro}, k_{rw}, and P_{cow} curves for an oil-water system (assume a completely water-wet system).
b) Sketch typical imbibition and drainage k_{ro}, k_{rg}, and P_{cog} curves for an oil-gas system Label axes, characteristic points and areas.
c) Sketch a typical imbibition capillary pressure curve for a oil-water system of mixed wetting. Label axes, characteristic points and areas.

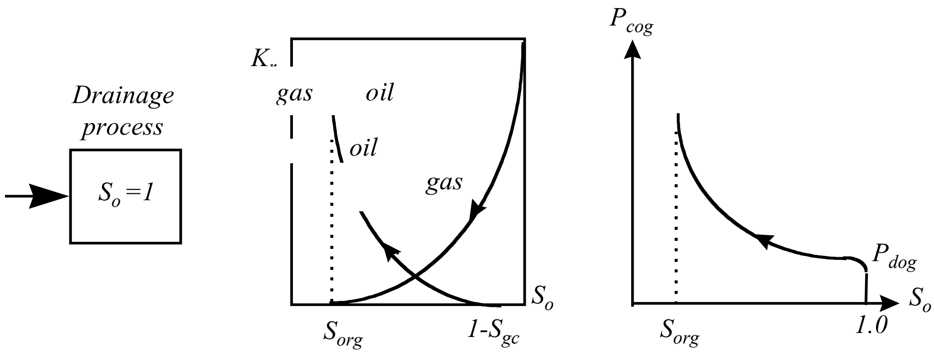
Solution

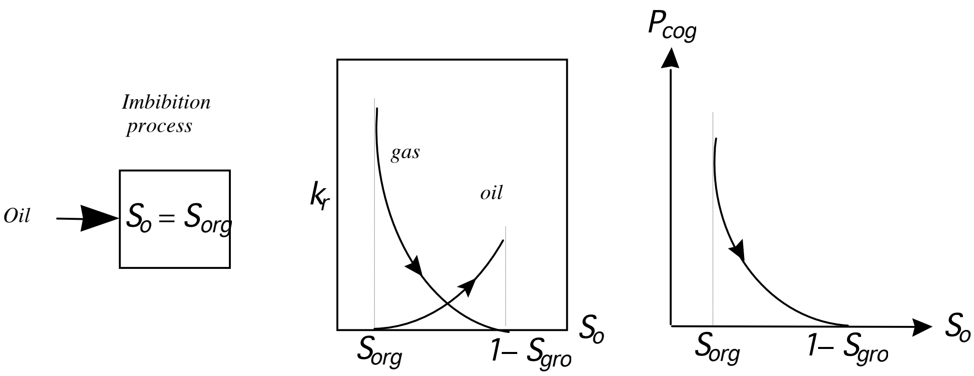
a)


Imbibition process
water
$S_w = S_{wir}$
K_r
oil
water
S_w
S_{wir}
$1-S_{or}$
P_c
S_w
S_{wir}
$1-S_{or}$

b)

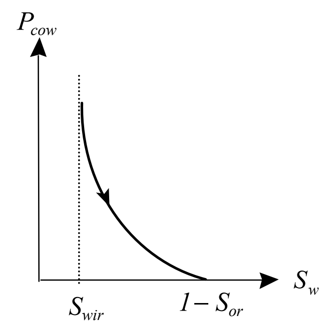

Drainage process
$S_o=1$
K_r
gas
oil
S_o
S_{org}
$1-S_{gc}$
P_{cog}
P_{dog}
S_o
S_{org}
1.0


Drainage process
$S_o=1$
gas
oil
oil
gas
S_o
S_{org}
$1-S_{gc}$
P_{cog}
P_{dog}
S_o
S_{org}
1.0

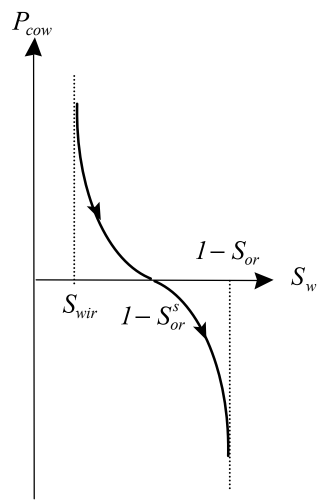

Imbibition process
Oil
$S_o = S_{org}$
k_r
gas
oil
S_o
S_{org}
$1-S_{gro}$
P_{cog}
S_o
S_{org}
$1-S_{gro}$

c)

- A typical imbibition curve for a 100% water wetted system



- A typical imbibition curve for a system that is partly oil wet.



