Exercise 4 - Recovery calculations

Part 1

Starting with Darcy´s equations for updip displacement of oil by water in a system of dip angle α :

$$q_o = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial P_o}{\partial x} + \rho_o g\, sin\alpha\right)$$

$$q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial (P_o - P_c)}{\partial x} + \rho_w g\, sin\alpha\right)$$

show all steps in the derivation of the following expression for fraction of water flowing, including capillary pressure and gravity:

$$f_w = \frac{q_w}{q_w + q_o} = \frac{1 + \frac{kk_{ro}A}{q\mu_o}\left(\frac{\partial P_c}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \frac{k_{ro}}{\mu_o}\frac{\mu_w}{k_{rw}}}$$

where $q = q_w + q_o$.

(Hint: start by elimination of oil pressure by combination of the two equations)

Part 2

Oil is being displaced by water in a horizontal, linear system under diffuse flow conditions. The relative permeabilities are given by

$$k_{ro} = k'_{ro}\left((S_o - S_{or})/(1 - S_{or} - S_{wir})\right)^2$$

$$k_{rw} = k'_{rw}\left((S_w - S_{wir})/(1 - S_{or} - S_{wir})\right)^2$$

where endpoint relative permeabilities are

$$k'_{ro} = 0.75, \quad k'_{rw} = 0.25$$

and endpoint saturations are

$$S_{or} = 0.2, \quad S_{wir} = 0.2$$

The pressure is maintained during the displacement process, and the formation volume factors and the viscosities are

$$B_o = 1.5, \quad \mu_o = 1\,cP, \quad B_w = 1.1, \quad \mu_w = 0.5\,cP$$

Use Excel for the following calculations and plots:

a) Neglect capillary pressure and make plots of f_w for the case above and for a case where the oil viscosity is 10 times higher. Find the oil recoveries at the time of water break-through for the two cases, and also water cuts at both reservoir and surface conditions.

b) What are the positions (in fractions of the reservoir length) of water saturations of 0.7, 0.6 and 0.5 at the time of water break-through?
Hint: Use the Buckley-Leverett equation:

$$x_{S_w} = \frac{q_{inj}t}{A\phi}\left(\frac{df_w}{dS_w}\right)_{S_w}$$

c) Make a f_w plot for the base case but with vertical displacement (from bottom) and determine the oil recovery at water break-through. Compare with the horizontal case. Use the following additional data:

$$\rho_o = 0.6, \quad \rho_w = 1 \quad (g/cm^3)$$

$$g = 9.81 \quad (m/s^2)$$

Assume an injection velocity of $u = \frac{q}{A} = 0{,}0005\ cm/s$ and k=1 D

NB: Make sure units are consistent!

 Professor Jon Kleppe