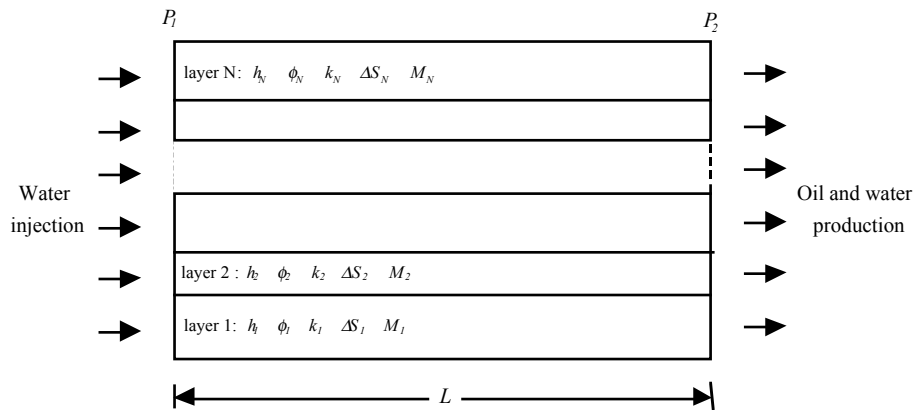


Dykstra-Parson's method for simplified analysis of oil displacement by water in a layered reservoir

The Dykstra-Parson's method applies to a non-communicating layered reservoir, which may be represented schematically as follows:

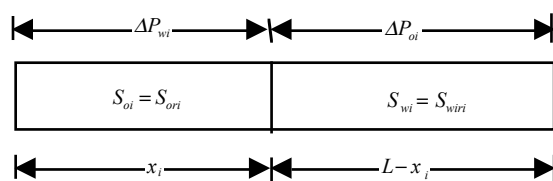


Here, each layer has different height (h), porosity (ϕ), permeability (k), end point saturations (ΔS), and end point mobility ratio (M). The specific assumptions of the method are:

- Pressure drop across layers is constant and equal ($\Delta P = P_2 - P_1 = \text{constant}$)
- Complete displacement efficiency (piston displacement)
- No communication between layers ($k_v = 0$)

Derivation of equations

Consider layer i , where the water-oil front position is at a distance of x_i from the injection side:



We may now write Darcy's equations for this layer as:

oil equation (ahead of the front)

$$u_{oi} = -k_i \lambda'_{oi} \frac{\Delta P_{oi}}{L - x_i} \quad (1)$$

water equation (behind the front)

$$u_{wi} = -k_i \lambda'_{wi} \frac{\Delta P_{wi}}{x_i}, \quad (2)$$

where the end-point mobilities are defined as

$$\lambda'_{oi} = \left(\frac{k'_{ro}}{\mu_o} \right)_i \quad (3)$$

and

$$\lambda'_{wi} = \left(\frac{k'_{rw}}{\mu_w} \right)_i. \quad (4)$$

For an incompressible system, the two velocities are equal, i.e. $u_i = u_{oi} = u_{wi}$. These Darcy-velocities are average velocities over the total flow area. Actual front velocity (derived used mass balance at the front) may be expressed as:

$$\frac{dx_i}{dt} = \frac{u_i}{\Delta S_i \phi_i}. \quad (5)$$

In addition, the sum of the pressure drops ahead and behind the front is equal to the total imposed pressure drop across the layer:

$$\Delta P = \Delta P_w + \Delta P_o,$$

By combination of the Equations (1), (2) and (5), we get the expression for the frontal velocity in a layer:

$$\frac{dx_i}{dt} = - \frac{k_i}{\phi_i \Delta S_i} \frac{\Delta P}{\frac{x_i}{\lambda'_{wi}} + \frac{L - x_i}{\lambda'_{oi}}} \quad (6)$$