Question 5 (15 points)

The general form of the Material Balance Equation may be written as (se attached definitions of the symbols used):

$$N_p\left[B_{o2}+\left(R_p-R_{so2}\right)B_{g2}\right]+W_pB_{w2}=$$
$$N\left[\left(B_{o2}-B_{o1}\right)+\left(R_{so1}-R_{so2}\right)B_{g2}+mB_{o1}\left(\frac{B_{g2}}{B_{g1}}-1\right)-\left(1+m\right)B_{o1}\frac{C_r+C_wS_{w1}}{1-S_{w1}}\left(P_2-P_1\right)\right]$$
$$+\left(W_i+W_e\right)B_{w2}+G_iB_{g2}$$

a) What is the primary assumption behind the use of the Material Balance Equation, and which "driving mechanisms" or "energies" are included in the equation?

b) Identify by words the physical meaning of each of the terms below (be brief, 3-5 words for each term is sufficient)

b1) $N_p B_{o2}$

b2) $N_p\left(R_p - R_{so2}\right)B_{g2}$

b3) $W_p B_{w2}$

b4) $N\left(B_{o2} - B_{o1}\right)$

b5) $N\left(R_{so1} - R_{so2}\right)B_{g2}$

b6) $NmB_{o1}\left(\dfrac{B_{g2}}{B_{g1}} - 1\right)$

b7) $-N\left(1+m\right)B_{o1}\dfrac{C_r + C_w S_{w1}}{1 - S_{w1}}\left(P_2 - P_1\right)$

b8) $\left(W_i + W_e\right)B_{w2}$

b9) $G_i B_{g2}$

c) Simplify the equation and write the expression for oil recovery factor (N_p / N) for the following reservoir system:

- Initially undersaturated oil
- Production stream consists of oil and gas
- Gas injection only
- No aquifer

d) Sketch a typical curve of GOR vs. time for the reservoir above (c). Explain details of the curve.

Solution

a)

Assumption: Zero-dimensional flow

Energies:	**Fluid expansion**
	Rock compaction (pore compaction)
	Fluid injection
	Aquifer influx

b)

b1) $N_p B_{o2}$	**Reservoir volume of produced oil**
b2) $N_p\left(R_p - R_{so2}\right)B_{g2}$	**Reservoir volume of produced free gas**
b3) $W_p B_{w2}$	**Reservoir volume of produced water**
b4) $N\left(B_{o2} - B_{o1}\right)$	**Reservoir expansion of OOIP**
b5) $N\left(R_{so1} - R_{so2}\right)B_{g2}$	**Reservoir volume change of solution gas in place**

b6) $NmB_{o1}\left(\dfrac{B_{g2}}{B_{g1}}-1\right)$ **Reservoir expansion of gas cap**

b7) $-N(1+m)B_{o1}\dfrac{C_r+C_wS_{w1}}{1-S_{w1}}(P_2-P_1)$ **Volume change of pores and water**

