Exercise 3 - Derivation and solution of single phase flow equations

The simplest form of the flow equation was derived in the classroom (see handout) as

$$\frac{\partial^2 P}{\partial x^2} = (\frac{\phi \mu c}{k}) \frac{\partial P}{\partial t}.$$

Part 1

List all the assumptions made in the derivation of this equation.

Part 2

Make similar derivations for one-dimensional, radial flow, and for one-dimensional spherical flow.

Part 3

The transient linear equation was solved using IC and BC´s of $P(x, t = 0) = P_R$, $P(x = 0, t) = P_L$ and $P(x = L, t) = P_R$, to yield the following series solution:

$$P(x,t) = P_L + (P_R - P_L)\left[\frac{x}{L} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} exp(-\frac{n^2\pi^2}{L^2}\frac{k}{\phi\mu c}t) sin(\frac{n\pi x}{L})\right]$$

The steady state form of the simple, linear equation is

$$\frac{d^2P}{dx^2} = 0$$

which was solved by integration and using the two boundary conditions to determine the integration constants, to yield the steady state solution:

$$P(x,t) = P_L + (P_R - P_L)\frac{x}{L}.$$

Solve the steady state equations for the following conditions:

1. Radial flow
   a) $P(r = r_e) = P_e$ and $q(r = r_w) = q_w$
   b) $P(r = r_e) = P_e$ and $P(r = r_w) = P_w$
2. Spherical flow (neglect gravity) :
   a) $P(r \to \infty) = P_i$ and $P(r = r_w) = P_w$
   b) $P(r = r_e) = P_e$ and $q(r = r_w) = q_w$

Part 4

Solve the transient equations (if possible) for the following conditions:

1. Radial flow:
   - $P(r, t = 0) = P_i$, $P(r \to \infty, t) = P_i$ and $q(r = r_w) = q_w$
2. Spherical flow (neglect gravity):
   - $P(r, t = 0) = P_i$, $P(r \to \infty, t) = P_i$ and $q(r = r_w) = q_w$