As will be shown below, we are interested in finding expressions for relative front positions, and therefore we will use index R to denote a reference layer. Then, taking the ratio of frontal velocities in layers i and R , we obtain the following relationship:

$$\frac{d\dot{x}_i}{d\dot{x}_R} = F_i \frac{\dot{x}_R + M_R(1 - \dot{x}_R)}{\dot{x}_i + M_i(1 - \dot{x}_i)}. \quad (7)$$

Here, the end point mobility ratio is defined as

$$M_i = \left(\frac{k_{rw}}{\mu_w} \frac{\mu_o}{k_{ro}} \right)_i, \quad (8)$$

the heterogeneity factor as

$$F_i = \frac{k_i \phi_R \Delta S_R \lambda'_{wi}}{k_R \phi_i \Delta S_i \lambda'_{wR}}, \quad (9)$$

and the dimensionless distance as

$$\dot{x}_i = \frac{x_i}{L}. \quad (10)$$

By simple integration of Equation (7):

$$\int_0^{\dot{x}_i} [\dot{x}_i + M_i(1 - \dot{x}_i)] d\dot{x}_i = F_i \int_0^{\dot{x}_R} [\dot{x}_R + M_R(1 - \dot{x}_R)] d\dot{x}_R, \quad (11)$$

and solving the resulting quadratic equation for front position in layer i , we get:

$$\dot{x}_i = \frac{M_i - \sqrt{M_i^2 + F_i(1 - M_i)[\dot{x}_R^2(1 - M_R) + 2M_R\dot{x}_R]}}{M_i - 1} \quad (M_i \neq 1). \quad (12)$$

For the special case of $M_i = 1$, the integration yields the following expression:

$$\dot{x}_i = \frac{1}{2} F_i [\dot{x}_R^2(1 - M_R) + 2M_R\dot{x}_R] \quad (M_i = 1) \quad (13)$$

Finally, for the case of $M_i = M_R = 1$, the integration yields the following simple expression:

$$\dot{x}_i = \dot{x}_R F_i \quad (M_i = M_R = 1) \quad (14)$$

Equation (14) represents the *Stiles Method* (see Dake page 410), which is similar to the Dykstra-Parson's Method, except that it assumes that the end point mobility ratio is 1.

Now, based on Equation (12), we would like to find the position of the front in layer i at the time when break-through occurs in layer R . Thus, for $\dot{x}_R = 1$ (break-through in layer R) the expression reduces to (similar to Equation 4.59 in the Monograph, except that here $M_i \neq M_R$):

$$\dot{x}_i = \frac{M_i - \sqrt{M_i^2 + F_i(1 - M_i)(1 + M_R)}}{M_i - 1} \quad (\dot{x}_R = 1), \quad (15)$$

or, in case $M_i = 1$

$$\dot{x}_i = \frac{1}{2} F_i (1 + M_R) \quad (M_i = 1, \dot{x}_R = 1) \quad (16)$$

Equations (12) and (13) cover all cases of $\dot{x}_i \leq 1$. They may be used to determine relative front position of any layer. For a reservoir with N layers, we start by selecting the layer where water break-through will occur first as the reference layer. Then, as long as $\dot{x}_R \leq 1$ we may compute the relative front positions in all other layers. When $\dot{x}_R = 1$, we select the layer where the break-through will occur next as the new reference layer, and compute relative positions in the rest of the layers until \dot{x}_R again has reached 1. This procedure is repeated until until all layers have been flooded completely. From the Equation (12), we see that front

advancement in a layer depends on mobility ratios and the heterogeneity function F_i . By inspection, we see that the same sorting criterion apply for all values of M_i :

$$\text{break-through occurs first in a layer of higher value of } \frac{k_i \lambda'_{wi}}{\phi_i \Delta S_i} \frac{1}{(1 + M_i)} \quad (17)$$

Having computed the front positions, the corresponding layer flow rates are (obtained by combining the two Darcy-equations):

$$q_i = -\frac{k_i \lambda'_{wi} A_i}{\dot{x}_i + M_i(1 - \dot{x}_i)} \frac{\Delta P}{L}. \quad (18)$$