b8) $(W_i+W_e)B_{w2}$ **Reservoir volume of added water**

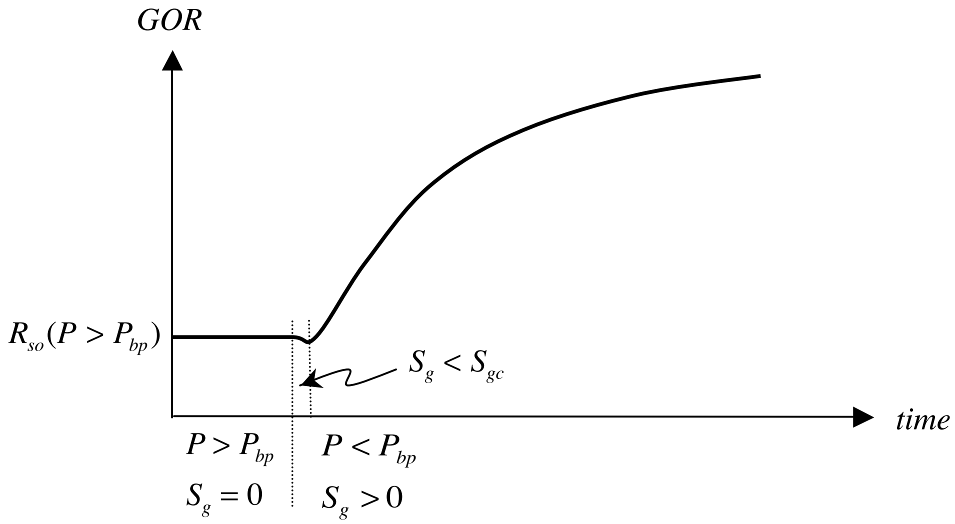
b9) G_iB_{g2} **Reservoir volume of injected gas**

<u>**c)**</u>

$$N_p\left[B_{o2}+(R_p-R_{so2})B_{g2}\right]=N\left[(B_{o2}-B_{o1})+(R_{so1}-R_{so2})B_{g2}-B_{o1}\frac{C_r+C_wS_{w1}}{1-S_{w1}}(P_2-P_1)\right]+G_iB_{g2}$$

$$\frac{N_p}{N}=\frac{\left[(B_{o2}-B_{o1})+(R_{so1}-R_{so2})B_{g2}-B_{o1}\dfrac{C_r+C_wS_{w1}}{1-S_{w1}}(P_2-P_1)\right]+G_iB_{g2}/N}{\left[B_{o2}+(R_p-R_{so2})B_{g2}\right]}$$

<u>**d)**</u>



<u>**Question 6**</u> (12 points)

Start with Darcy´s equations for oil and gas (neglect capillary pressure), and

a) Derive an expression for *GOR* (gas-oil ratio) at surface conditions for a well that perforates one layer in a horizontal, <u>undersaturated</u> reservoir.
b) Derive an expression for *GOR* (gas-oil ratio) at surface for a well that perforates one layer in a horizontal, <u>saturated</u> reservoir (Neglect capillary pressure).
c) Sketch a typical curve of GOR vs. time for an initially undersaturated oil reservoir that is produced through pressure depletion. Explain all details.

The producing GOR of a well is 1100 (sm^3 gas/sm^3 oil) and the solution gas-oil ratio (R_{so}) is 100 (sm^3 gas/sm^3 oil). Formation-volume factors for oil and gas are: $B_o = 2$ and $B_g = 0{,}005$.

d) What is GOR at reservoir conditions (rm^3 gas/rm^3 oil)?
e) What is the fraction of the surface-GOR (sm^3 gas/sm^3 oil) coming from the free gas in the reservoir?
f) What is the gas-oil mobility ratio in the reservoir?

Solution

a)

$$q_o = -\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$q_g = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g}\frac{\partial P}{\partial r} = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$\boxed{GOR = \frac{q_g}{q_o} = R_{so}}$$

b)

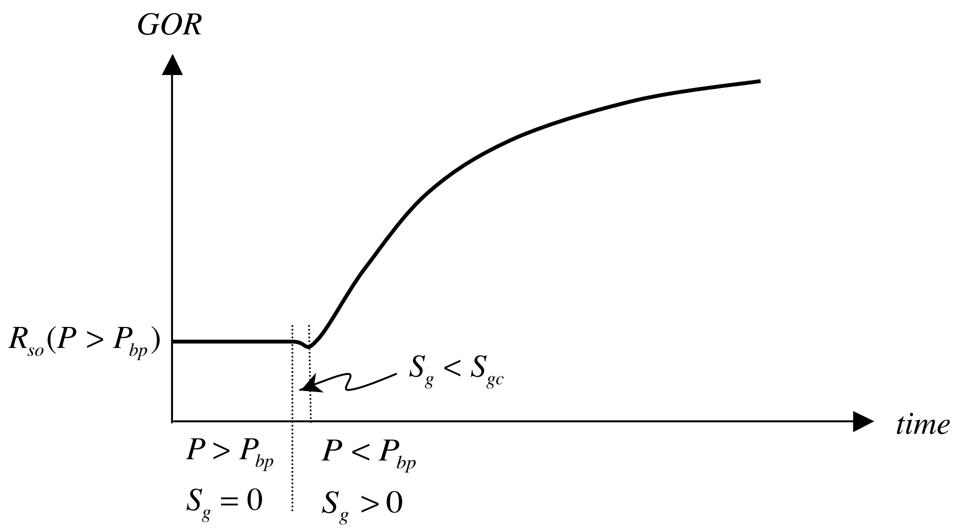
$$q_o = -\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}$$

$$q_g = -R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g B_g}\frac{\partial P}{\partial r}$$

$$GOR = \frac{q_g}{q_o} = \frac{-R_{so}\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r} - \frac{k_g A}{\mu_g B_g}\frac{\partial P}{\partial r}}{-\frac{k_o A}{\mu_o B_o}\frac{\partial P}{\partial r}}$$

$$\boxed{GOR = R_{so} + \frac{k_g \mu_o B_o}{\mu_g k_o B_g}}$$

c)



The producing GOR of a well is 1100 (sm^3 gas/sm^3 oil) and the solution gas-oil ratio (R_{so}) is 100 (sm^3 gas/sm^3 oil). Formation-volume factors for oil and gas are: $B_o = 2$ and $B_g = 0{,}005$.

d)

$$GOR_s = \left(\frac{q_g}{q_o}\right)_s$$

$$GOR_r = \left(\frac{q_g}{q_o}\right)_r = \frac{(q_{gs} - q_{os}R_{so})B_g}{q_{os}B_o} = (GOR_s - R_{so})\frac{B_g}{B_o} = 1000\frac{0{,}005}{2} = 2{,}5$$

e)

$$GOR_{total} = R_{so} + \frac{k_g\mu_o B_o}{\mu_g k_o B_g} = 1100$$

$$GOR_{free\ gas} = 1100 - R_{so} = 1000\ sm^3\ gas\ /\ sm^3\ oil$$

f)

$$GOR_{total} = GOR_{solution\ gas} + GOR_{free\ gas} = R_{so} + \frac{k_g\mu_o B_o}{\mu_g k_o B_g} = 1100$$

$$GOR_{free\ gas} = \frac{k_g\mu_o B_o}{\mu_g k_o B_g} = 1000 \Rightarrow M_{go} = \frac{k_g\mu_o}{\mu_g k_o} = 1000\frac{B_g}{B_o} = 1000\frac{0{,}005}{2} = 2{,}5$$

Question 7 (12 points)

a) List all steps and formulas/equations/definitions used in the derivation of a (one-phase) fluid flow equation.

Which coordinate systems are used for the following flow equations?

b) $\frac{\partial^2 P}{\partial x^2} = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$

c) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$

d) $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial P}{\partial r}\right) = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$

e) Which two main types of boundary conditions are normally used to represent reservoir fluid production and injection?

f) Write the steady-state form of equation d) above, and solve for pressure as a function of radius for boundary conditions $P(r = r_e) = P_e$ and $P(r = r_w) = P_w$

Solution

a)

- **Continuity equation**
- **Darcy's equation (velocity as function of pressure)**
- **Fluid description (density as function of pressure)**
- **Pore description (porosity as function of pressure)**

b) $$\frac{\partial^2 P}{\partial x^2} = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$ **Cartesian (linear)**

c) $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$ **Cylindrical (radial)**

d) $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial P}{\partial r}\right) = (\frac{\phi\mu c}{k})\frac{\partial P}{\partial t}$$ **Spherical**

e)

- **Bottom hole pressure specified**
- **Production rate specified**

g)