Thus, for layers with water break-through ($\dot{x}_i = 1$), the total water rate is:

$$Q_w = -\frac{\Delta P}{L} \sum_1^N k_i \lambda'_{wi} A_i \quad (\text{for layers where } \dot{x}_i = 1) \quad (19)$$

For layers that still are producing oil ($\dot{x}_i < 1$), the total oil rate is:

$$Q_o = -\frac{\Delta P}{L} \sum_1^N \frac{k_i \lambda'_{wi} A_i}{\dot{x}_i + M_i(1 - \dot{x}_i)} \quad (\text{for layers where } \dot{x}_i < 1) \quad (20)$$

The water cut may now be computed as function of front position in reference layer R :

$$WC = \frac{Q_w}{Q_w + Q_o}. \quad (21)$$

If we prefer to compute water-cut as function of pore volumes injected, we may use the following expressions for amounts of water injected:

$$WI = \sum_1^N A_i \dot{x}_i L \phi_i \Delta S_i, \quad (22)$$

or, in terms of number of pore volumes:

$$WI^* = \frac{\sum_1^N A_i \dot{x}_i L \phi_i \Delta S_i}{\sum_1^N A_i L \phi_i}. \quad (23)$$

However, these formulas apply only to layers where $\dot{x}_i \leq 1$. After break-through in a layer, we may, of course, compute the water rate using Darcy's equation:

$$q_{wi} = -k_i \lambda'_{wi} A_i \frac{\Delta P}{L}, \quad (24)$$

and compute the total amount of injected water in the layer at a given time after break-through, Δt_i , as:

$$WI_i = A_i \dot{x}_i L \phi_i \Delta S_i - k_i \lambda'_{wi} A_i \frac{\Delta P}{L} \Delta t_i \quad (25)$$

A disadvantage is that this expression includes pressure drop and a time term. We will therefore use a different procedure for layers that have had water break-through. Let us repeat the integration of Equation (11), but now let $\dot{x}_R \leq 1$ and $\dot{x}_i \geq 1$ (\dot{x}_i is now an imaginary front position in a layer where water is displacing water):

$$\int_0^1 [\bar{x}_i + M_i(1 - \dot{x}_i)] d\dot{x}_i + \int_1^{\dot{x}_i} d\dot{x}_i = F_i \int_0^{\dot{x}_R} [\dot{x}_R + M_i(1 - \dot{x}_R)] d\dot{x}_R \quad (26)$$

We then solve the resulting equation for the imaginary front position in layer i :

$$\dot{x}_i = \frac{1}{2}(1 - M_i) + \frac{1}{2} F_i [\dot{x}_R^2(1 - M_R) + 2M_R \dot{x}_R] \quad (27)$$

The imaginary front positions for all layers that have had water break-through may then be used in the formulas above for computing the amounts of water injected.

Application of formulas to a layered reservoir

First we sort the layers using Equation (17), with new layer 1 at the bottom. Next, the sorted layer 1 is assigned as reference layer, and front positions are computed for the remaining layers using Equations (12)-(13), for pre-selected intervals along layer 1. If all we need to compute is the front positions when break-through occurs in layer one, we only need to use Equations (12)-(13). After break-through in layer 1, layer 2 becomes reference layer, and front positions are computed for remaining layers for selected positions along layer 2. This procedure is repeated until all layers have water break-through. For the last layer, all we need is to assign computational points along the remainder of the layer.

The above procedure is sufficient if we assume that a layer is immediately shut in after water break-through has occurred. In case the layers are still producing (water), we need to extend the procedure to account for this.

In this case, we may use Equation (27) to compute imaginary front positions for all layers that have had water break-through. We use the last layer to have break-through (at the point of time of the computation) as reference layer, and compute imaginary positions for all other layers, for $\dot{x}_i > 1$.

Finally, we compute flow rates and water cuts using Equations (19)-(21), and corresponding water injection volumes using Equation (22) or (23).

See Exercise 6 for application.