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial P}{\partial r}\right) = 0 \quad \Rightarrow \quad r^2\frac{\partial P}{\partial r} = A \quad \Rightarrow \quad \frac{\partial P}{\partial r} = \frac{A}{r^2} \quad \Rightarrow \quad P = -\frac{A}{r} + B$$

$$\left.\begin{aligned} P(r = r_e) = P_e &= -\frac{A}{r_e} + B \\ P(r = r_w) = P_w &= -\frac{A}{r_w} + B \end{aligned}\right\} \quad A = \frac{P_e - P_w}{\left(\frac{1}{r_w} - \frac{1}{r_e}\right)}, \quad B = P_e - \frac{P_e - P_w}{\left(\frac{1}{r_w} - \frac{1}{r_e}\right)r_e}$$

$$P = P_e - \frac{P_e - P_w}{\left(\frac{1}{r_w} - \frac{1}{r_e}\right)}\left(\frac{1}{r} - \frac{1}{r_e}\right)$$

Question 8 (8 points)

For displacement of oil by water in a porous rod of constant cross-sectional area, answer following questions:

a) Which assumptions are made for the Buckley-Leverett method (in a linear system)?
b) Define fraction of water flowing, f_w (definition only)
c) Make sketches of typical f_w vs. S_w, and S_w vs. x curves, for displacement in a horizontal porous rod with and without capillary forces, respectively. Explain differences.
d) Make sketches of f_w vs. S_w, and S_w vs. x, for displacement in a horizontal and a vertical rod (injection at bottom), respectively. Explain differences.

Solution

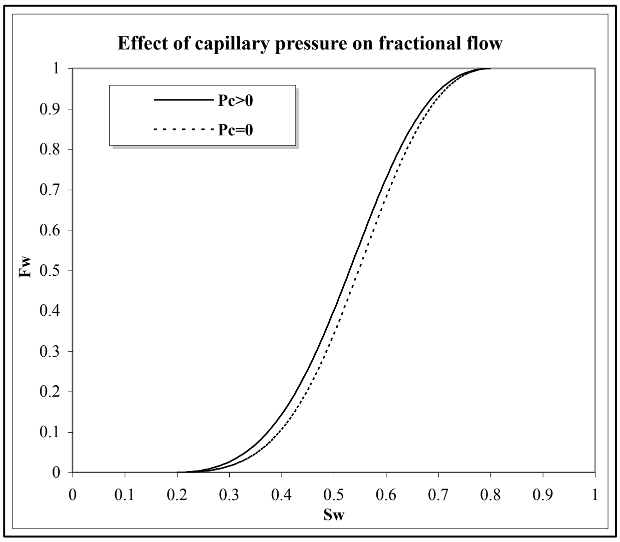
a)

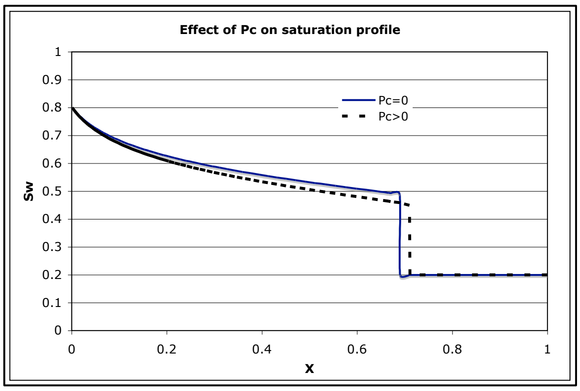
diffuse flow

b)

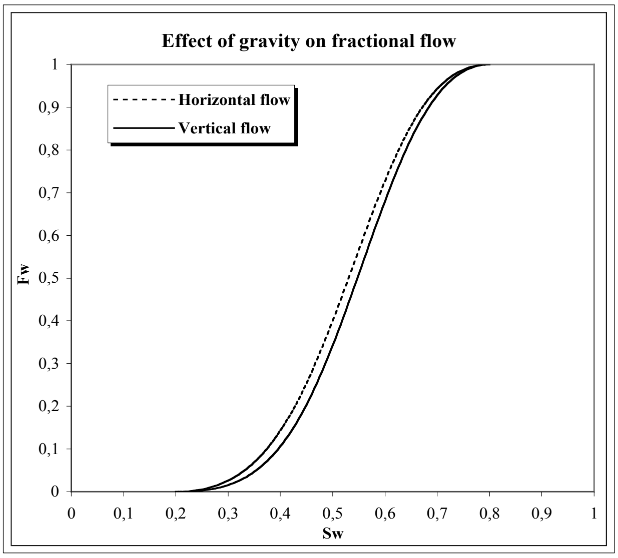
$$f_w = \frac{q_w}{q_w + q_o}$$

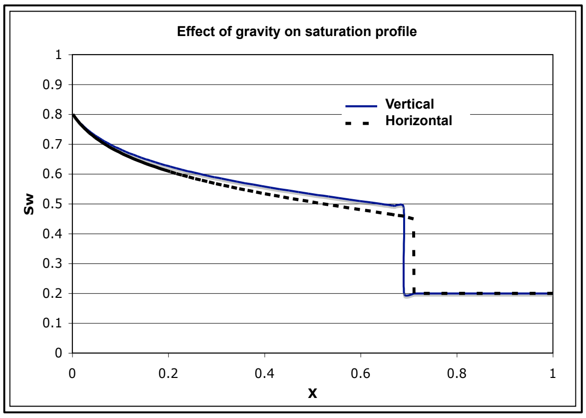
c)





d)





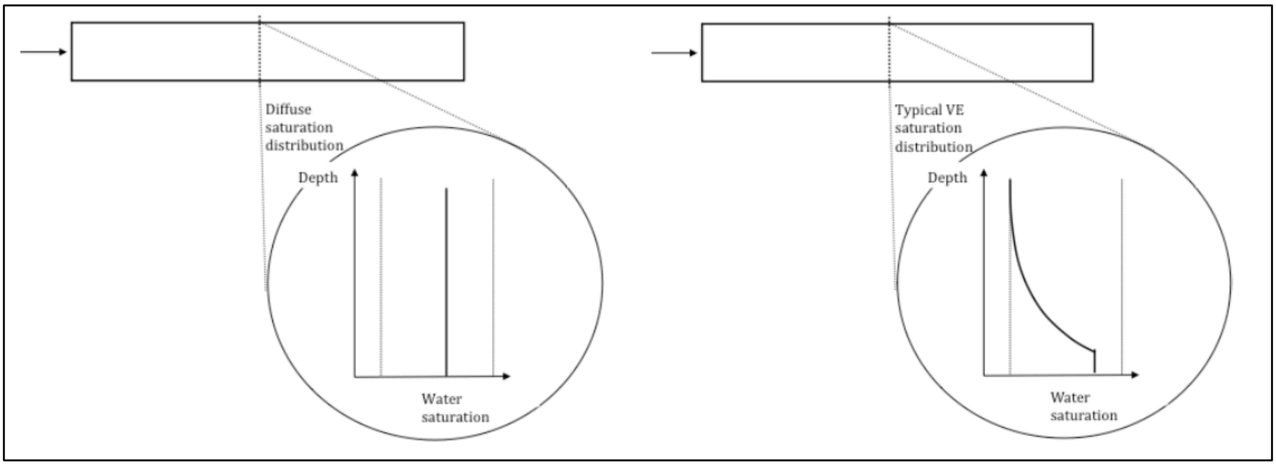
Question 9 (10 points)

For displacement of oil by water in a reservoir cross-section, answer following questions:

a) Sketch saturation profiles (in vertical direction) for "diffuse flow" conditions and "segregated flow" conditions.
b) What do the terms "diffuse flow" and "segregated flow" mean, and which factors determine these flow conditions?
c) What does the term "vertical equilibrium" mean in reservoir analysis, and when is it a realistic assumption?
d) What does the term "piston displacement" mean in reservoir analysis, and when is it a realistic assumption?
e) What is the Dykstra-Parsons method used for, and which assumptions are made for the method?

Solution

a)



b)

"Diffuse flow" means that the viscous (dynamic) forces are dominating the flow over the gravity forces, so that vertical variation in saturations may be neglected. Low permeability vertically, combined with high pressure gradient horizontally, characterise this type of flow. "Segregated flow" means that the fluids are segregated vertically. High vertical permeability, and/or low flow rates would be typical for this condition.

c)

-"Vertical equilibrium" means that the fluids instantly are segregated in the vertical direction
-It´s realistic when the gravity forces ar dominating the flow over the viscous (dynamic) forces

d)

-"Piston displacement" means 100% efficiency of displacement (no movable oil behind fluid front
-It´s realistic when the mobility ratio of the fluids is very low (favourable), ie. when the fractional flow curve has no inflection point

e)

-It´s used for computing displacement behavior in a layered system
-Assumptions: **no communication between layers**
piston displacement
no capillary pressure
constant ΔP over all layers

Question 10 (8 points)

Applying Dietz' stability analysis to displacement of oil by water or by gas in an inclined layer (angle α), we may derive the following formula for the angle (β) between the fluid interface and the layer:

$$\tan(\beta) = \tan(\alpha) + \frac{1 - M_e}{M_e N_{ge} \cos(\alpha)}$$

where the gravity number is defined as

$$N_{ge} = \frac{(k'_{ro}/\mu_o)kA\Delta\gamma}{q_{inj}}$$

and M_e is the end-point mobility ratio, both computed using endpoint relative permeabilities.

a) What are the assumptions behind the Dietz' method?
b) What is the criterion for the stability of the fluid front?
c) When is the front completely stable (in the equation above)?
d) When is the front conditionally stable?

Solution

a)

segregated flow
piston displacement
no capillary pressure

b)

$\beta > 0$

c)

$M_e \leq 1$

d)

$M_e > 1$

Question 11 (6 points)

The vertical capillary continuity (contact) between matrix blocks in a fractured reservoir may in some cases affect the recovery of oil significantly. Explain shortly in which situations this is the case (consider both gas displacement and water displacement of oil)

Solution

- Water displacement
    a. *For a 100% water-wet reservoir, there is no influence of vertical contact on oil recovery, since all movable oil is recovered by spontaneous imbibition*
    b. *For mixed-wet reservoir, spontaneous imbibition recovers oil only until* $P_{cow} = 0$ *ie. until* $S_o = S_{or}^s$*. Thereafter, oil is recovered by forced imbibition by gravity for* $P_{cow} < 0$*. The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller blocks*
- Gas displacement
  Since the process is a drainage process, for gas to enter the matrix blocks, and thus replace oil, the difference in phase pressures must exceed the displacement capillary pressure, ie. $P_g - P_o > P_{cogd}$*. Thus, oil is recovered by means of gravity forces. The taller the block, the higher recovery. Capillary continuity between blocks will have the same effect as taller block.s*

Attachment - Definition of symbols

B_g	Formation volume factor for gas (res.vol./st.vol.)
B_o	Formation volume factor for oil (res.vol./st.vol.)
B_w	Formation volume factor for water (res.vol./st.vol.)
C_r	Pore compressibility (pressure^{-1})
C_w	Water compressibility (pressure^{-1})
ΔP	$P_2 - P_1$
G_i	Cumulative gas injected (st.vol.)
GOR	Producing gas-oil ratio (st.vol./st.vol.)
G_p	Cumulative gas produced (st.vol.)
k	Absolute permeability
k_{ro}	Relative permeability to oil
k_{rw}	Relative permeability to oil
k_{rg}	Relative permeability to oil
m	Initial gas cap size (res.vol. of gas cap)/(res.vol. of oil zone)
M_e	End point mobility ratio
N	Original oil in place (st.vol.)
N_{ge}	Gravity number
N_p	Cumulative oil produced (st.vol.)
P	Pressure
P_{cow}	Capillary pressure between oil and water
P_{cog}	Capillary pressure between oil and gas
q_{inj}	Injection rate (res.vol./time)
R_p	Cumulative producing gas-oil ratio (st.vol./st.vol) = G_p / N_p
R_{so}	Solution gas-oil ratio (st.vol. gas/st.vol. oil)
S_g	Gas saturation
S_o	Oil saturation
S_w	Water saturation
T	Temperature
V_b	Bulk volume (res.vol.)
V_p	Pore volume (res.vol.)
WC	Producing water cut (st.vol./st.vol.)
W_e	Cumulative aquifer influx (st.vol.)
W_i	Cumulative water injected (st.vol.)
W_p	Cumulative water produced (st.vol.)
ρ	Density (mass/vol.)
ϕ	Porosity
μ_g	Gas viscosity
μ_o	Oil viscosity
μ_w	Water viscosity
γ	Hydrostatic pressure gradient (pressure/distance)